Summer 2008 **CMSC 351: Homework 0** Clyde Kruskal

Problem 1. Use mathematical induction to show that

$$(a) \quad \sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3} \qquad\qquad (b) \quad \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

Problem 2. See bottom of page 53 of CLRS (bottom of page 34 in CLR) and/or the bottom of this sheet.

(a) Assume $b^x = a$. What is x (in terms of a and b)?

(b) Using only part (a), show that $\log_c(ab) = \log_c a + \log_c b$.

(c) Show that $a^{\log_b n} = n^{\log_b a}$

Problem 3. Differentiate the following functions:

(a) $\ln(x^2 + 5)$

(b) $\lg(x^2 + 5)$

(c) $\frac{1}{\ln(x^2+5)}$

Problem 4. Integrate the following functions:

(a) $\frac{1}{x}$

(b) $\frac{1}{3x+7}$

(c) $\ln x$ [HINT: Use integration by parts.]

(d) $x \ln x$ [HINT: Use integration by parts.]

(e) $x \lg x$

$$\begin{aligned} \lg n &= \log_2 n \\ \ln n &= \log_e n \\ \lg^k n &= (\lg n)^k \\ \lg\lg n &= \lg(\lg n) \end{aligned}$$

For all real $a > 0$, $b > 0$, $c > 0$, and n,

$$\begin{aligned} a &= b^{\log_b a} \\ \log_c(ab) &= \log_c a + \log_c b \\ \log_b a^n &= n \log_b a \\ \log_b a &= \frac{\log_c a}{\log_c b} \\ \log_b(1/a) &= -\log_b a \\ \log_b a &= \frac{1}{\log_a b} \\ a^{\log_b n} &= n^{\log_b a} \end{aligned}$$