CMSC 132: Object-Oriented Programming II

Recursive Algorithms

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Recursion

- Recursion is a strategy for solving problems
  - A procedure that calls itself
- Approach

If (problem instance is simple / trivial)

Solve it directly

Else

1. Simplify problem instance into smaller instance(s) of the original problem
2. Solve smaller instance using same algorithm
3. Combine solution(s) to solve original problem

Recursive Algorithm Format

1. **Base case**
   - Solve small problem directly
2. **Recursive step**
   - Simplify problem into smaller subproblem(s)
   - Recursively apply algorithm to subproblem(s)
   - Calculate overall solution

Example – Factorial

- Factorial definition
  - $n! = n \times n\text{-}1 \times n\text{-}2 \times n\text{-}3 \times \ldots \times 3 \times 2 \times 1$
  - $0! = 1$
- To calculate factorial of n
  - Base case
    - If n = 0, return 1
  - Recursive step
    - Calculate the factorial of n-1
    - Return n × (the factorial of n-1)

Example – Factorial

- **Code**

```
int fact ( int n ) {
    if ( n == 0 ) return 1;            // base case
    return n * fact(n-1);              // recursive step
}
```

Properties

- Recursion relies on the call stack
  - State of current procedure is saved when procedure is recursively invoked
  - Every procedure invocation gets own stack space
- Any problem solvable with recursion may be solved with iteration (and vice versa)
  - Use iteration with explicit stack to store state
  - Algorithm may be simpler for one approach

Recursion vs. Iteration

- Recursive algorithm

```
int fact ( int n ) {
    if ( n == 0 ) return 1;
    return n * fact(n-1);
}
```

- Iterative algorithm

```
int fact ( int n ) {
    int i, res;
    res = 1;
    for (i=n; i>0; i--) {
        res = res * i;
    }
    return res;
}
```

Recursive algorithm is closer to factorial definition

Example – Find

- To find an element in an array
  - Base case
    - If array is empty, return false
  - Recursive step
    - If 1st element of array is given value, return true
    - Skip 1st element and recur on remainder of array

Example – Count

- To count # of elements in an array
  - Base case
    - If array is empty, return 0
  - Recursive step
    - Skip 1st element and recur on remainder of array
    - Add 1 to result

Auxiliary/Helper Functions

- Some recursive problems require an auxiliary function
- Auxiliary function – the one that actually is recursive
- Example: ArrayExamples.java

Recursion vs. Iteration

- **Iterative algorithms**
  - **May be more efficient**
    - **No additional function calls**
    - **Run faster, use less memory**

Recursion vs. Iteration

- **Recursive algorithms**
  - **Higher overhead**
    - **Time to perform function call**
    - **Memory for call stack**
  - **May be simpler algorithm**
    - **Easier to understand, debug, maintain**
  - **Natural for backtracking searches**
  - **Suited for recursive data structures**
    - **Trees, graphs…**

Making Recursion Work

- Designing a correct recursive algorithm
- Verify
  1. Base case is
     - Recognized correctly
     - Solved correctly
  2. Recursive case
     - Solves 1 or more simpler subproblems
     - Can calculate solution from solution(s) to subproblems
- Uses principle of proof by induction

Requirements

- **Must have**
  - **Small version of problem solvable without recursion**
  - **Strategy to simplify problem into 1 or more smaller subproblems**
  - **Ability to calculate overall solution from solution(s) to subproblem(s)**

Proof By Induction

- Mathematical technique
- A theorem is true for all $n \geq 0$ if
  1. Base case
     - Prove theorem is true for $n = 0$, and
  2. Inductive step
     - Assume theorem is true for n
       (inductive hypothesis)
     - Prove theorem must be true for $n+1$

Types of Recursion

- **Tail recursion**
  - **Single recursive call at end of function**
  - **Example**

```
int tail( int n ) {
    …
    return function( tail(n-1) );
}
```

  - **Can easily transform to iteration (loop)**

Types of Recursion

- **Non-tail recursion**
  - **Recursive call(s) not at end of function**
  - **Example**

```
int nontail( int n ) {
    …
    x =  nontail(n-1) ;
    y =  nontail(n-2) ;
    z = x + y;
    return z;
}
```

  - **Can transform to iteration using explicit stack**

Possible Problems – Infinite Loop

- **Infinite recursion**
  - **If recursion not applied to simpler problem**

```
int bad ( int n ) {
    if ( n == 0 ) return 1;
    return bad(n);
}
```

  - **Will infinite loop**
  - **Eventually halt when runs out of (stack) memory**
    - **Stack overflow**

Possible Problems – Efficiency

- **May perform excessive computation**
  - **If recomputing solutions for subproblems**
- **Example**
  - **Fibonacci numbers**
    - **fibonacci(0) = 1**
    - **fibonacci(1) = 1**
    - **fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)**

Possible Problems – Efficiency

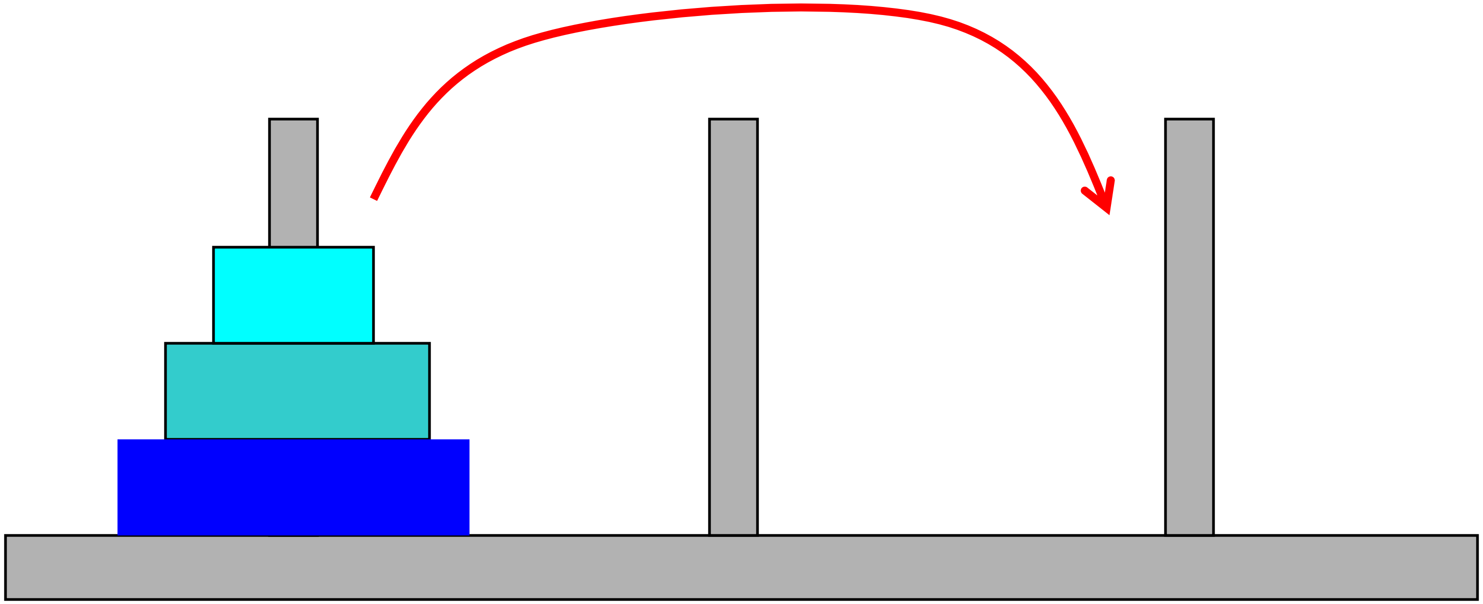
- **Recursive algorithm to calculate fibonacci(n)**
  - **If n is 0 or 1, return 1**
  - **Else compute fibonacci(n-1) and fibonacci(n-2)**
  - **Return their sum**
- **Simple algorithm $\Rightarrow$ exponential time $O(2^n)$**
  - **Computes fibonacci(1) 2^n times**
- **Can solve efficiently using**
  - **Iteration**
  - **Dynamic programming**
  - **Will examine different algorithm strategies later…**

Examples of Recursive Algorithms

- Towers of Hanoi
- Binary search
- Quicksort
- N-queens
- Fractals

Example – Towers of Hanoi

- Problem
  - Move stack of disks between pegs
  - Can only move top disk in stack
  - Only allowed to place disk on top of larger disk



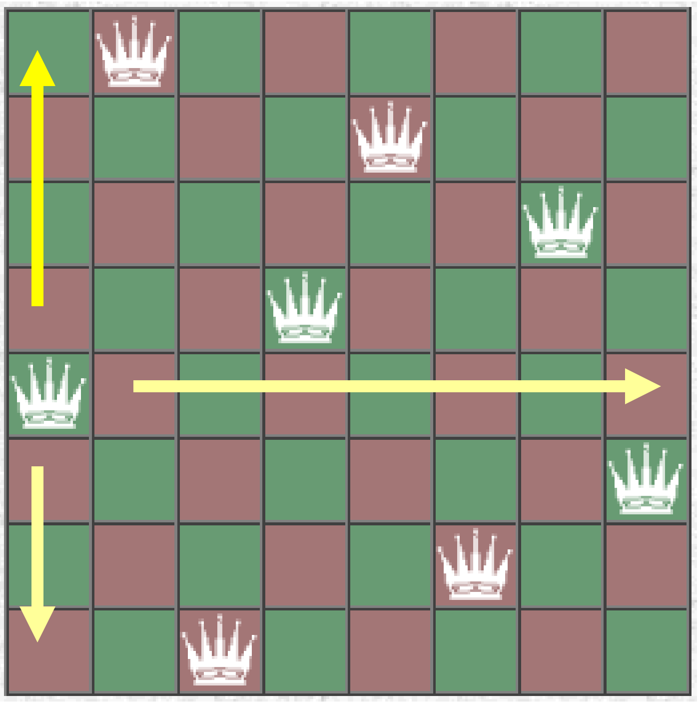
Example – Towers of Hanoi

- **To move a stack of n disks from peg X to Y**
  - **Base case**
    - **If n = 1, move disk from X to Y**
  - **Recursive step**
    1. **Move top n-1 disks from X to 3rd peg**
    2. **Move bottom disk from X to Y**
    3. **Move top n-1 disks from 3rd peg to Y**

Iterative algorithm would take much longer to describe!

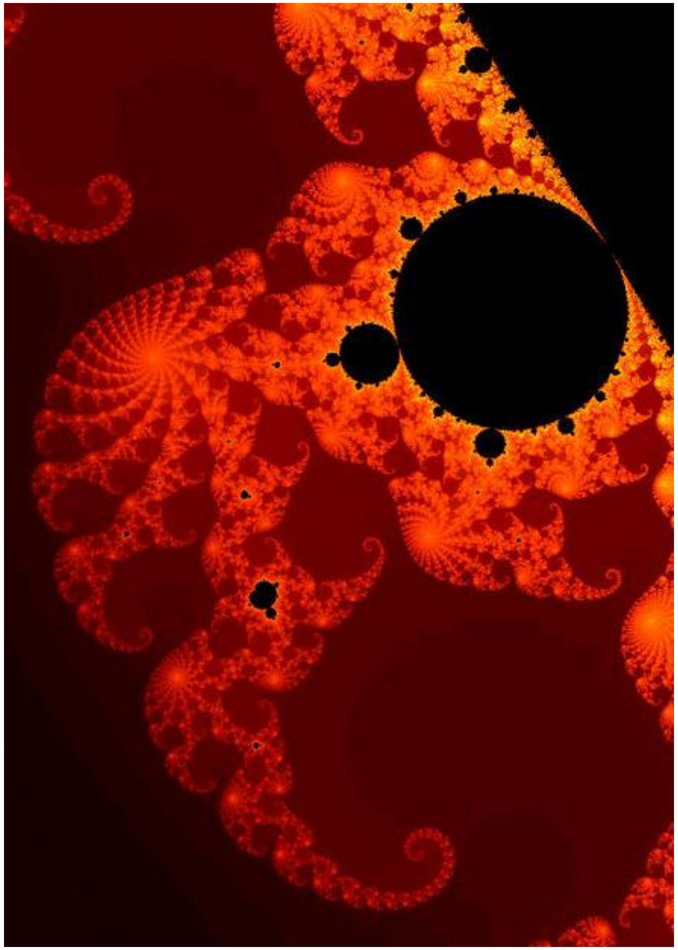
N-Queens

- **Goal**
  - **Place queens on a board such that every row and column contains one queen, but no queen can attack another queen**
- **Recursive approach**
  - **To place queens on NxN board**
  - **Assume you've already placed K queens**



Fractals

- Goal
  - Construct shapes using a simple recursive definition with a natural appearance
- Properties
  - Appears similar at all scales of magnification
    - Therefore “infinitely complex”
  - Not easily described in Euclidean geometry



Mandelbrot Set