CMSC 132:
Object-Oriented Programming II

Advanced Tree Structures

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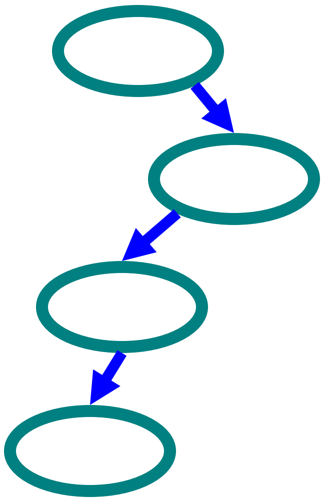
Overview

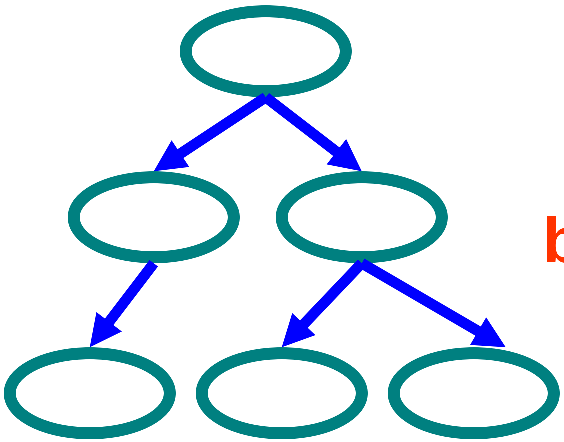
- Binary trees
  - Balance
  - Rotation
- Multi-way trees
  - Search
  - Insert
- Indexed tries

Tree Balance

- **Degenerate**
  - **Worst case**
  - **Search in O(n) time**
- **Balanced**
  - **Average case**
  - **Search in O(log(n)) time**



Degenerate binary tree



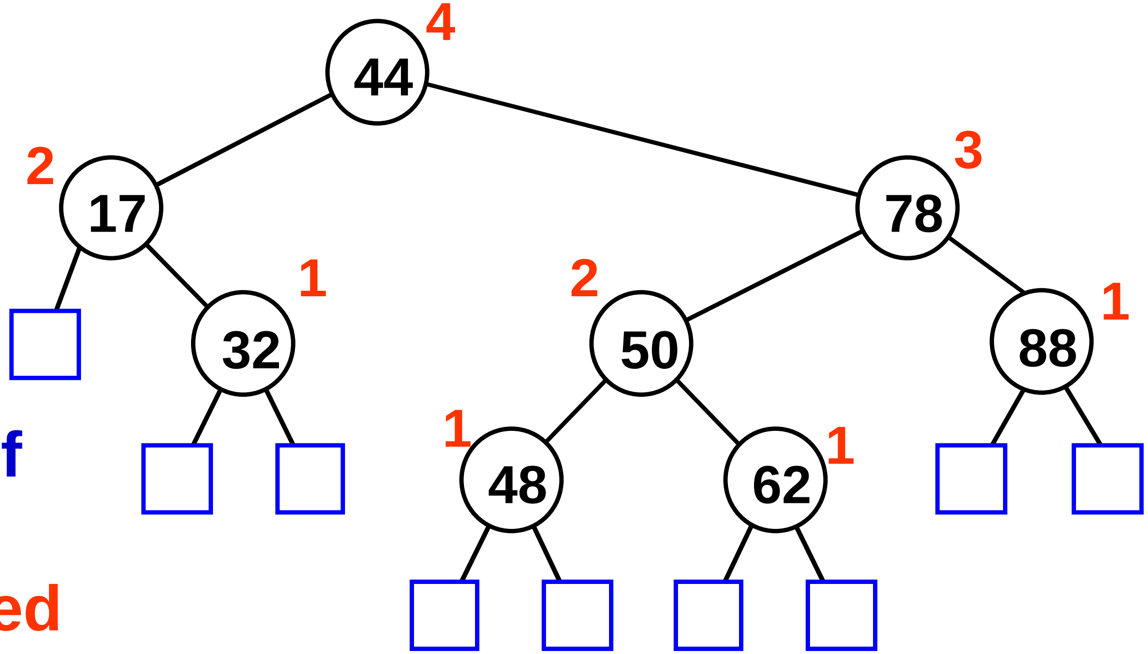
Balanced binary tree

Tree Balance

- Question
  - Can we keep tree (mostly) balanced?
- Self-balancing binary search trees
  - AVL trees
  - Red-black trees
- Approach
  - Select invariant (that keeps tree balanced)
  - Fix tree after each insertion / deletion
    - Maintain invariant using rotations
  - Provides operations with O(log(n)) worst case

AVL Trees

- Properties
  - Binary search tree
  - Heights of children for node differ by at most 1
- Example



Heights of children shown in red

AVL Trees

- History
  - Discovered in 1962 by two Russian mathematicians, Adelson-Velskii & Landis
- Algorithm

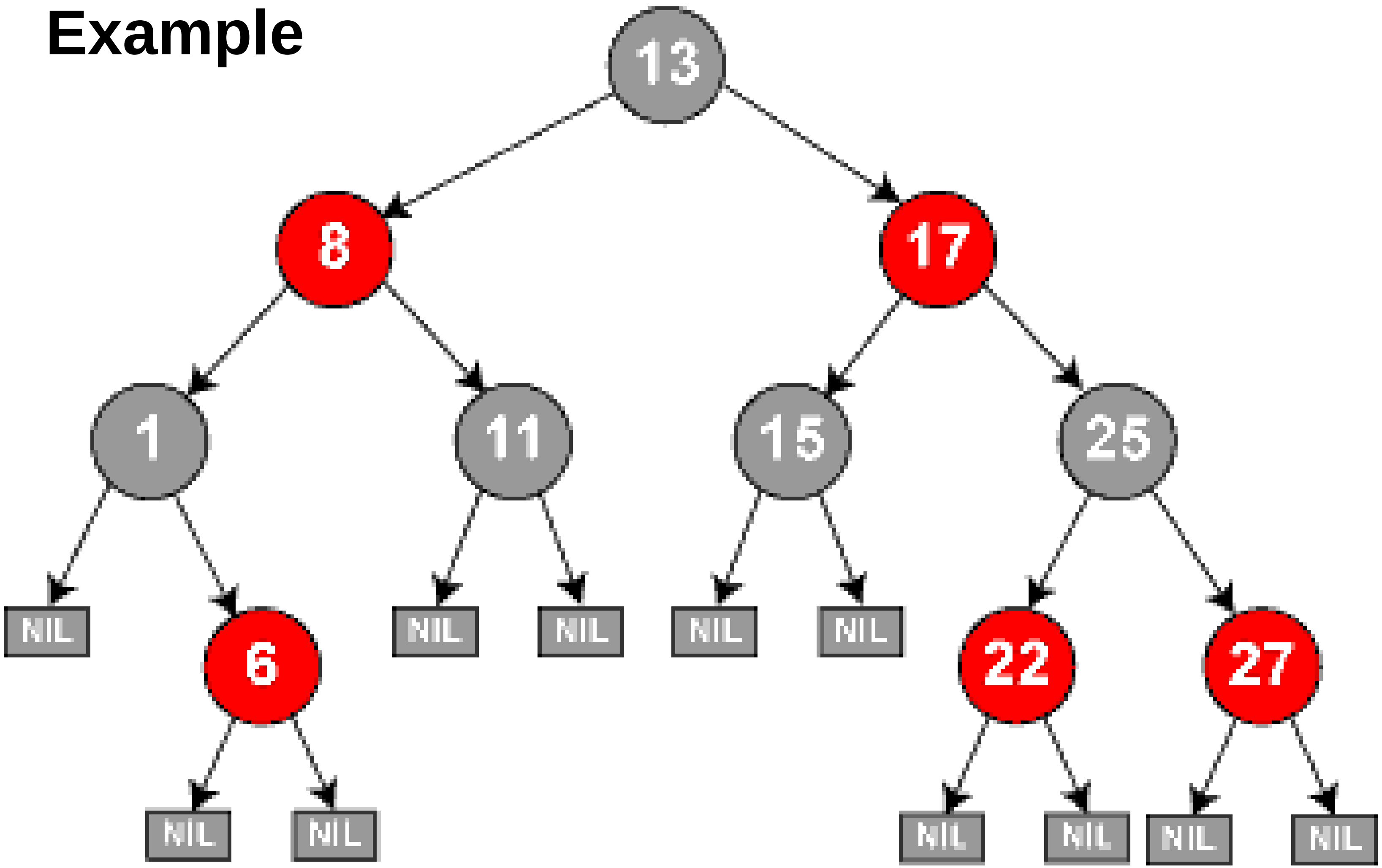
1. Find / insert / delete as a binary search tree
2. After each insertion / deletion
   a) If height of children differ by more than 1
   b) Rotate children until subtrees are balanced
   c) Repeat check for parent (until root reached)

Red-black Trees

- **Properties**
  - **Binary search tree**
  - **Every node is red or black**
  - **The root is black**
  - **Every leaf is black**
  - **All children of red nodes are black**
  - **For each leaf, same # of black nodes on path to root**
- **Characteristics**
  - **Properties ensures no leaf is twice as far from root as another leaf**

Red-black Trees

- **Example**

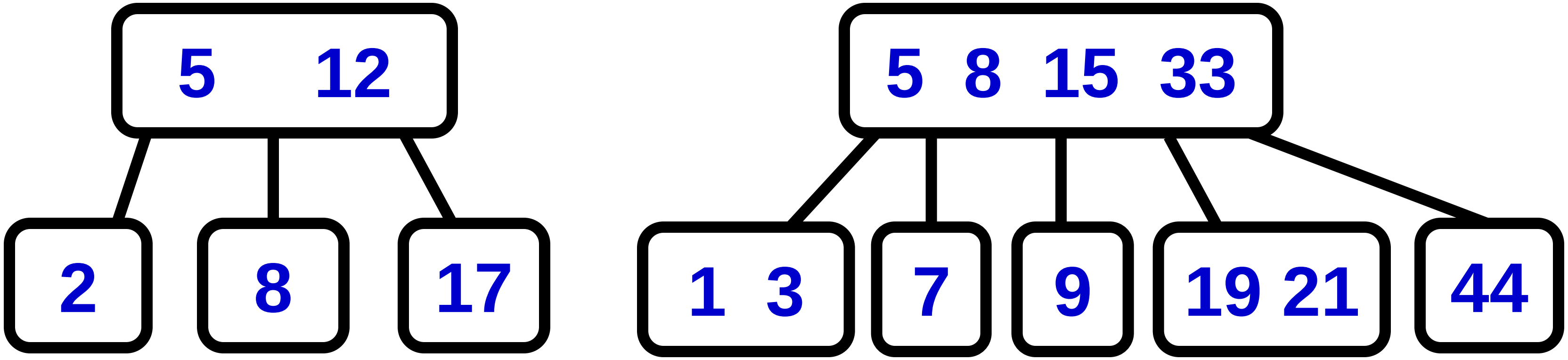


Red-black Trees

- History
  - Discovered in 1972 by Rudolf Bayer
- Algorithm
  - Insert / delete may require complicated bookkeeping & rotations
- Java collections
  - TreeMap, TreeSet use red-black trees

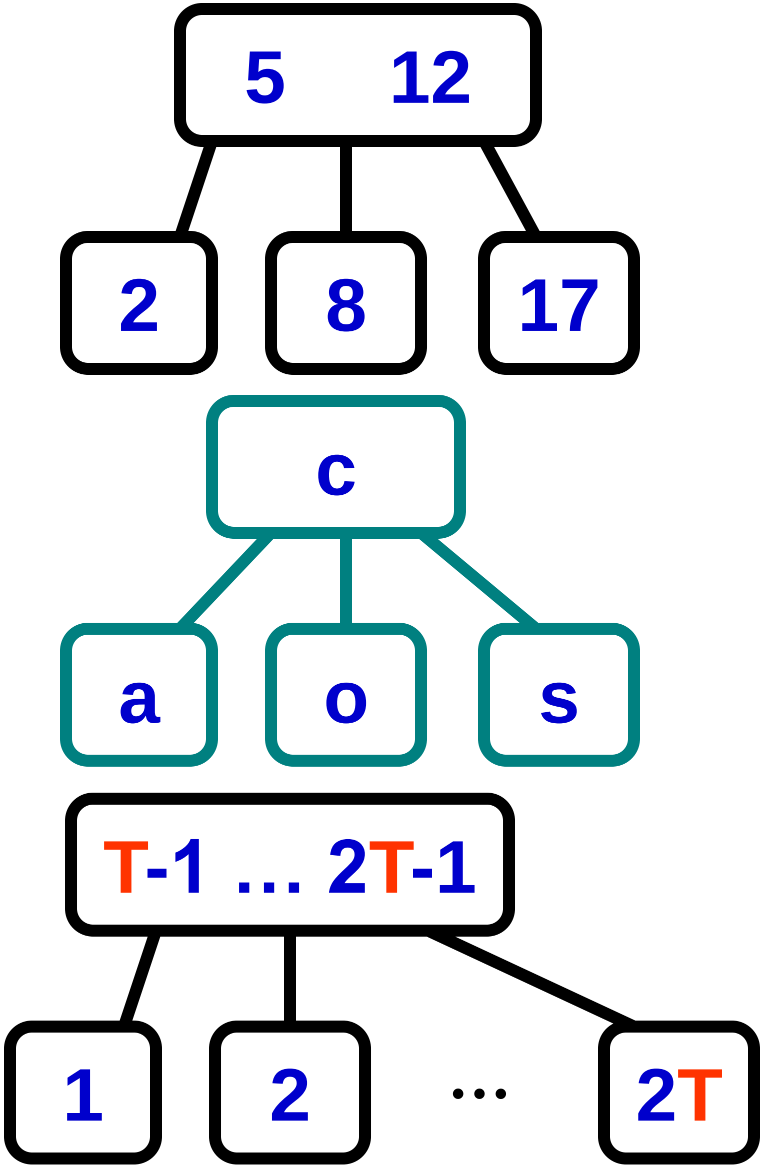
Multi-way Search Trees

- **Properties**
  - Generalization of binary search tree
  - Node contains 1…k keys (in sorted order)
  - Node contains 2…k+1 children
  - Keys in j^{th} child < j^{th} key < keys in $(j+1)^{th}$ child
- **Examples**



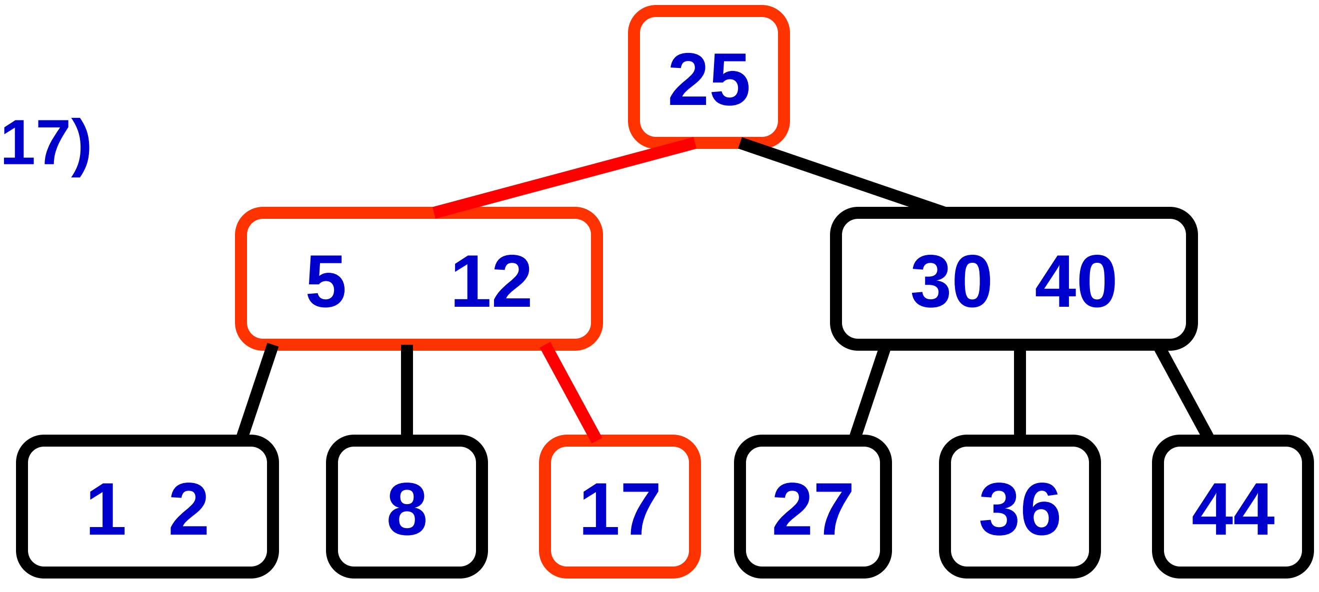
Types of Multi-way Search Trees

- 2-3 tree
  - Internal nodes have 2 or 3 children
- Index search trie
  - Internal nodes have up to 26 children (for strings)
- B-tree
  - T = minimum degree
  - Non-root internal nodes have T-1 to 2T-1 children
  - All leaves have same depth



Multi-way Search Trees

- **Search algorithm**
  1. **Compare key x to 1…k keys in node**
  2. **If x = some key then return node**
  3. **Else if (x < key j) search child j**
  4. **Else if (x > all keys) search child k+1**
- **Example**
  - **Search(17)**

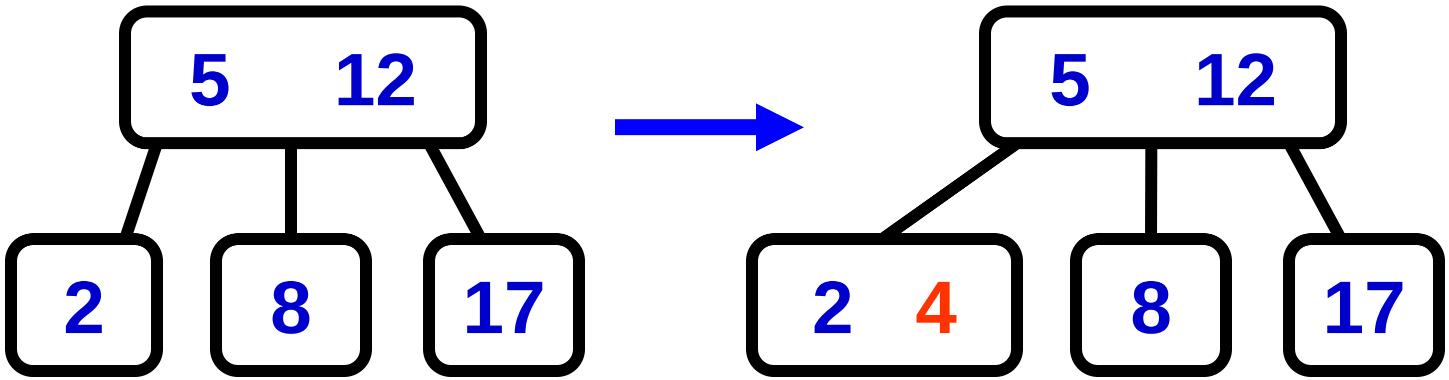


Multi-way Search Trees

- **Insert algorithm**
  1. Search key x to find node n
  2. If (n not full) insert x in n
  3. Else if (n is full)
     a) Split n into two nodes
     b) Move middle key from n to n's parent
     c) Insert x in n
     d) Recursively split n's parent(s) if necessary

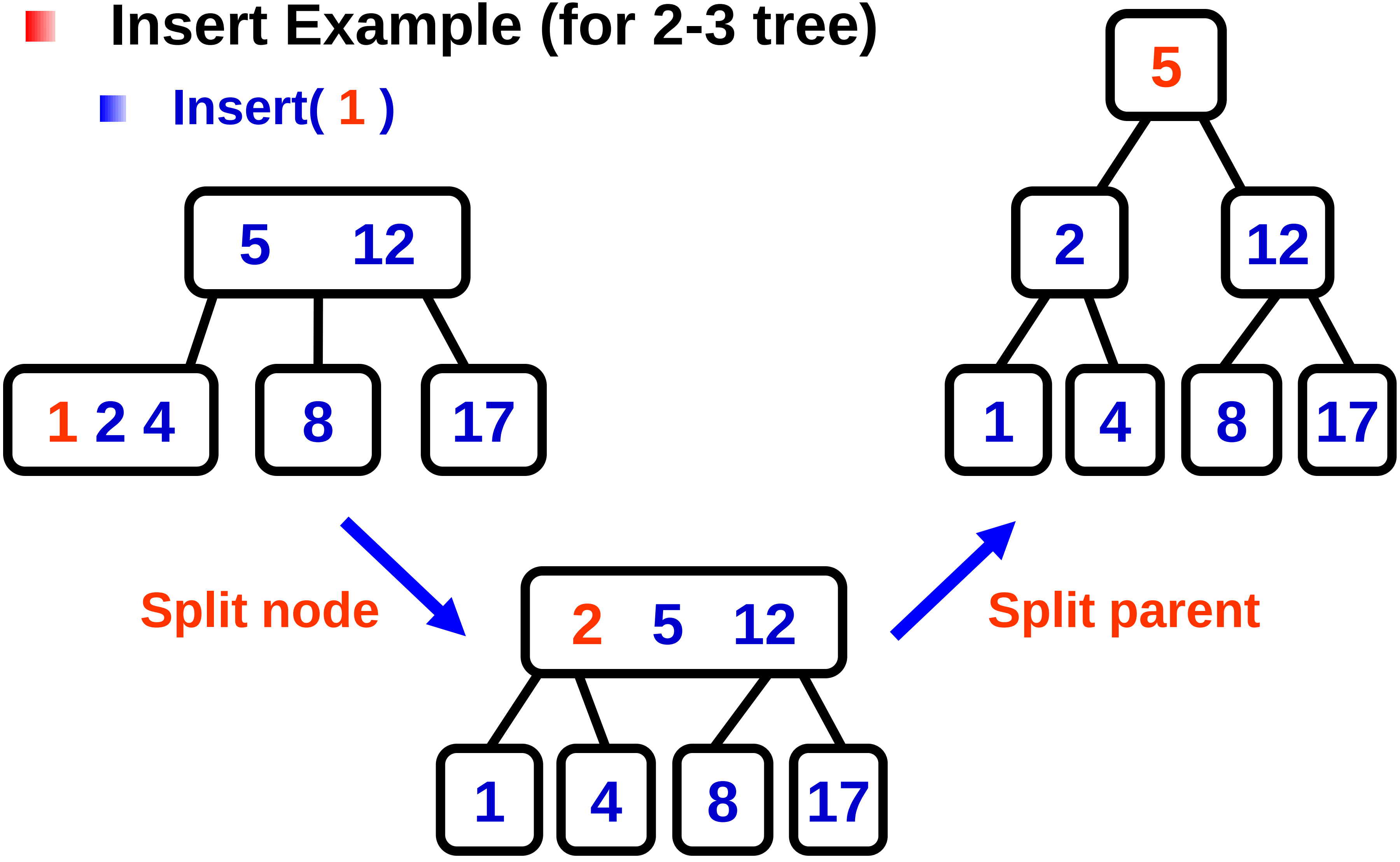
Multi-way Search Trees

- **Insert Example (for 2-3 tree)**
  - Insert(4)



Multi-way Search Trees

- **Insert Example (for 2-3 tree)**
  - Insert(1)

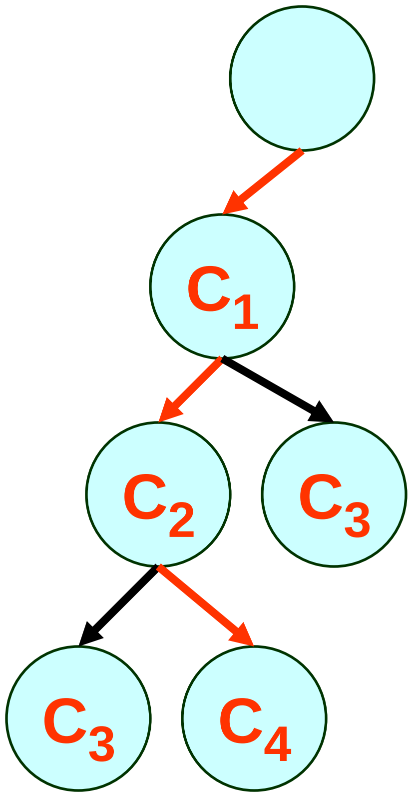


B-Trees

- **Characteristics**
  - **Height of tree is $O(\log_T(n))$**
  - **Reduces number of nodes accessed**
  - **Wasted space for non-full nodes**
- **Popular for large databases**
  - **1 node = 1 disk block**
  - **Reduces number of disk blocks read**

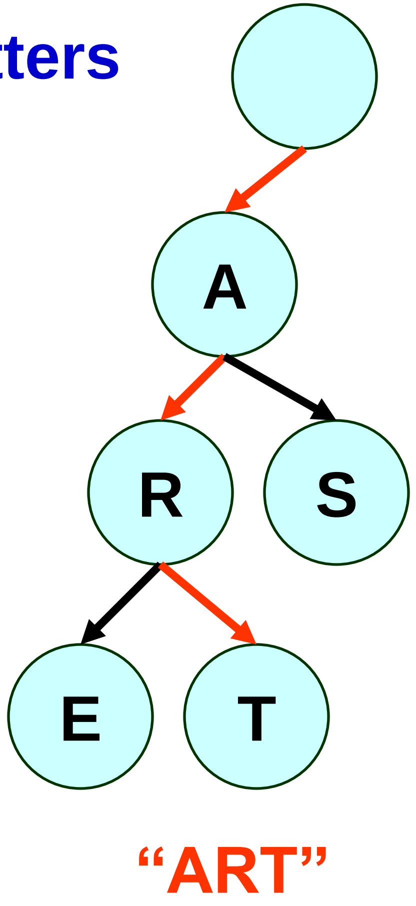
Indexed Search Tree (Trie)

- Special case of tree
- Applicable when
  - Key C can be decomposed into a sequence of subkeys C_1, C_2, … C_n
  - Redundancy exists between subkeys
- Approach
  - Store subkey at each node
  - Path through trie yields full key
- Example
  - Huffman tree



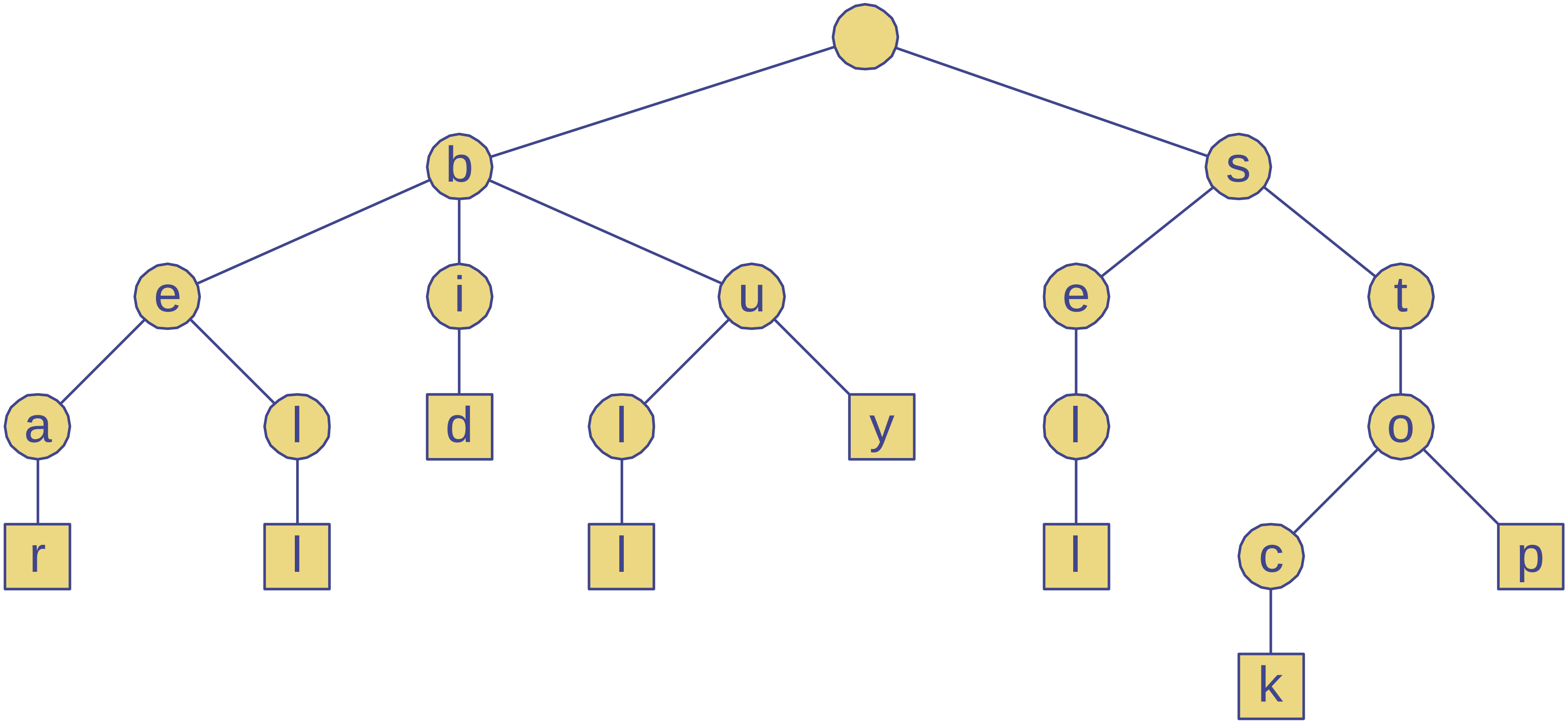
Tries

- Useful for searching strings
  - String decomposes into sequence of letters
  - Example
    - “ART” ⇒ “A” “R” “T”
- Can be very fast
  - Less overhead than hashing
- May reduce memory
  - Exploiting redundancy
- May require more memory
  - Explicitly storing substrings



Standard Trie Example

- **For strings**
  - **{ bear, bell, bid, bull, buy, sell, stock, stop }**



Word Matching Trie

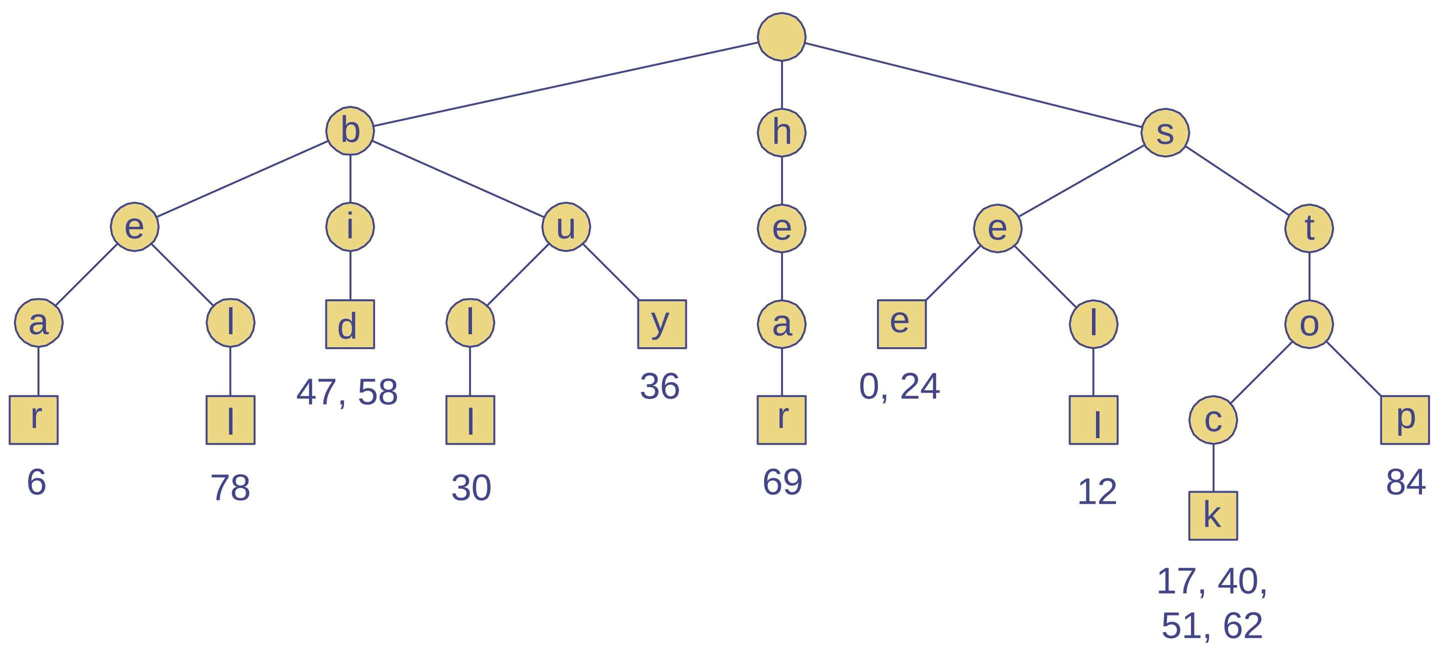
- **Insert words into trie**
- **Each leaf stores occurrences of word in the text**

s	e	e		a		b	e	a	r	?		s	e	l	l		s	t	o	c	k	!	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

s	e	e		a		b	u	l	l	?		b	u	y		s	t	o	c	k	!	
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46

b	i	d		s	t	o	c	k	!		b	i	d		s	t	o	c	k	!	
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68

h	e	a	r		t	h	e		b	e	l	l	?		s	t	o	p	!
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88



Tries and Web Search Engines

- Search engine index
  - Collection of all searchable words
  - Stored in compressed trie
- Each leaf of trie
  - Associated with a word
  - List of pages (URLs) containing that word
    - Called occurrence list
- Trie is kept in memory (fast)
- Occurrence lists kept in external memory
  - Ranked by relevance

Computational Biology

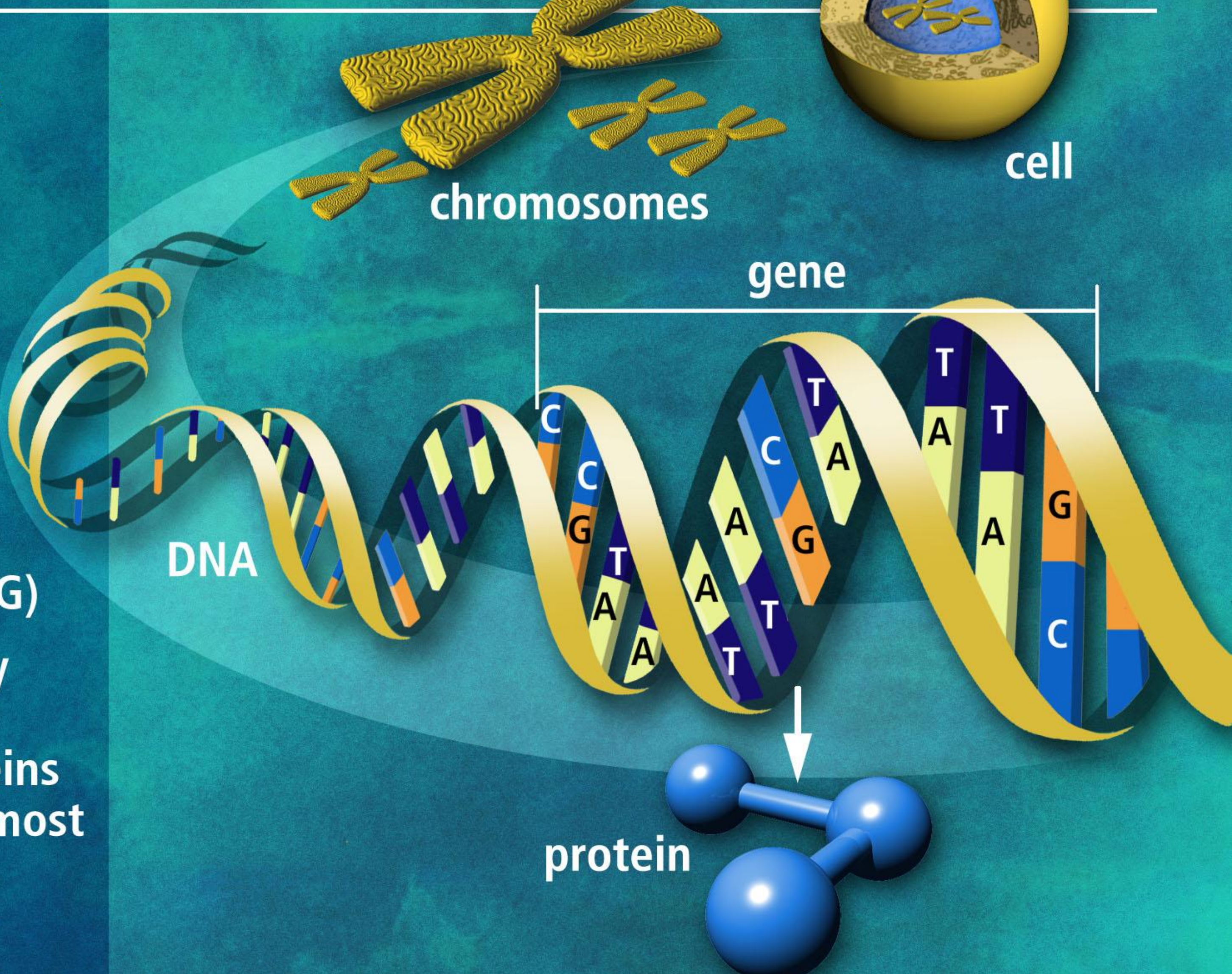
- DNA
  - Sequence of 4 different nucleotides (ATCG)
  - Portions of DNA sequence produce proteins (genes)
- Genome
  - Master DNA sequence for organism
  - For Human
    - 46 chromosomes
    - 3 billion nucleotides

DNA the molecule of life

Trillions of cells

Each cell:

- 46 human chromosomes
- 2 meters of DNA
- 3 billion DNA subunits (the bases: A, T, C, G)
- Approximately 30,000 genes code for proteins that perform most life functions



Tries and Computational Biology

- ESTs
  - Fragments of expressed DNA
  - Indicator for genes (& location)
  - 5.5 million sequences at NIH
- ESTmapper
  - Build suffix trie of genome
    - 8 hours, 60 Gbytes
  - Search for ESTs in suffix trie
    - 11 hours w/ 8 processor Sun
- Search genome w/ BLAST
  - 5^+ years (predicted)