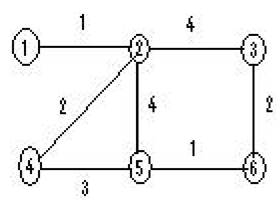
CMSC451: Midterm Exam

Monday, August 6

1. Given the undirected weighted graph G below



   (a) Draw the BFS produced when starting at node 1.
   (b) Draw the meta graph of G
   (c) Draw the MST produced by Prim's algorithm, starting at node 1, and label the edges with the order that they would be added.
   (d) Draw the MST produced by Kruskal's algorithm and label the edges with the order that they would be added.

2. Dijkstras algorithm used the definition of the cost of a path as being the SUM of the edge weights. We define the cost of a path as being the NUMBER of edges on it. Design an algorithm for Single Source Shortest Path with this definition of distance. Prove that your algorithm is correct.

3. Suppose that we want to find an optimal prefix code where the encoding is going to use 3 characters instead of 2. So, instead of just having codes be binary strings, the codes will contain 0,1,2. Below is a table of characters and their frequencies in a message that you want to encode. Show how to construct a trinary(0,1,2) optimal prefix code for these characters.

a	.15
b	.07
c	.05
d	.30
e	.10
f	.13
g	.20

   (a) Draw the prefix Tree corresponding to the optimal prefix code for these characters.
   (b) Give an optimal prefix code equivalent to the tree from part a.

4. Dr. Llib has come up with a way to multiply two n-bits numbers by doing 24 pairs of multiplying pairs of numbers that are $n/5$ bits long (you can ignore floor and ceiling issues) and 1000 additions of such numbers.
   On the other hand, Dr. H.Crasag, has come up with a way to multiply two n-bit numbers using 9 multiplications of numbers that are $n/3$ bits long and no additions.

(a) Suggest how Dr.'s Llib and H.Crasag can use their algorithms to multiply n-bit numbers.

(b) Show the big-O running time of the algorithms these two prof's would produce.

(c) Which one will do better for large n?

5. Let Z_{13} be the integers mod 13. Note that $1, 5, 8$, and 12 are 4th roots of unity mod 13:

   $1^4 = 1 \equiv 1 \pmod{13}$.

   $5^4 = 25 \times 25 \equiv -1 \times -1 \equiv 1 \pmod{13}$.

   $8^4 = 65 \times 65 \equiv -1 \times -1 \equiv 1 \pmod{13}$.

   $12^4 \equiv (-1)^4 \equiv 1 \pmod{13}$.

   Also note that 1 and 12 are the only 2nd roots of unity. (You can use $12 \equiv -1$ for easier calculations.)

   Let $p(x) = 2x^3 + 5x^2 + 2x + 3$.

   (a) Evaluate $p(1) \pmod{13}$ and $p(12) \pmod{13}$. (You can do this anyway you want, no need to use anything fancy.)

   (b) Evaluate $p(1), p(5), p(8), p(12)$ using the values $p(1)$ and $p(12)$ and the FFT algorithm from class.