Homework 4

Due Friday, August 10

1. Prove that for any $a > 1, b > 1$ we have $a^{\log_2 b} = b^{\log_2 a}$
2. Consider the following problem. You are given n objects. The ONLY operation you can perform on them is, given two of them, tell if they are THE SAME or DIFFERENT. You want to determine if there exists $n/2$ objects that are the same. You COULD do this with $O(n^2)$ calls to the operation (ask for each pair SAME or DIFF). Devise an algorithm that takes $O(n \log n)$ calls to that OPERATION. The algorithm should be clear and you need to do an analysis to prove its $O(n \log n)$.
3. Exercise 2 on p.246 of Kleinberg and Tardos
4. Assume that scientists found a way to multiply 4×4 matrices using 40 multiplications and 80 additions. Design an algorithms to multiply $n \times n$ matrices. (You may assume that $n = 4^l$ is a power of 4). Analyze the asymptotic running time of your algorithm. How will your algorithm compare to Strassen's algorithm for large values of n?
5. Let Z_7 be the integers mod 7. Answer the following questions.
   (a) List the 6^{th} roots of unity in Z_7
   (b) List the 3^{rd} roots of unity in Z_7
   (c) Use the FFT to evaluate $A(x) = 3x^5 + 2x^4 + 6x^2 + 4x + 3$ in Z_7 at all the 6^{th} roots of unity. (Note, you only need to show what happens at the first level of the recursion).