Homework 6

Due Thursday, August 23

1. The 15-Coloring problem is defined similarly to the 3-Coloring problem presented in class. Given, graph G, is there a way to color the nodes using 15 colors s.t no 2 nodes that share an edge have the same color.
   (a) Prove that 3-Coloring $\leq_p$ 15-Coloring
   (b) Prove that 15-Coloring $\leq_p$ 3-Coloring (Note: You may use the result from part a for this)
2. This problem will show the difference between Karp reductions and Turing reductions. Notation: $\leq_K$ - Karp reduction, $\leq_T$ - Turing reduction (both are poly-time)
   (a) Prove that if $X \in NP$ and $Y \leq_K X$ then $Y \in NP$
   (b) If $NP \neq coNP$, is the following statement true, why or why not?
   if $X \in NP$ and $Y \leq_T X$ then $Y \in NP$
3. Prove that the complements of NP-complete problems are coNP-complete. A problem X is coNP-complete if $X \in coNP$ and for all $Y \in coNP$, $Y \leq_p X$.
4. Exercise 5 on pp. 506-507 of Kleinberg and Tardos
5. Exercise 17 on p. 513 of Kleinberg and Tardos