CMSC 132:
Object-Oriented Programming II

Trees & Binary Search Trees

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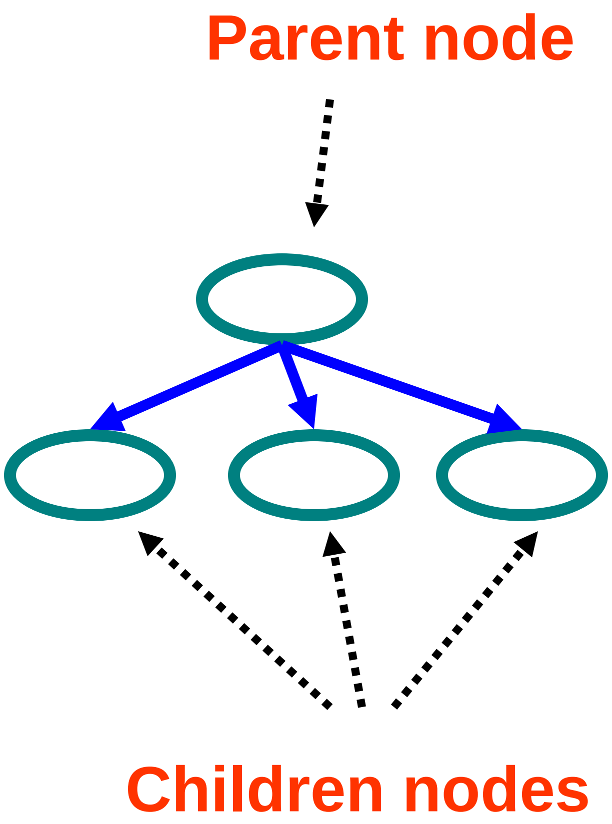
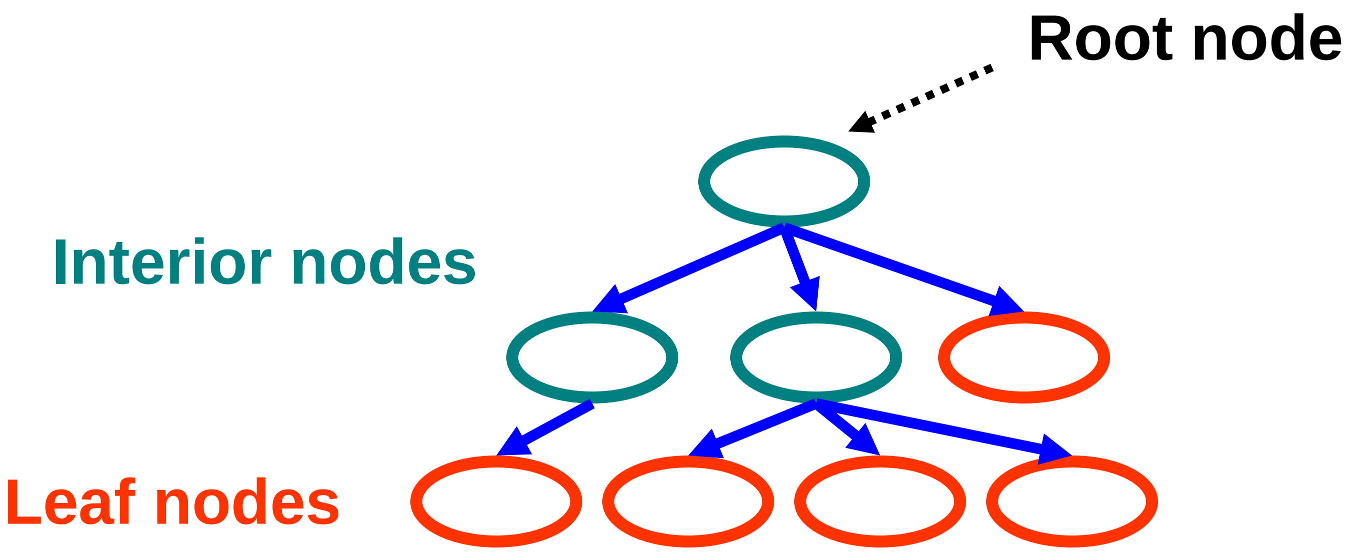
Trees

- **Trees are hierarchical data structures**
  - **One-to-many relationship between elements**
- **Tree node / element**
  - **Contains data**
  - **Referred to by only 1 (parent) node**
  - **Contains links to any number of (children) nodes**



Trees

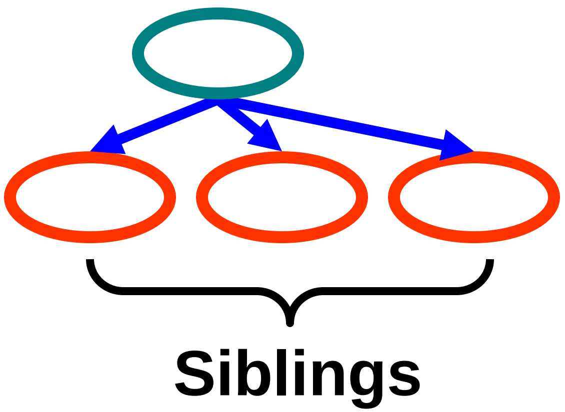
- **Terminology**
  - **Root ⇒ node with no parent**
  - **Leaf ⇒ all nodes with no children**
  - **Interior ⇒ all nodes with children**

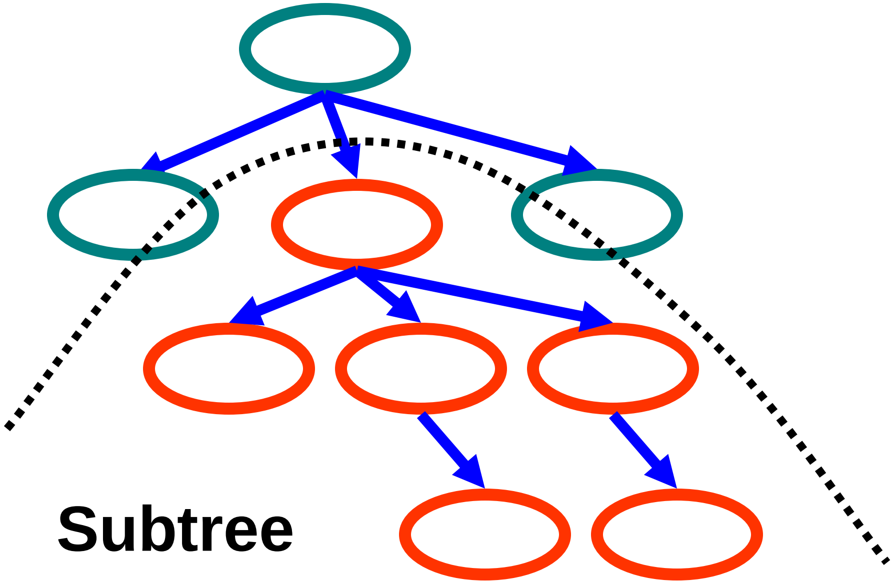


Trees

- **Terminology**
  - **Sibling ⇒ node with same parent**
  - **Descendent ⇒ children nodes & their descendents**
  - **Subtree ⇒ portion of tree that is a tree by itself**

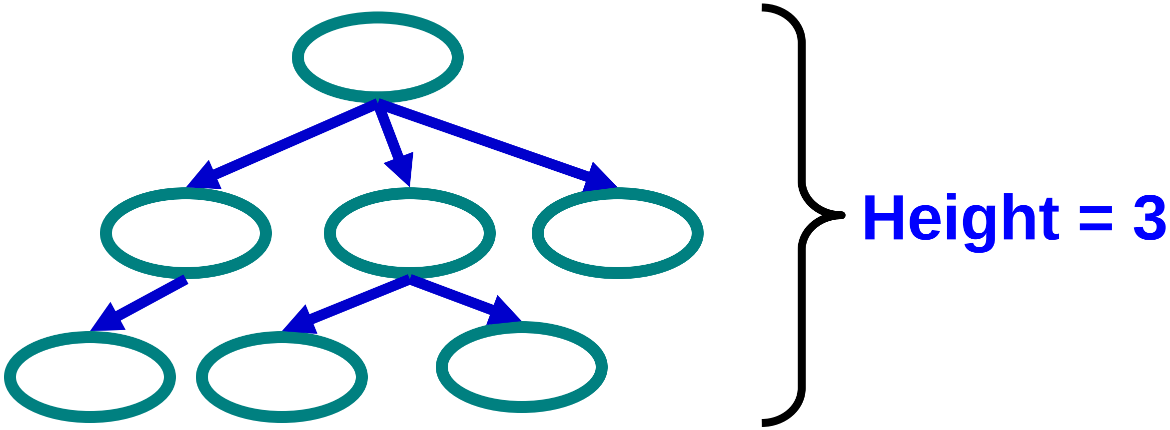
    ⇒ a node and its descendents





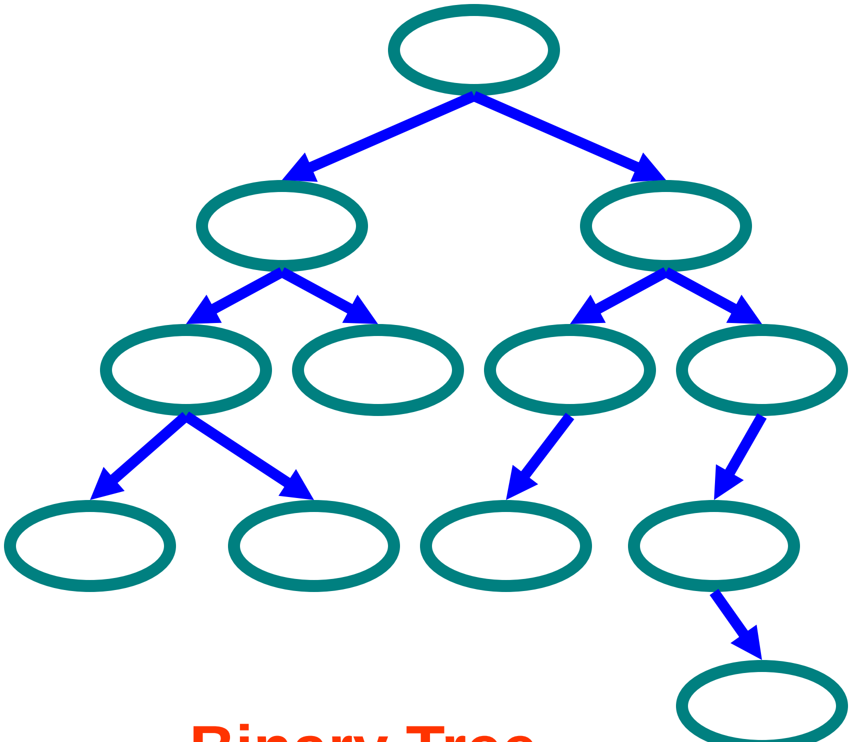
Trees

- Terminology
  - Level ⇒ is a measure of a node's distance from root
  - Definition of level
    - If node is the root of the tree, its level is 1
    - Else, the node's level is 1 + its parent's level
  - Height (depth) ⇒ max level of any node in tree

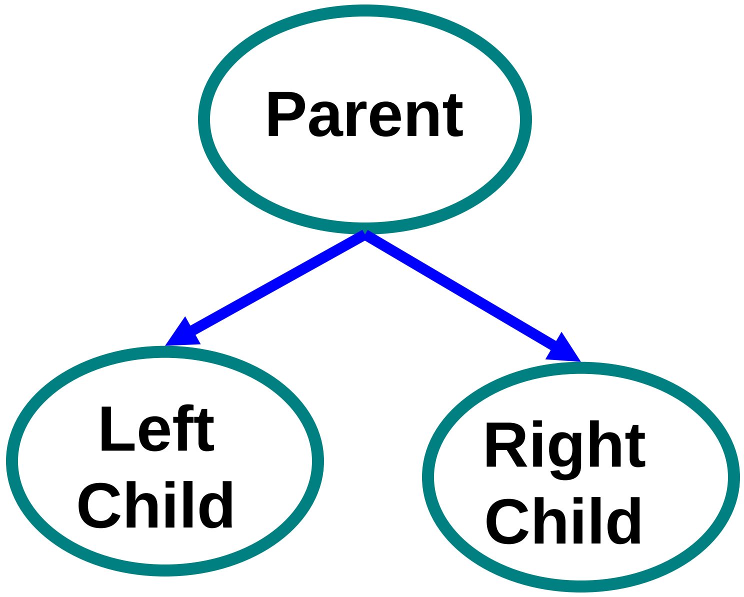


Binary Trees

- **Binary tree**
  - **Tree with 0–2 children per node**
    - **Left & right child / subtree**



Binary Tree



Tree Traversal

- **Often we want to**
  1. **Find all nodes in tree**
  2. **Determine their relationship**

- **Can do this by**
  1. **Walking through the tree in a prescribed order**
  2. **Visiting the nodes as they are encountered**

- **Process is called tree traversal**

Tree Traversal

- **Goal**
  - **Visit every node in binary tree**

- **Approaches**
  - **Depth first**
    - **Preorder $\Rightarrow$ parent before children**
    - **Inorder $\Rightarrow$ left child, parent, right child**
    - **Postorder $\Rightarrow$ children before parent**
  - **Breadth first $\Rightarrow$ closer nodes first**

Tree Traversal Methods

- **Pre-order**
  1. **Visit node // first**
  2. **Recursively visit left subtree**
  3. **Recursively visit right subtree**
- **In-order**
  1. **Recursively visit left subtree**
  2. **Visit node // second**
  3. **Recursively right subtree**
- **Post-order**
  1. **Recursively visit left subtree**
  2. **Recursively visit right subtree**
  3. **Visit node // last**

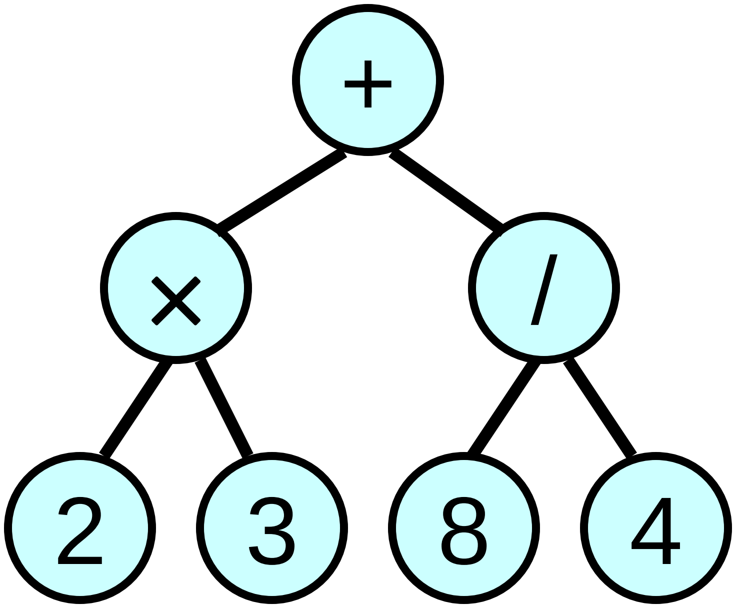
Tree Traversal Methods

- **Breadth-first**

```
BFS(Node n) {
    Queue Q = new Queue();
    Q.enqueue(n);                  // insert node into Q
    while ( !Q.empty()) {
        n = Q.dequeue();           // remove next node
        if ( !n.isEmpty()) {
            visit(n);              // visit node
            Q.enqueue(n.Left());   // insert left subtree in Q
            Q.enqueue(n.Right());// insert right subtree in Q
  } }
```

Tree Traversal Examples

- **Pre-order (prefix)**
  - **+ × 2 3 / 8 4**
- **In-order (infix)**
  - **2 × 3 + 8 / 4**
- **Post-order (postfix)**
  - **2 3 × 8 4 / +**
- **Breadth-first**
  - **+ × / 2 3 8 4**



Expression tree

Binary Tree Implementation

- **Using a class to represent a Node**

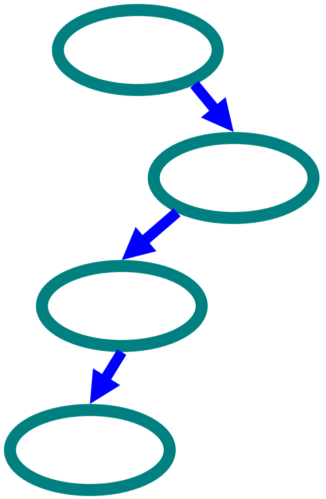
```
Class Node {
    KeyType key;
    Node left, right;    // null if empty
}

Node root = null; // Empty Tree
```

- **Using a Polymorphic Binary Tree**
  - **We will talk about this implementation later on**

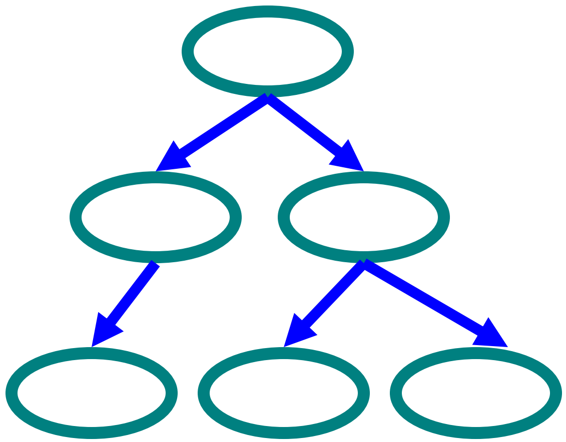
Types of Binary Trees

- **Degenerate**
  - Mostly 1 child / node
  - Height = O(n)
  - Similar to linear list

- **Balanced**
  - Mostly 2 child / node
  - Height = O(log(n))
  - $2^{Height} - 1 = n$ (# of nodes)
  - Useful for searches



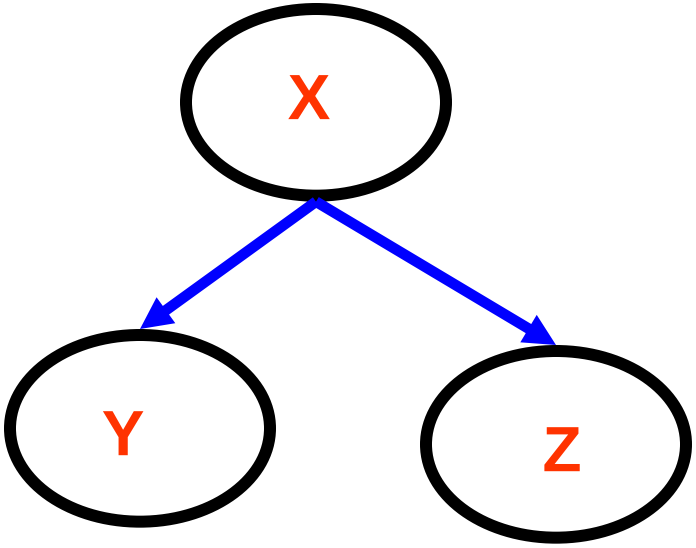
Degenerate binary tree



Balanced binary tree

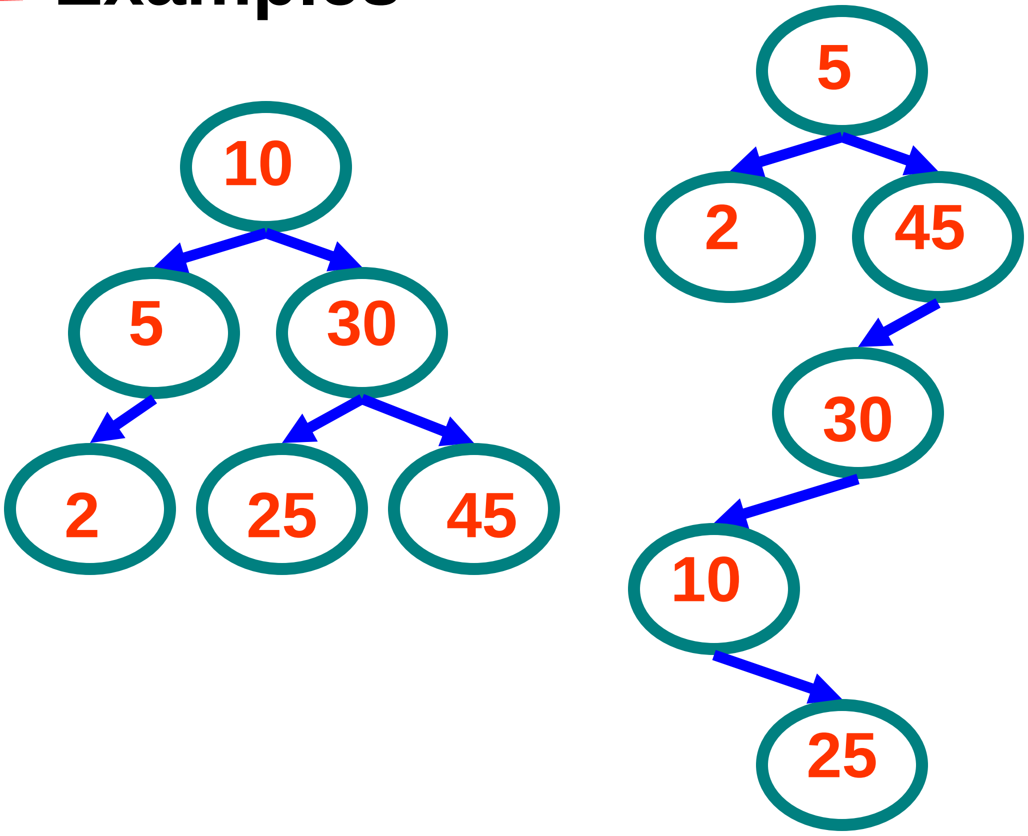
Binary Search Trees

- Key property
  - Value at node
    - Smaller values in left subtree
    - Larger values in right subtree
  - Example
    - X > Y
    - X < Z

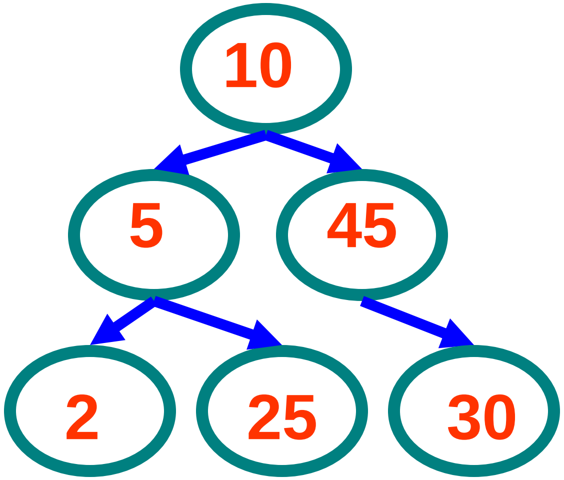


Binary Search Trees

- **Examples**



Binary search trees

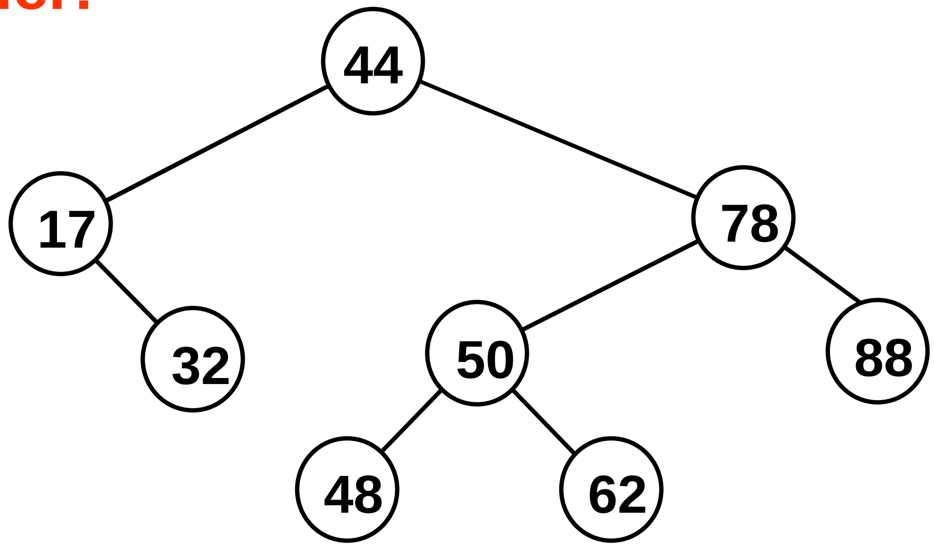


Non-binary search tree

Tree Traversal Examples

- Pre-order
  - 44, 17, 32, 78, 50, 48, 62, 88
- In-order
  - 17, 32, 44, 48, 50, 62, 78, 88
- Post-order
  - 32, 17, 48, 62, 50, 88, 78, 44
- Breadth-first
  - 44, 17, 78, 32, 50, 88, 48, 62

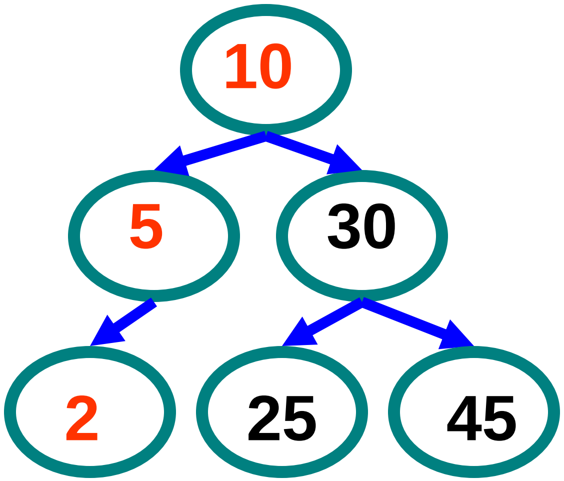
Sorted order!



Binary search tree

Example Binary Searches

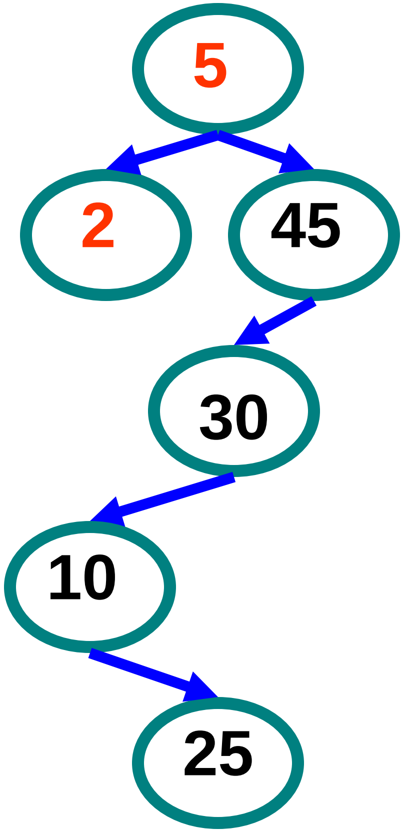
- **Find (2)**



2 < 10, left

2 < 5, left

2 = 2, found

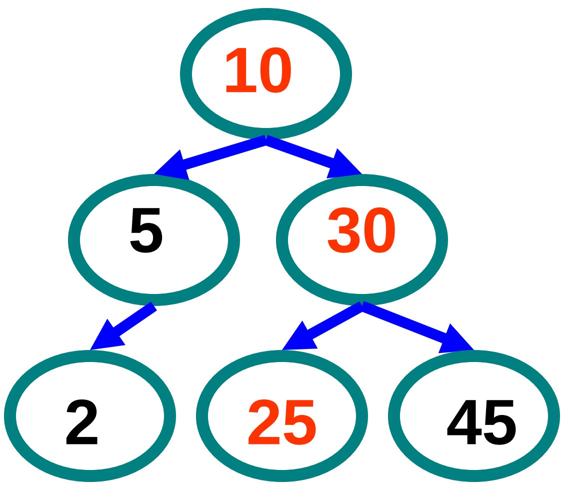


2 < 5, left

2 = 2, found

Example Binary Searches

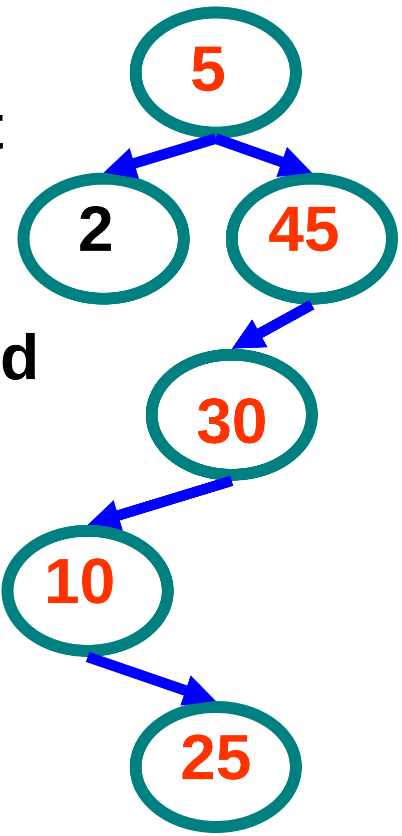
- **Find (25)**



25 > 10, right

25 < 30, left

25 = 25, found



25 > 5, right

25 < 45, left

25 < 30, left

25 > 10, right

25 = 25, found

Binary Search Properties

- **Time of search**
  - **Proportional to height of tree**
  - **Balanced binary tree**
    - **O(log(n)) time**
  - **Degenerate tree**
    - **O(n) time**
    - **Like searching linked list / unsorted array**
- **Requires**
  - **Ability to compare key values**

Binary Search Tree Construction

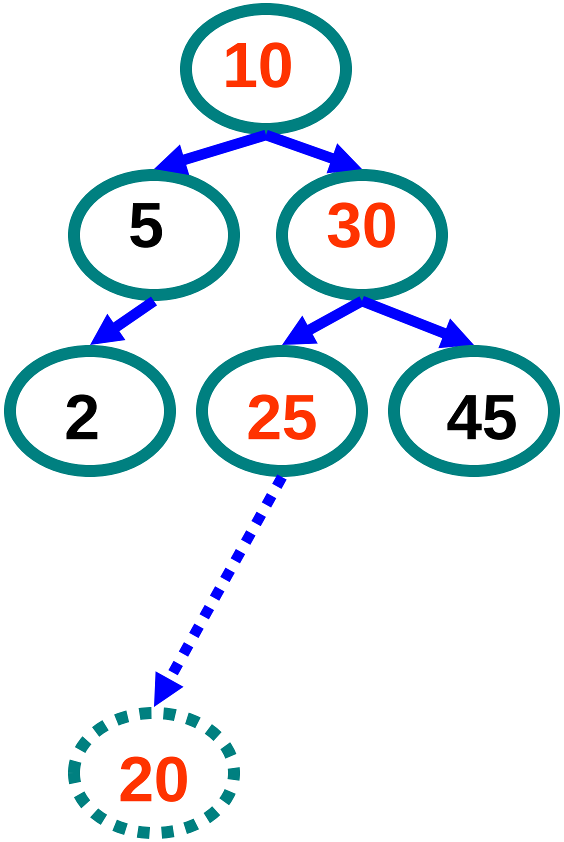
- How to build & maintain binary trees?
  - Insertion
  - Deletion
- Maintain key property (invariant)
  - Smaller values in left subtree
  - Larger values in right subtree

Binary Search Tree – Insertion

- **Algorithm**
  1. **Perform search for value X**
  2. **Search will end at node Y (if X not in tree)**
  3. **If X < Y, insert new leaf X as new left subtree for Y**
  4. **If X > Y, insert new leaf X as new right subtree for Y**
- **Observations**
  - **O(log(n)) operation for balanced tree**
  - **Insertions may unbalance tree**

Example Insertion

- **Insert (20)**



20 > 10, right

20 < 30, left

20 < 25, left

Insert 20 on left

Binary Search Tree – Deletion

- **Algorithm**
  1. **Perform search for value X**
  2. **If X is a leaf, delete X**
  3. **Else // must delete internal node**

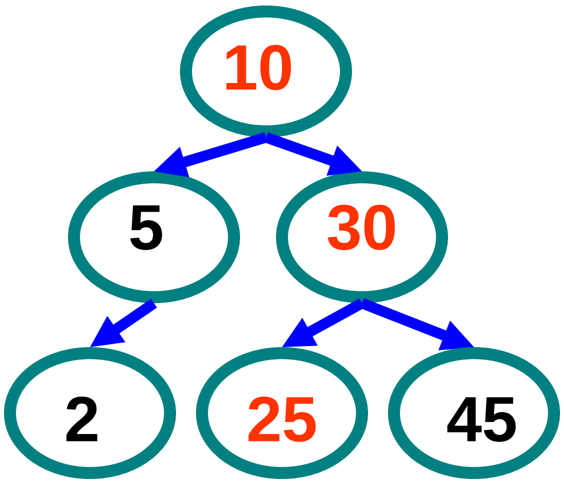
     a) Replace with largest value Y on left subtree

     OR smallest value Z on right subtree

     b) Delete replacement value (Y or Z) from subtree
- **Observation**
  - **O(log(n)) operation for balanced tree**
  - **Deletions may unbalance tree**

Example Deletion (Leaf)

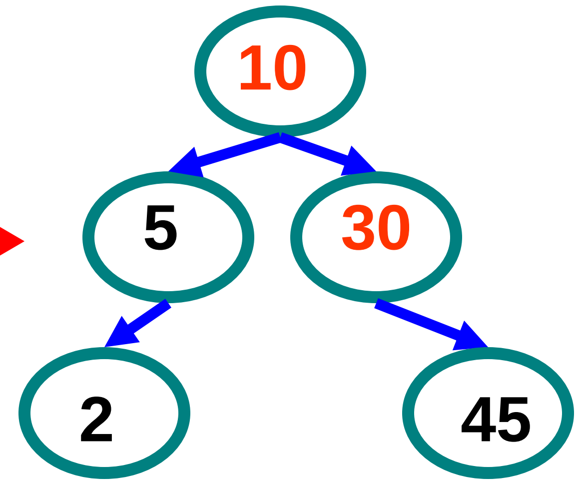
- **Delete (25)**



25 > 10, right

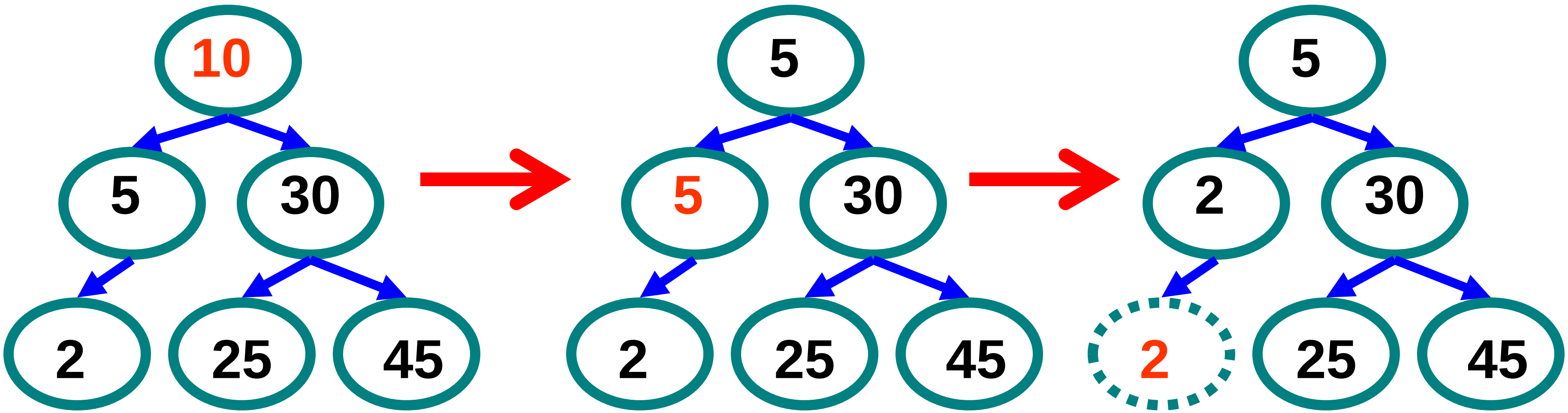
25 < 30, left

25 = 25, delete



Example Deletion (Internal Node)

- **Delete (10)**



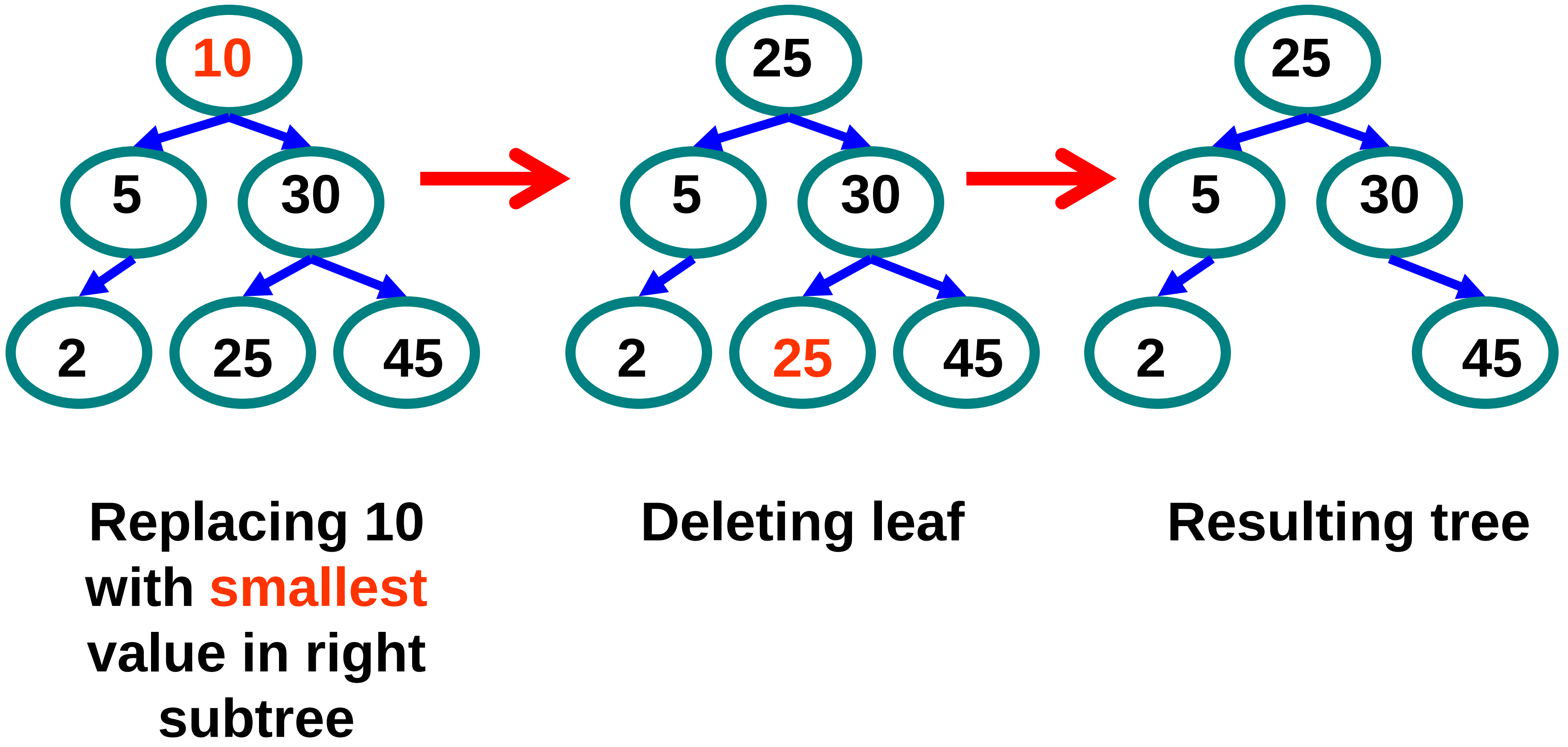
Replacing 10 with largest value in left subtree

Replacing 5 with largest value in left subtree

Deleting leaf

Example Deletion (Internal Node)

- **Delete (10)**



Building Maps w/ Search Trees

- Binary Search trees often used to implement maps
  - Each non-empty node contains
    - Key
    - Value
    - Left and right child

- Need to be able to compare keys
  - Generic type <K extends Comparable<K>>
    - Denotes any type K that can be compared to K's

BST (Binary Search Tree) Implementation

- **Implementing Tree using traditional approach**
- **Based on the BST definition below let's see how to implement typical BST Operations (constructor, add, print, find, isEmpty, isFull, size, height, etc.)**

```
public class BinarySearchTree <K extends Comparable<K>, V> {
    private class Node {
        private K key;
        private V data;
        private Node left, right;
        public Node(K key, V data) {
            this.key = key;
            this.data = data;
        }
    }
    private Node root;
}
```

- **See code distribution BinaryTreeCode.zip**

BST Testing

- **How can we test the correctness of BST Methods?**
- **What is the best approach?**

Polymorphic Binary Search Trees

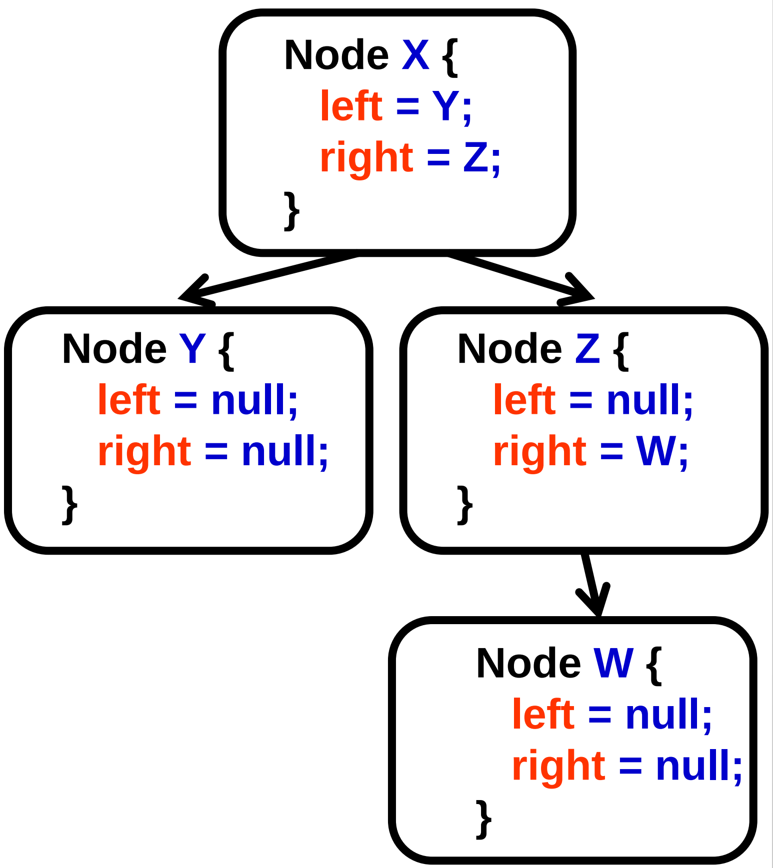
- **Second approach to implement BST**
- **What do we mean by polymorphic?**
- **Implement two subtypes of Tree**
  1. **EmptyTree**
  2. **NonEmptyTree**
- **Use EmptyTree to represent the empty tree**
  - **Rather than null**
- **Invoke methods on tree nodes**
  - **Without checking for null (IMPORTANT!)**

Polymorphic Binary Tree Implementation

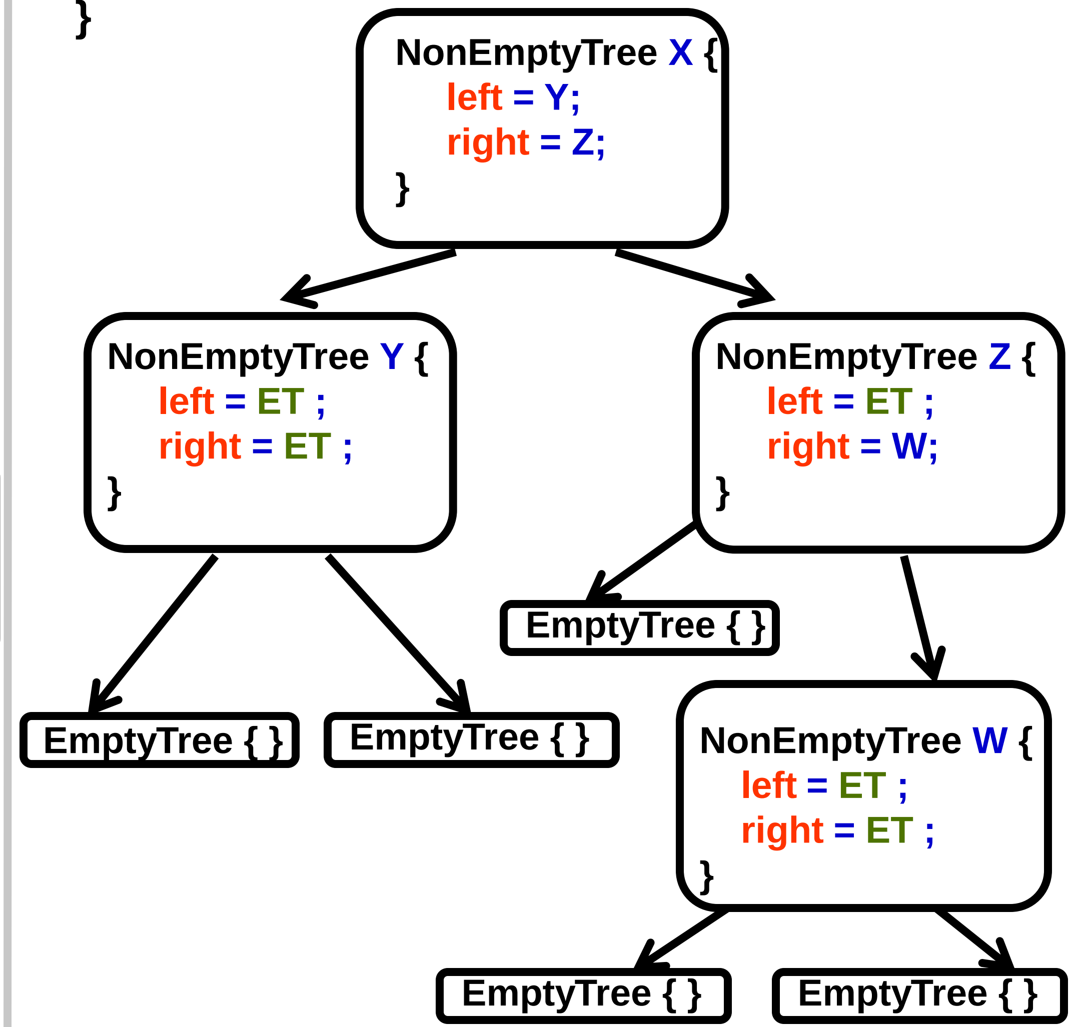
```
Interface Tree {
    Tree insert ( Value data1 ) { … }
}
Class EmptyTree implements Tree {
    Tree insert ( Value data1 ) { … }
}
Class NonEmptyTree implements Tree {
    Value data;
    Tree left, right; // Either Empty or NonEmpty
    Tree insert ( Value data1 ) { … }
}
```

Standard vs. Polymorphic Binary Tree

```
Class Node {
        Node left, right;
}
```




```
Class EmptyTree {}
Class NonEmptyTree {
        Tree left, right;
}
```



Singleton Design Pattern

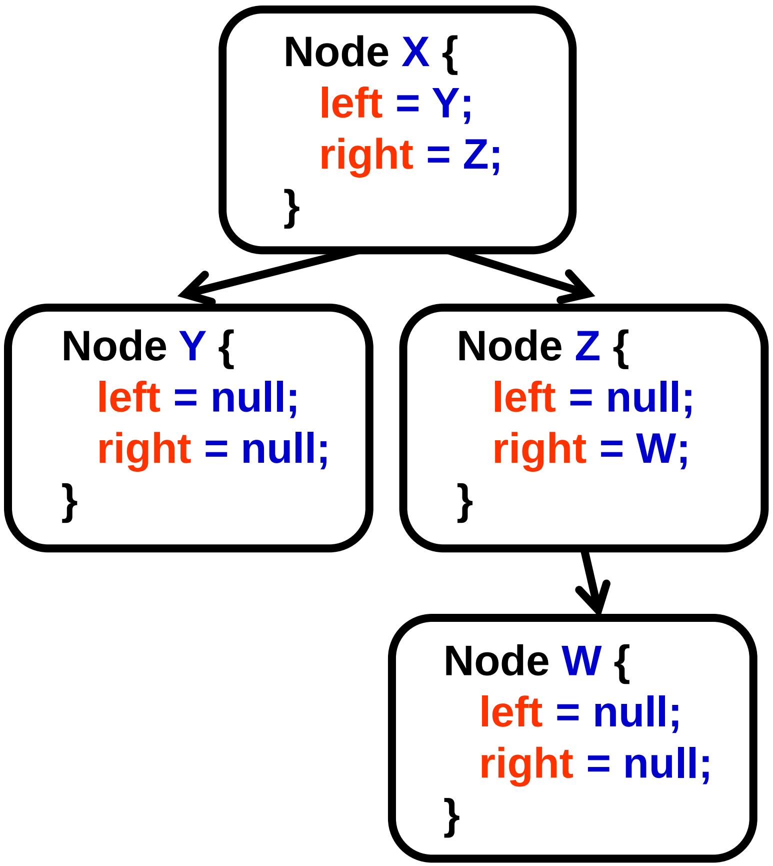
- **Definition**
  - **One instance of a class or value accessible globally**
- **Where to use & benefits**
  - **Ensure unique instance by defining class final**
  - **Access to the instance only via methods provided**
- **EmptyTree class will be a singleton class**

Singleton Example

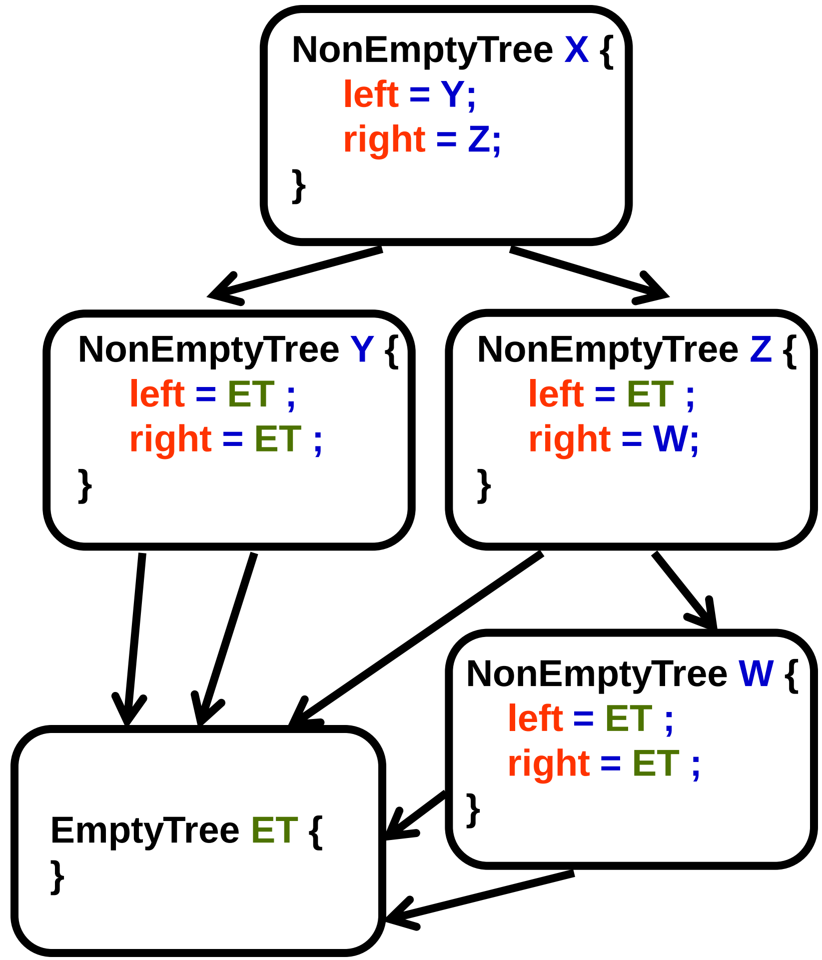
```
public final class MySingleton {
    // declare the unique instance of the class
    private static MySingleton uniq = new MySingleton();
    // private constructor only accessed from this class
    private MySingleton() { … }
    // return reference to unique instance of class
    public static MySingleton getInstance() {
        return uniq;
    }
}
```

Using Singleton EmptyTree

```
Class Node {
        Node left, right;
}
```




```
Class EmptyTree {}
Class NonEmptyTree {
        Tree left, right;
}
```



Polymorphic List Implementation

- Let’s see a polymorphic list implementation
- See code distribution PolymorphicListCode.zip