

CMSC 132: Object-Oriented Programming II



Recursive Algorithms

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Recursion

- Recursion is a strategy for solving problems

- A procedure that calls itself

- Approach

If (problem instance is simple / trivial)

Solve it directly

Else

1. Simplify problem instance into **smaller** instance(s) of the original problem
2. Solve smaller instance using same algorithm
3. Combine solution(s) to solve original problem

Recursive Algorithm Format

1. Base case

- Solve small problem directly

2. Recursive step

- Simplify problem into smaller subproblem(s)
- Recursively apply algorithm to subproblem(s)
- Calculate overall solution

Example – Find

- To **find** an element in an array
 - Base case
 - If array is empty, return false
 - Recursive step
 - If 1st element of array is given value, return true
 - Skip 1st element and **recur** on remainder of array

Example – Count

- To **count** # of elements in an array
 - Base case
 - If array is empty, return **0**
 - Recursive step
 - Skip 1st element and **recur** on remainder of array
 - Add **1** to result

Auxiliary/Helper Functions

- **Some recursive problems require an auxiliary function**
- **Auxiliary function – the one that actually is recursive**
- **Example: `ArrayExamples.java`**

Example – Factorial

■ Factorial definition

- $n! = n \times n-1 \times n-2 \times n-3 \times \dots \times 3 \times 2 \times 1$

- $0! = 1$

■ To calculate factorial of n

- Base case

- If $n = 0$, return 1

- Recursive step

- Calculate the factorial of $n-1$

- Return $n \times (\text{the factorial of } n-1)$

Example – Factorial

■ Code

```
int fact ( int n ) {  
    if ( n == 0 ) return 1;           // base case  
    return n * fact(n-1);             // recursive step  
}
```

Properties

- **Recursion relies on the call stack**
 - **State of current procedure is saved when procedure is recursively invoked**
 - **Every procedure invocation gets own stack space**
- **Any problem solvable with recursion may be solved with iteration (and vice versa)**
 - **Use iteration with explicit stack to store state**
 - **Algorithm may be simpler for one approach**

Recursion vs. Iteration

■ Recursive algorithm

```
int fact ( int n ) {  
    if ( n == 0 ) return 1;  
    return n * fact(n-1);  
}
```

■ Iterative algorithm

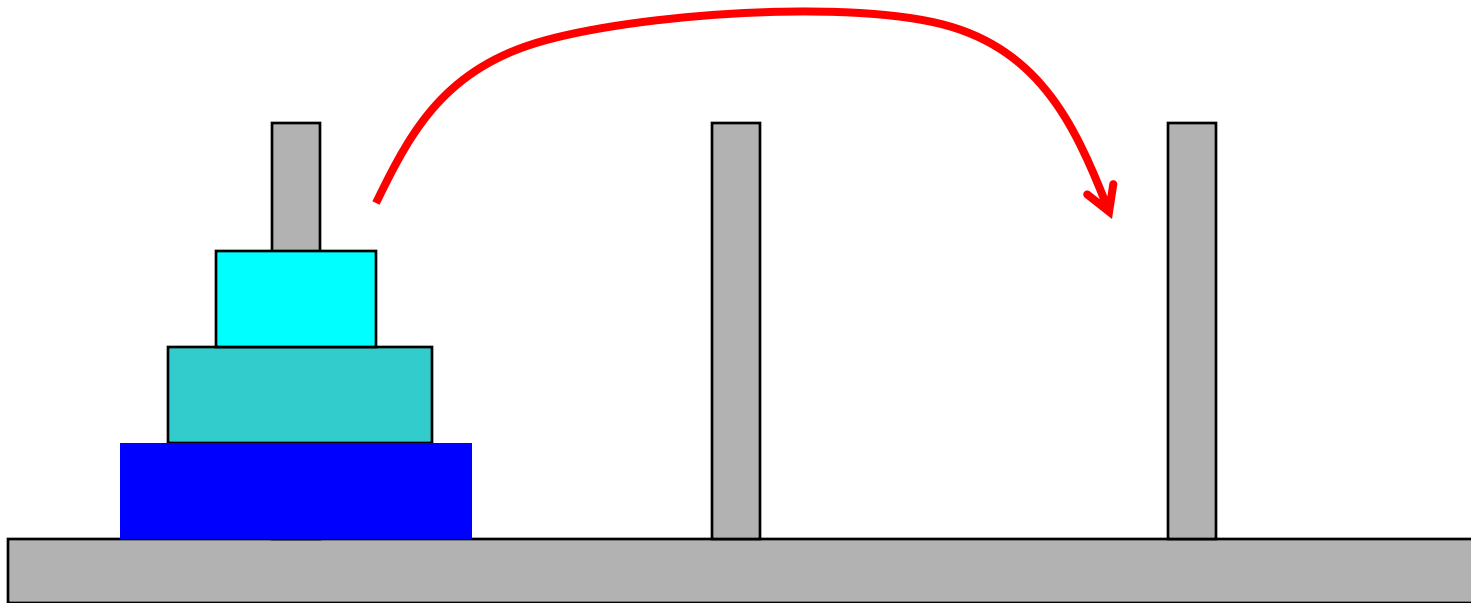
```
int fact ( int n ) {  
    int i, res;  
    res = 1;  
    for (i=n; i>0; i--) {  
        res = res * i;  
    }  
    return res;  
}
```

Recursive algorithm is closer to factorial definition

Example – Towers of Hanoi

■ Problem

- Move stack of disks between pegs
- Can only move top disk in stack
- Only allowed to place disk on top of larger disk



Example – Towers of Hanoi

- To move a stack of n disks from peg X to Y
 - Base case
 - If $n = 1$, move disk from X to Y
 - Recursive step
 1. Move top $n-1$ disks from X to 3rd peg
 2. Move bottom disk from X to Y
 3. Move top $n-1$ disks from 3rd peg to Y

Iterative algorithm would take much longer to describe!

Recursion vs. Iteration

■ Iterative algorithms

■ May be more efficient

- No additional function calls

- Run faster, use less memory

Recursion vs. Iteration

■ Recursive algorithms

■ Higher overhead

- Time to perform function call
- Memory for call stack

■ May be simpler algorithm

- Easier to understand, debug, maintain

■ Natural for backtracking searches

■ Suited for recursive data structures

- Trees, graphs...

Making Recursion Work

- Designing a correct recursive algorithm
- Verify
 1. Base case is
 - Recognized correctly
 - Solved correctly
 2. Recursive case
 - Solves 1 or more simpler subproblems
 - Can calculate solution from solution(s) to subproblems
- Uses principle of **proof by induction**

Requirements

■ Must have

- Small version of problem solvable without recursion
- Strategy to simplify problem into 1 or more smaller subproblems
- Ability to calculate overall solution from solution(s) to subproblem(s)

Proof By Induction

- Mathematical technique
- A theorem is true for all $n \geq 0$ if
 1. Base case
 - Prove theorem is true for $n = 0$, and
 2. Inductive step
 - Assume theorem is true for n
(inductive hypothesis)
 - Prove theorem must be true for $n+1$

Types of Recursion

■ Tail recursion

- Single recursive call at end of function

- Example

```
int tail( int n ) {  
    ...  
    return function( tail(n-1) );  
}
```

- Can easily transform to iteration (loop)

Types of Recursion

■ Non-tail recursion

- Recursive call(s) not at end of function

- Example

```
int nontail( int n ) {  
    ...  
    x = nontail(n-1) ;  
    y = nontail(n-2) ;  
    z = x + y;  
    return z;  
}
```

- Can transform to iteration using explicit stack

Possible Problems – Infinite Loop

■ Infinite recursion

- If recursion not applied to simpler problem

```
int bad ( int n ) {  
    if ( n == 0 ) return 1;  
    return bad(n);  
}
```

- Will infinite loop
- Eventually halt when runs out of (stack) memory
 - Stack overflow

Possible Problems – Efficiency

- May perform excessive computation
 - If recomputing solutions for subproblems

- Example

- Fibonacci numbers

- $\text{fibonacci}(0) = 1$

- $\text{fibonacci}(1) = 1$

- $\text{fibonacci}(n) = \text{fibonacci}(n-1) + \text{fibonacci}(n-2)$

Possible Problems – Efficiency

- **Recursive algorithm to calculate fibonacci(n)**
 - If n is 0 or 1, return 1
 - Else compute fibonacci(n-1) and fibonacci(n-2)
 - Return their sum
- **Simple algorithm $\Rightarrow$ exponential time $O(2^n)$**
 - Computes fibonacci(1) 2^n times
- **Can solve efficiently using**
 - Iteration
 - Dynamic programming
 - Will examine different algorithm strategies later...

Examples of Recursive Algorithms

- **Binary search**
- **Quicksort**
- **N-queens**
- **Fractals**

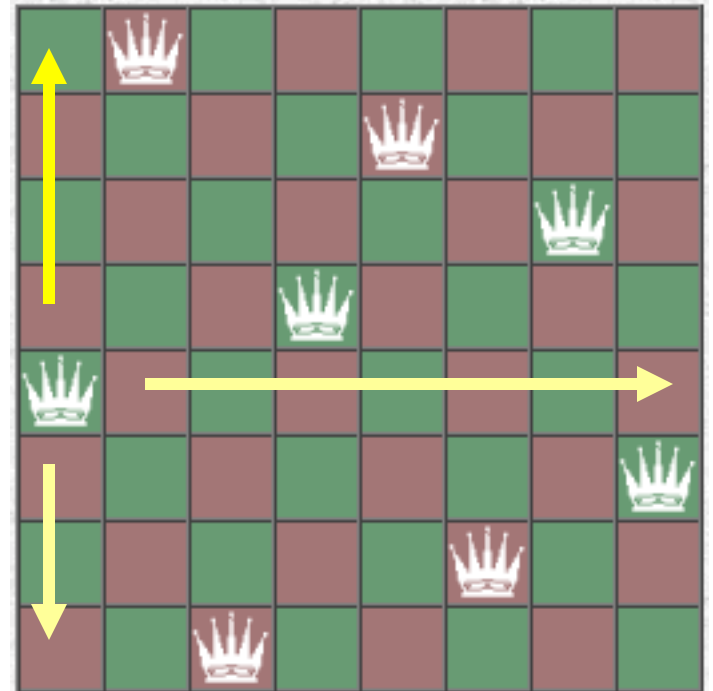
N-Queens

■ Goal

- Place queens on a board such that every row and column contains one queen, but no queen can attack another queen

■ Recursive approach

- To place queens on $N \times N$ board
- Assume you've already placed K queens



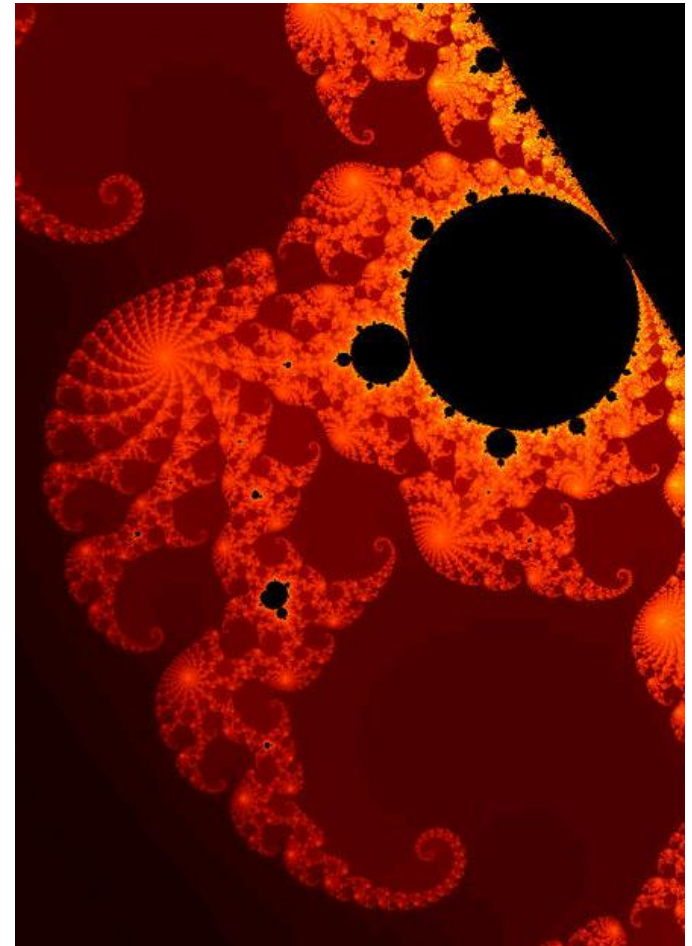
Fractals

■ Goal

- Construct shapes using a simple recursive definition with a natural appearance

■ Properties

- Appears similar at all scales of magnification
 - Therefore “infinitely complex”
- Not easily described in Euclidean geometry



Mandelbrot Set