Homework 1

Due Friday, July 20

1. Exercise 4, on pp. 67-68 of Kleinberg and Tardos
2. Exercise 5, on p. 68 of Kleinberg and Tardos
3. Prove using induction that $\sum_{i=1}^{n} i = \frac{n^2+n}{2}$
4. G is an undirected graph on n nodes. Prove that any 2 of the following statements imply the third.
   (a) G is connected
   (b) G does not contain a cycle
   (c) G has $n-1$ edges
5. The following undirected graph is given in adjacency matrix format

$$G \quad = \quad \begin{array}{cc} & \begin{array}{ccccccccc} a & b & c & d & e & f & g & h & i \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

   (a) Draw the graph (use labeled circles for nodes and lines for edges)
   (b) Give the adjacency list representation of this graph
   (c) Draw a BFS tree produced when BFS is run starting at node a
   (d) Draw a DFS tree produced when DFS is run starting at node a