CMSC 132:
Object-Oriented Programming II

Algorithm Strategies

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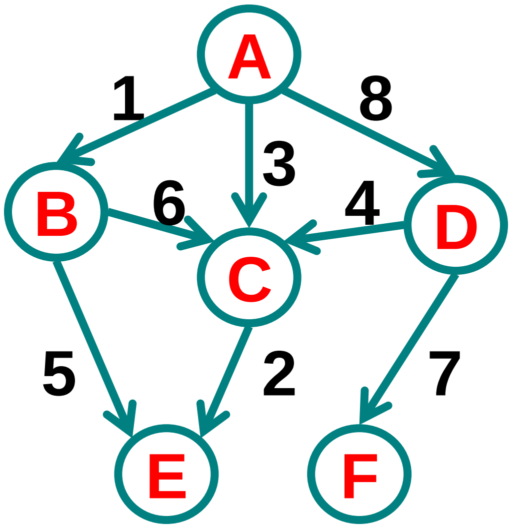
General Concepts

- **Algorithm strategy**
  - **Approach to solving a problem**
  - **May combine several approaches**
- **Algorithm structure**
  - **Iterative** $\Rightarrow$ **execute action in loop**
  - **Recursive** $\Rightarrow$ **reapply action to subproblem(s)**
- **Problem type**

Problem Type

- **Satisfying**
  - **Find any satisfactory solution**
  - **Example → Find path from A to F**



- **Optimization**
  - **Find best solution (vs. cost metric)**
  - **Example → Find shortest path from A to E**

Some Algorithm Strategies

- Recursive algorithms
- Backtracking algorithms
- Divide and conquer algorithms
- Dynamic programming algorithms
- Greedy algorithms
- Brute force algorithms
- Branch and bound algorithms
- Heuristic algorithms

Recursive Algorithm

- Based on reapplying algorithm to subproblem
- Approach
  1. Solves base case(s) directly
  2. Recurs with a simpler subproblem
  3. May need to convert solution(s) to subproblems

Backtracking Algorithm

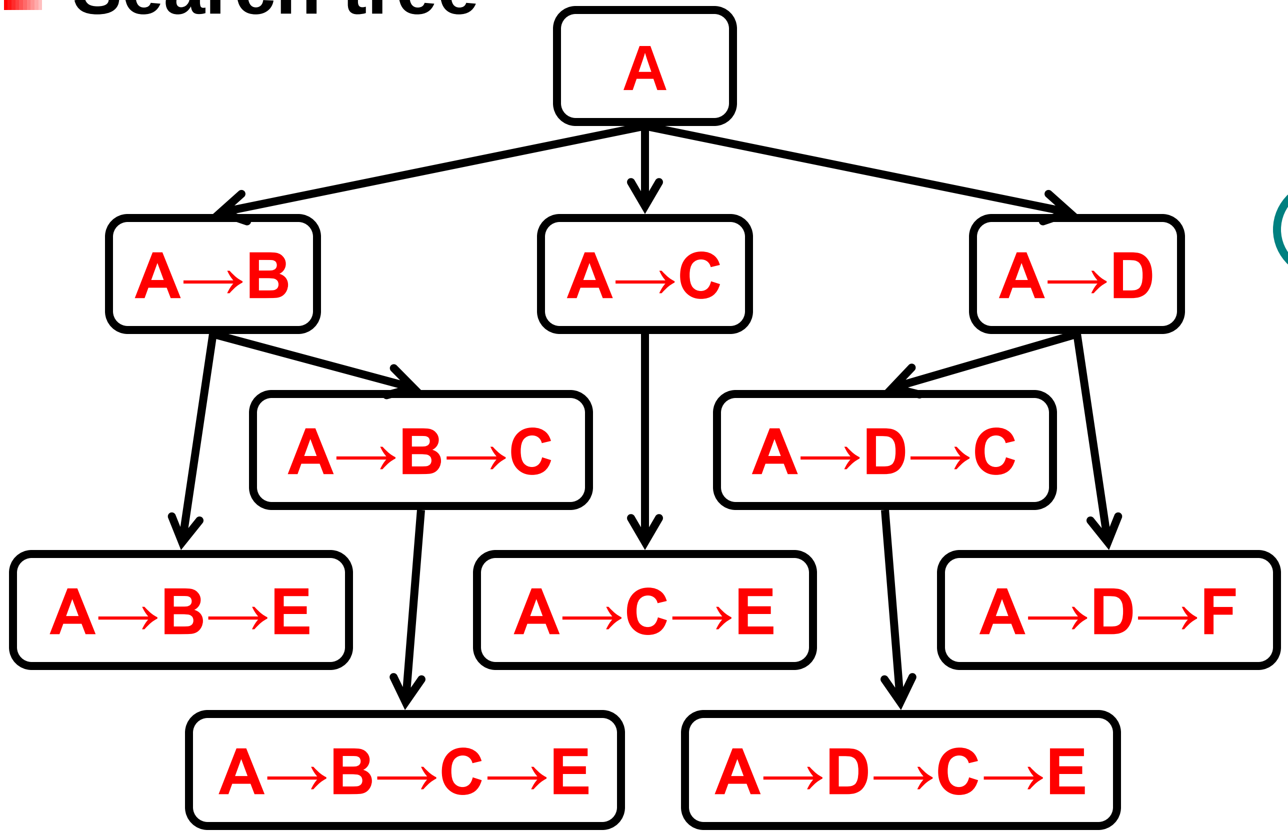
- Based on depth-first recursive search
- Approach
  1. Tests whether solution has been found
  2. If found solution, return it
  3. Else for each choice that can be made
     a) Make that choice
     b) Recur
     c) If recursion returns a solution, return it
  4. If no choices remain, return failure
- Tree of possible solutions → search tree

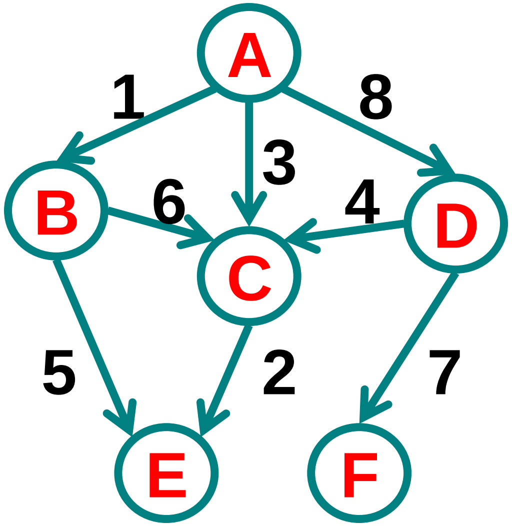
Backtracking Algorithm – Reachability

- **Find path in graph from A to F**
  1. **Start with currentNode = A**
  2. **If currentNode has edge to F, return path**
  3. **Else select neighbor node X for currentNode**
     - **Recursively find path from X to F**
       - **If path found, return path**
       - **Else repeat for different X**
     - **Return false if no path from any neighbor X**

Backtracking Algorithm – Path Finding

- **Search tree**





- **Internal nodes → partial solutions**
- **Leaves → complete solutions**

Backtracking Algorithm – Map Coloring

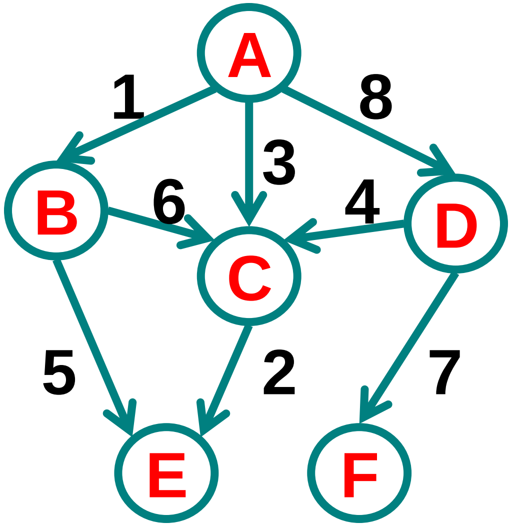
- Color a map with no more than four colors
  - If all countries have been colored return success
  - Else for each color c of four colors and country n
    - If country n is not adjacent to a country that has been colored c
      - Color country n with color c
      - Recursively color country n+1
      - If successful, return success
  - Return failure

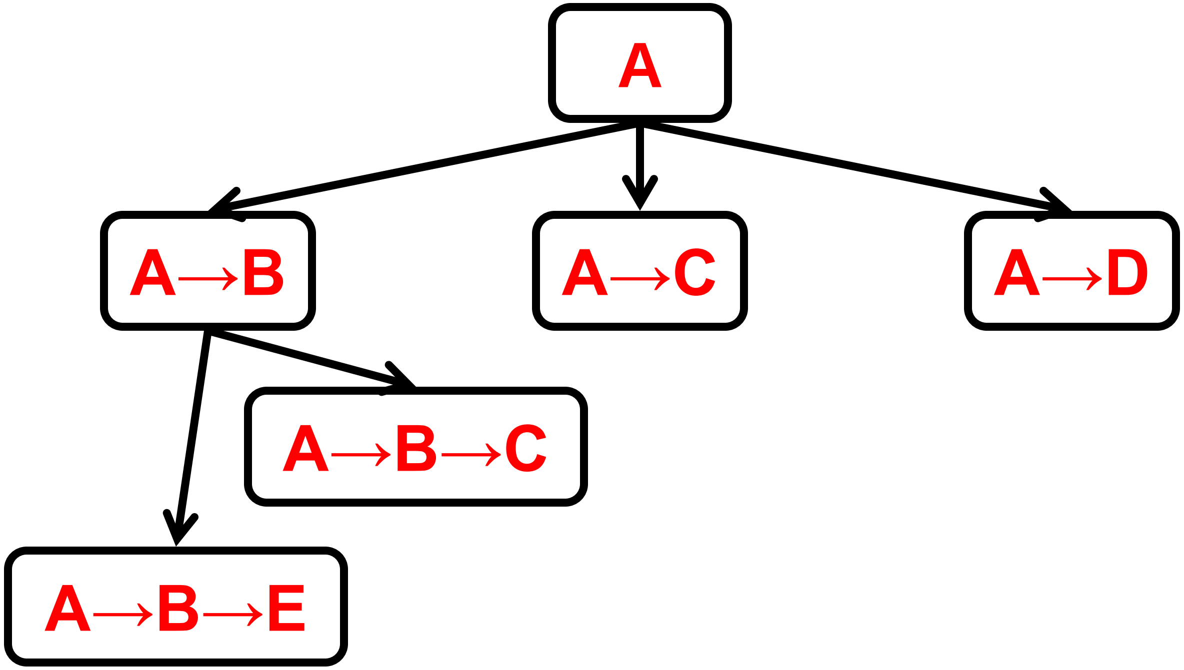
Divide and Conquer

- **Based on dividing problem into subproblems**
- **Approach**
  1. **Divide problem into smaller subproblems**
     - **Subproblems must be of same type**
     - **Subproblems do not need to overlap**
  2. **Solve each subproblem recursively**
  3. **Combine solutions to solve original problem**
- **Usually contains two or more recursive calls**

Divide and Conquer – Shortest Path

- **Example**
  1. Divide into subproblem for each neighboring node
  2. Solve subproblems (recursively)
  3. Combine by comparing costs





Divide and Conquer – Sorting

- Quicksort
  - Partition array into two parts around pivot
  - Recursively quicksort each part of array
  - Concatenate solutions

- Mergesort
  - Partition array into two parts
  - Recursively mergesort each half
  - Merge two sorted arrays into single sorted array

Dynamic Programming Algorithm

- Based on remembering past results
- Approach
  1. Divide problem into smaller subproblems
     - Subproblems must be of same type
     - Subproblems must overlap
  2. Solve each subproblem recursively
     - May simply look up solution (if previously solved)
  3. Combine solutions into to solve original problem
  4. Store solution to problem
- Generally applied to optimization problems

Fibonacci Algorithm

- Fibonacci numbers
  - fibonacci(0) = 1
  - fibonacci(1) = 1
  - fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)
- Recursive algorithm to calculate fibonacci(n)
  - If n is 0 or 1, return 1
  - Else compute fibonacci(n-1) and fibonacci(n-2)
  - Return their sum
- Simple algorithm $\Rightarrow$ exponential time $O(2^n)$

Dynamic Programming – Fibonacci

- Dynamic programming version of fibonacci(n)
  - If n is 0 or 1, return 1
  - Else solve fibonacci(n-1) and fibonacci(n-2)
    - Look up value if previously computed
    - Else recursively compute
  - Find their sum and store
  - Return result
- Dynamic programming algorithm $\Rightarrow$ O(n) time
  - Since solving fibonacci(n-2) is just looking up value

Dynamic Programming – Shortest Path

Djikstra's Shortest Path Algorithm

S = ∅

C[X] = 0

C[Y] = ∞ for all other nodes

while (not all nodes in S)

find node K not in S with smallest C[K]

add K to S

for each node M not in S adjacent to K

$\boxed{\text{C[M]}}$ = min (C[M] , C[K] + cost of (K,M))

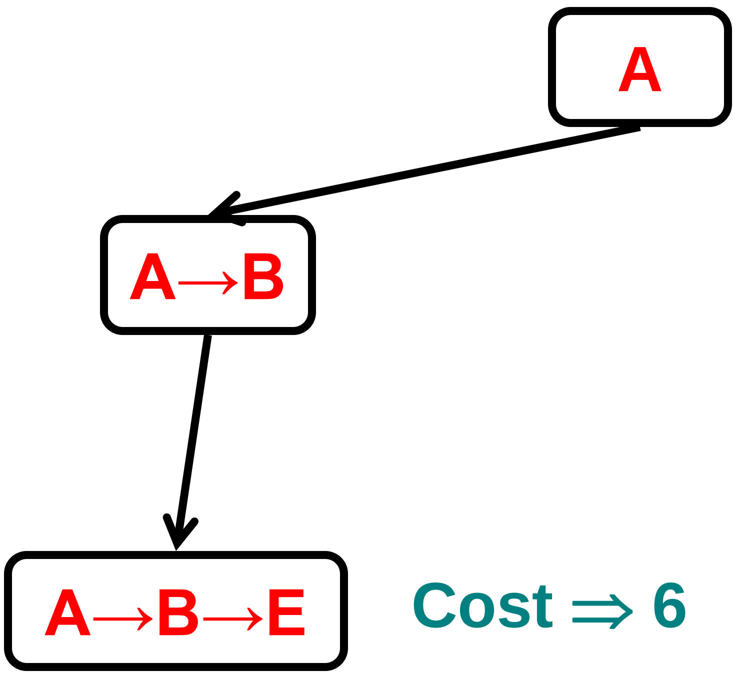
Stores results of smaller subproblems

Greedy Algorithm

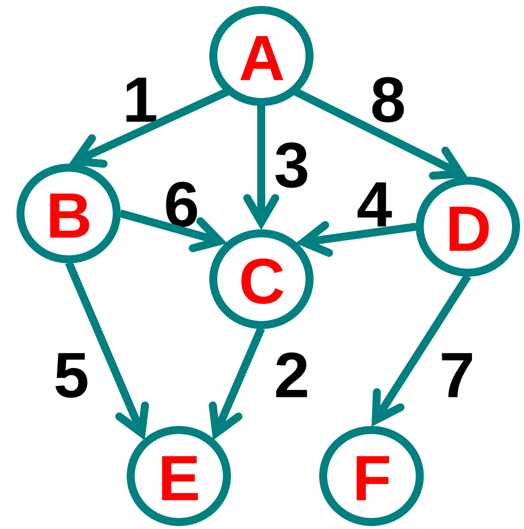
- Based on trying best current (local) choice
- Approach
  - At each step of algorithm
  - Choose best local solution
- Avoid backtracking, exponential time $O(2^n)$
- Hope local optimum lead to global optimum

Greedy Algorithm – Shortest Path

- **Example**
  - **Choose lowest-cost neighbor**



Cost ⇒ 6



  - **Does not yield overall (global) shortest path**

Greedy Algorithm – MST

Kruskal's Minimal Spanning Tree Algorithm

sort edges by weight (from least to most)

tree = ∅

for each edge (X,Y) in order

if it does not create a cycle

add (X,Y) to tree

stop when tree has N–1 edges

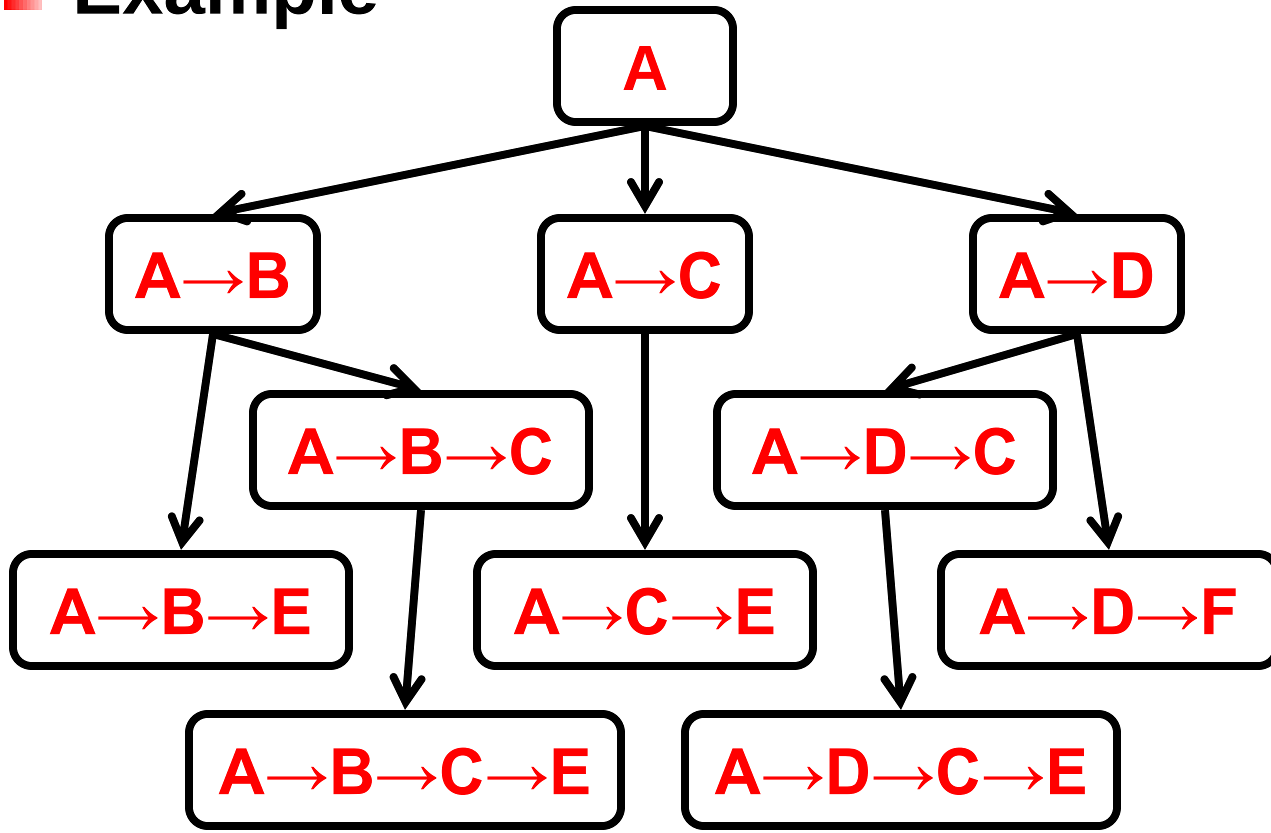
Picks best local solution at each step

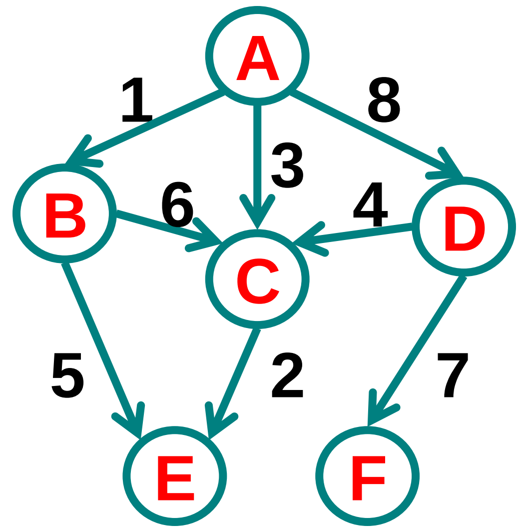
Brute Force Algorithm

- Based on trying all possible solutions
- Approach
  - Generate and evaluate possible solutions until
    - Satisfactory solution is found
    - Best solution is found (if can be determined)
    - All possible solutions found
      - Return best solution
      - Return failure if no satisfactory solution
- Generally most expensive approach

Brute Force Algorithm – Shortest Path

- **Example**





- **Examines all paths in graph**

Brute Force Algorithm – TSP

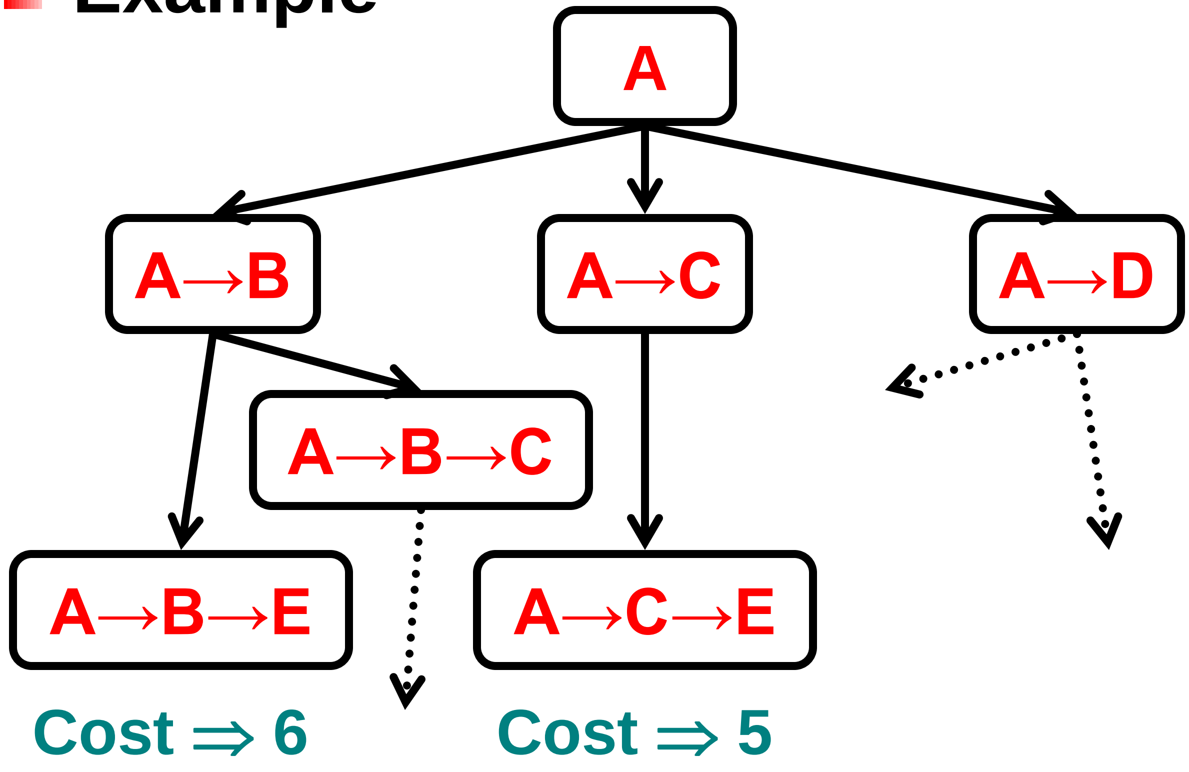
- Traveling Salesman Problem (TSP)
  - Given weighted undirected graph (map of cities)
  - Find lowest cost path visiting all nodes (cities) once
  - No known polynomial-time general solution
- Brute force approach
  - Find all possible paths using recursive backtracking
  - Calculate cost of each path
  - Return lowest cost path
- Requires exponential time $O(2^n)$

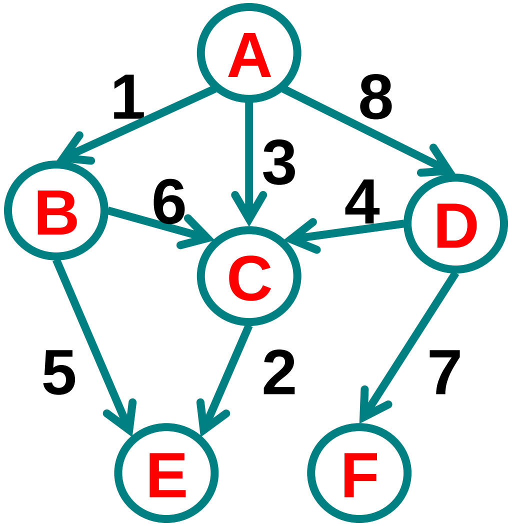
Branch and Bound Algorithm

- **Based on limiting search using current solution**
- **Approach**
  - **Track best current solution found**
  - **Eliminate (prune) partial solutions that can not improve upon best current solution**
- **Reduces amount of backtracking**
  - **Not guaranteed to avoid exponential time $O(2^n)$**

Branch & Bound Alg. – Shortest Path

- Example





- Pruned paths beginning with A→B→C & A→D

Branch and Bound – TSP

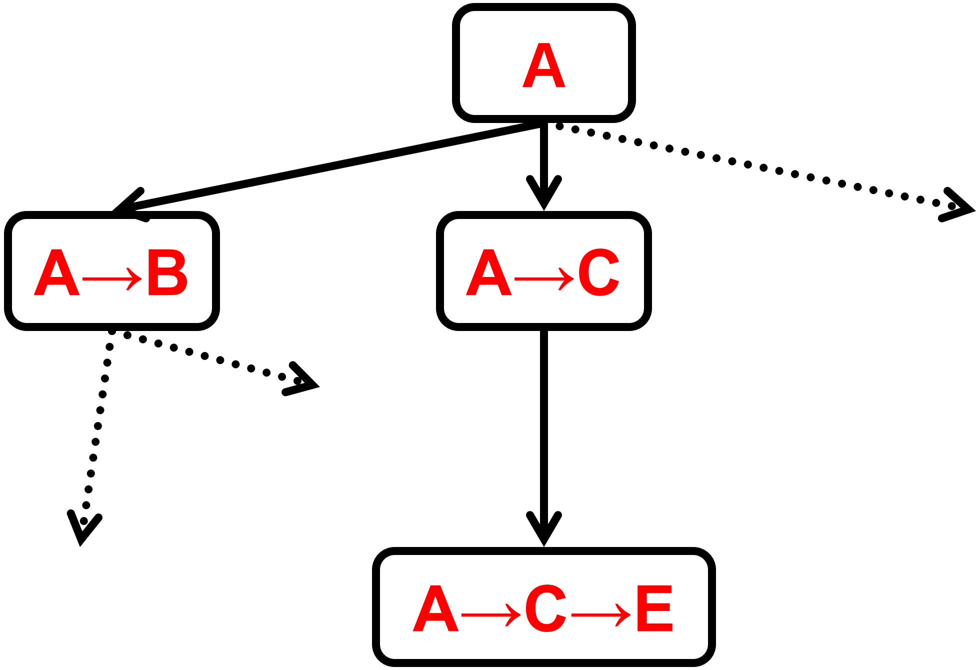
- Branch and bound algorithm for TSP
  - Find possible paths using recursive backtracking
  - Track cost of best current solution found
  - Stop searching path if cost > best current solution
  - Return lowest cost path
- If good solution found early, can reduce search
- May still require exponential time $O(2^n)$

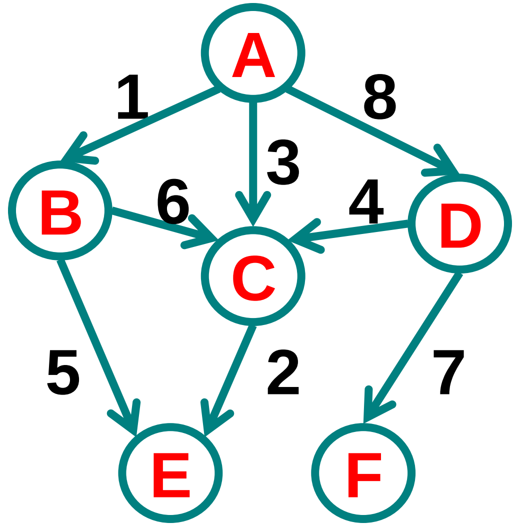
Heuristic Algorithm

- Based on trying to guide search for solution
- Heuristic $\Rightarrow$ "rule of thumb"
- Approach
  - Generate and evaluate possible solutions
    - Using "rule of thumb"
    - Stop if satisfactory solution is found
- Can reduce complexity
- Not guaranteed to yield best solution

Heuristic – Shortest Path

- **Example**
  - **Try only edges with cost < 5**





  - **Worked…in this case**

Heuristic Algorithm – TSP

- Heuristic algorithm for TSP
  - Find possible paths using recursive backtracking
    - Search 2 lowest cost edges at each node first
  - Calculate cost of each path
  - Return lowest cost path from first 100 solutions
- Not guaranteed to find best solution
- Heuristics used frequently in real applications

Summary

- **Wide range of strategies**
- **Choice depends on**
  - **Properties of problem**
  - **Expected problem size**
  - **Available resources**