CMSC330

Ocaml Features, Recursion, and Higher-order functions

Yesterday

- Ocaml intro
  - Lists
  - `let`
  - Types
  - Functions
  - Pattern matching

Today

- More Ocaml
  - Tuples
  - Recursion
  - Higher order functions

OCaml Functions Take One Argument

- Recall this example

```
let plus (x, y) = x + y;;
plus (3, 4);;
```

  – It looks like you're passing in two arguments

- Actually, you're passing in a tuple instead

```
let plus t = match t with
    (x, y) = x + y;;
plus (3, 4);;
```

  – And using pattern matching to extract its contents

Tuples

- Constructed using `(e1, ..., en)`
- Deconstructed using pattern matching
- Tuples are like C structs
  - But without field labels
  - Allocated on the heap
- Tuples can be heterogenous
  - Unlike lists, which must be homogenous
  - `(1, ["string1"; "string2"])` is a valid tuple

Tuples - examples

- `let plusThree (x, y, z) = x+y+z`
  `let addOne (x, y, z) = (x+1, y+1, z+1)`
  - `plusThree (addOne (3,4,5)) = 15`
- `let sum ((a, b), c) = (a+c, b+c)`
  - `sum ((1, 2), 3) = (4,5)`
- `let plusFirstTwo (x::y::_, a) = (x+a, y+a)`
  - `plusFirstTwo ([1; 2; 3], 4) = (5,6)`

Tuples – more examples

- `let tls (_::xs, _::ys) = (xs, ys)`
  - `tls ([1;2;3],[4;5;6;7]) =`
    `([2;3],[5;6;7])`

- Remember
  - Semicolon for lists
  - Comma for tuples
- Example
  - `[1, 2]` = `[(1, 2)]` = a list of size one
  - `(1; 2)` = a syntax error

Another tuple example

- Given
  - `let f l = match l with x::(_::y) -> (x,y)`
- What is the value of
  - `f [1;2;3;4]`
- Possibilities
  - `([1],[3])`
  - `(1,3)`
  - `(1,[3])`
  - `(1,4)`
  - `(1,[3;4])`

List and Tuple Types

- Tuple types use **`*`** to separate components
- Examples
  - **`(1,2) :                int * int`**
  - **`(1,"string",3.5) :   int * string * float`**
  - **`(1,["a";"b"],'c'):   int * string list * char`**
  - **`[(1,2)] :           (int * int) list`**
  - **`[(1,2);(3, 4)] :    (int * int) list`**
  - **`[(1,2);(1,2,3)] :    ERROR`**

Type Declarations

- **`type`** can be used to create new names for types
  - Useful for combinations of lists and tuples

- Examples
  - **`type my_type = int * (int list)`**

    `(3, [1; 2]) : my_type`

  - **`type my_type2 = int * char * (int * float)`**

    `(3, 'a', (5, 3.0)) : my_type2`

Polymorphic Functions

- Some functions require specific list types
  - `let plusFirstTwo (x::y::_, a) =`
    `(x + a, y + a)`
  - `plusFirstTwo :`
    `int list * int -> (int * int)`
- But other functions work for a list of any type
  - `let hd (h::_) = h`
  - `hd [1; 2; 3]          (* returns 1   *)`
  - `hd ["a"; "b"; "c"]    (* returns "a" *)`
- These functions are polymorphic

Polymorphic Types

- OCaml gives such functions polymorphic types
  - `hd : 'a list -> 'a`
  - Read as
    - Function takes a list of any element type `'a`
    - And returns something of that type

- Example
  - `let tl (_::t) = t`

    `tl : 'a list -> 'a list`

Polymorphic Types (cont.)

- More Examples
  - `let swap (x, y) = (y, x)`
    `swap : 'a * 'b -> 'b * 'a`
  - `let tls (_::xs, _::ys) = (xs, ys)`
    `tls : 'a list * 'b list ->`
    `'a list * 'b list`

Tuples Are a Fixed Size

- This OCaml definition
  - ```
    # let foo x = match x with
      (a, b) -> a + b
      | (a, b, c) -> a + b + c;;
    ```
- Would yield this error message
  - This pattern matches values of type 'a * 'b * 'c
    but is here used to match values of type 'd * 'e
- Tuples of different size have different types
  - Thus never more than one match case with tuples
    ```

Conditionals

- Use **`if...then...else`** just like C/Java
  - No parentheses and no end

```
if grade >= 90 then
  print_string "You got an A"
else if grade >= 80 then
  print_string "You got a B"
else if grade >= 70 then
  print_string "You got a C"
else
  print_string "You’re not doing so well"
```

Conditionals (cont.)

- In OCaml, conditionals return a result
  - The value of whichever branch is true/false

```
# if 7 > 42 then "hello" else
  goodbye";;
- : string = "goodbye"
# let x = if true then 3 else 4;;
x : int = 3
# if false then 3 else 3.0;;
This expression has type float but is
  here used with type int
```

The Factorial Function

- Using conditionals & functions
  - Can you write **`fact`**, the factorial function?

```
let rec fact n =
  if n = 0 then
     1
  else
     n * fact (n-1);;
```

- Notice no return statements
  - This is pretty much how it needs to be written

let rec

- The **rec** part means “define a recursive function”
- Let vs. let rec
  - **let x = e1 in e2**
    - x in scope within e2
  - **let rec x = e1 in e2**
    - x in scope within e2 and e1
- Why use let rec?
  - If you used **let** instead of **let rec** to define fact

```
let fact n =
  if n = 0 then 1
  else n * fact (n-1) in e1
```

Fact is not bound here!

Examples - semicolon

- Definition
  - `e1 ; e2 (* evaluate e1, evaluate e2, return e2)`
- `1 ; 2 ;;`
  - `(* 2 – value of 2nd expression is returned *)`
- `(1 + 2) ; 4 ;;`
  - `(* 4 – value of 2nd expression is returned *)`
- `1 + (2 ; 4) ;;`
  - `(* 5 – value of 2nd expression is returned to 1 + *)`
- `1 + 2 ; 4 ;;`
  - `(* 4 – since + has higher precedence than ; *)`

Examples - `let`

- x;;
  - (* Unbound value x *)
- let x = 1 in x + 1;;
  - (* 2 *)
- let x = x in x + 1;;
  - (* Unbound value x *)

Examples – `let` (cont.)

- `let x = 1 in (x + 1 ; x ) ;;`
  - `(* 1 – ; has higher precedence than let … in *)`
- `(let x = 1 in x + 1) ; x;;`
  - `(* Unbound value x *)`
- `let x = 4 in (let x = x + 1 in x );;`
  - `(* 5 *)`

`let` – more examples

- `let f n = 10;;`
  ```
  let f n =
      if n = 0 then 1
      else n * f (n - 1);;
  ```
  - `f 0;; (* = 1 *)`
  - `f 1;; (* = 10 *)`

- `let f x = f x;;`
  - `(* Unbound value f *)`

Recursion

- Recursion is essentially the only way to iterate
  - The only way we're going to talk about, anyway
  - Feature of functional programming languages
- Another example

```
let rec print_up_to (n, m) =
  print_int n; print_string "\n";
  if n < m then print_up_to (n + 1, m)
```

Lists and recursion

- Lists have a recursive structure
  - And so most functions over lists will be recursive

```
let rec length l = match l with
    [] -> 0
  | (_::t) -> 1 + (length t)
```

  - This is just like an inductive definition
    - *The length of the empty list is zero*
    - *The length of a nonempty list is 1 plus the length of the tail*
  - Type of **`length`**?

Examples – recursive functions

- sum li (* sum of elements in li *)

```
let rec sum li = match li with
    [] -> 0
  | (x::xs) -> x + (sum xs)
```

- negate li (* negate elements in list *)

```
let rec negate li = match li with
    [] -> []
  | (x::xs) -> (-x) :: (negate xs)
```

Example – recursive functions

- last li (* last element of li *)

```
let rec last li = match li with
      [x] -> x
    | (_::xs) -> last xs
```

- append (li, m)

  (* list containing all elements in list li followed by all elements in list m *)

```
let rec append (li, m) = match li with
     [] -> m
   | (x::xs) -> x::(append (xs, m))
```

Example – recursive functions

- rev li (* reverse list; hint: use append *)

```
let rec rev li = match li with
    [] -> []
  | (x::xs) -> append ((rev xs),
 [x])
```

  – `rev` takes O(n^2) time. Can you do better?

A clever version of reverse

```
let rec rev_helper (li, a) = match li
  with
     [] -> a
   | (x::xs) -> rev_helper (xs, (x::a))
let rev li = rev_helper (li, [])
```

- Let’s give it a try

```
rev [1; 2; 3] →
rev_helper ([1;2;3], []) →
rev_helper ([2;3], [1]) →
rev_helper ([3], [2;1]) →
rev_helper ([], [3;2;1]) →
[3;2;1]
```

Example – recursive functions

- flattenPairs li (* ('a * 'a) list -> 'a list *)

```
let rec flattenPairs li = match li with
   [] -> []
  | ((a, b)::t)
          -> a :: b :: (flattenPairs t)
```

- take (n, li) (* return first n elements of li *)

```
let rec take (n, li) =
   if n = 0 then []
   else match l with
        [] -> []
      | (x::xs) -> x :: (take (n-1, xs))
```

Working with lists

- Several of these examples have the same flavor
  - Walk through the list and do something to every element
  - Walk through the list and keep track of something
- Recall the following example code from Ruby:

```
a = [1,2,3,4,5]
b = a.collect { |x| -x }
```

  - Here we passed a code block into the **`collect`** method
  - Wouldn't it be nice to do the same in OCaml?

Higher order functions

- In OCaml you can pass functions as arguments, and return functions as results

```
let plus_three x = x + 3
let twice (f, z) = f (f z)
  (* twice : ('a->'a) * 'a -> 'a *)
twice (plus_three, 5)

let plus_four x = x + 4
let pick_fn n =
  if n > 0 then plus_three else plus_four
(pick_fn 5) 0
(* pick_fn : int -> (int->int) *)
```

The **`map`** function

- Let’s write the **`map`** function (just like Ruby's **`collect`**)
  - Takes a function and a list, applies the function to each element of the list, and returns a list of the results

```
let rec map (f, l) = match l with
     [] -> []
   | (h::t) -> (f h)::(map (f, t))
```

```
let add_one x = x + 1
let negate x = -x
map (add_one, [1; 2; 3])
map (negate, [9; -5; 0])
```

The **map** function (cont.)

- What is the type of the **map** function?

```
let rec map (f, l) = match l with
      [] -> []
    | (h::t) -> (f h)::(map (f, t))
```

$$\underbrace{\texttt{('a -> 'b)}}_{\texttt{f}}\ \texttt{*}\ \underbrace{\texttt{'a list}}_{\texttt{l}}\ \texttt{-> 'b list}$$

Anonymous Functions

- Passing functions around is very common
  - So often we don't want to bother to give them names

- Use **`fun`** to make a function with no name

Parameter   Body

```
fun x -> x + 3
```

```
twice ((fun x -> x + 3), 5) = 11
map ((fun x -> x+1), [1; 2; 3]) = [2; 3; 4]
```

Pattern matching with `fun`

- `match` can be used within `fun`

```
map ((fun l -> match l with (h::_) -> h),
     [ [1; 2; 3]; [4; 5; 6; 7]; [8; 9] ])
```

- But use named functions for complicated matches
- May use standard pattern matching abbreviations

```
map ((fun (x, y) -> x+y), [(1,2); (3,4)])
= [3;7]
```

All functions are anonymous

- Functions are first-class, so you can bind them to other names as you like

```
let f x = x + 3
let g = f
g 5
```

- In fact, `let` for functions is just shorthand

```
let f x = body
```

↓ stands for

```
let f = fun x -> body
```

Examples – anonymous functions

- `let next x = x + 1`
  - Short for `let next = fun x -> x + 1`

- `let plus (x, y) = x + y`
  - Short for `let plus = fun (x, y) -> x + y`
  - Which is short for

```
let plus = fun z ->
           (match z with (x, y) -> x + y)
```

Examples – anonymous functions

- `let rec fact n =`
  ```
  if n = 0 then 1
  else n * fact (n-1)
  ```
  - Short for `let rec fact = fun n ->`
    ```
    (if n = 0 then 1
     else n * fact (n-1))
    ```

The `fold` function

- Common pattern
  - Iterate through list and apply function to each element, keeping track of partial results computed so far

```
let rec fold (f, a, l) = match l with
    [] -> a
  | (h::t) -> fold (f, f (a, h), t)
```

  - `a` = "accumulator"
  - Usually called `fold left` to remind us that `f` takes the accumulator as its first argument
- What's the type of `fold`?

`fold` example

```
let rec fold (f, a, l) = match l with
      [] -> a
    | (h::t) -> fold (f, f (a, h), t)
```

```
let add (a, x) = a + x
fold (add, 0, [1; 2; 3; 4]) →
fold (add, 1, [2; 3; 4]) →
fold (add, 3, [3; 4]) →
fold (add, 6, [4]) →
fold (add, 10, []) →
10
```

We just built the `sum` function!

Another `fold` example

```
let rec fold (f, a, l) = match l with
      [] -> a
   | (h::t) -> fold (f, f (a, h), t)
```

```
let next (a, _) = a + 1
fold (next, 0, [2; 3; 4; 5]) →
fold (next, 1, [3; 4; 5]) →
fold (next, 2, [4; 5]) →
fold (next, 3, [5]) →
fold (next, 4, []) →
4
```

We just built the `length` function!

Using **fold** to build reverse

```
let rec fold (f, a, l) = match l with
      [] -> a
    | (h::t) -> fold (f, f (a, h), t)
```

- Can you build the **rev** function with **fold**?

```
let prepend (a, x) = x::a
fold (prepend, [], [1; 2; 3; 4]) →
fold (prepend, [1], [2; 3; 4]) →
fold (prepend, [2; 1], [3; 4]) →
fold (prepend, [3; 2; 1], [4]) →
fold (prepend, [4; 3; 2; 1], []) →
[4; 3; 2; 1]
```

The call stack in Java/C++/etc

```
void f(void) {
  int x;
  x = g(3);
}
int g(int x) {
  int y;
  y = h(x);
  return y;
}
int h (int z) {
  return z + 1;
}
int main(){
  f();
  return 0;
}
```

x	4	**f**
x	3	**g**
y	4	
z	3	**h**

Nested functions

- In OCaml, you can define functions anywhere
  - Even inside of other functions

```
let sum l =
  fold ((fun (a, x) -> a + x), 0, l)
```

```
let pick_one n =
  if n > 0 then (fun x -> x + 1)
  else (fun x -> x - 1)
(pick_one -5) 6   (* returns 5 *)
```

Nested functions (cont.)

- You can also use **`let`** to define functions inside of other functions

```
let sum l =
  let add (a, x) = a + x in
  fold (add, 0, l)
```

```
let pick_one n =
  let add_one x = x + 1 in
  let sub_one x = x - 1 in
  if n > 0 then add_one else sub_one
```

How about this?

```
let addN (n, l) =
  let add x = n + x in
  map (add, l)
```

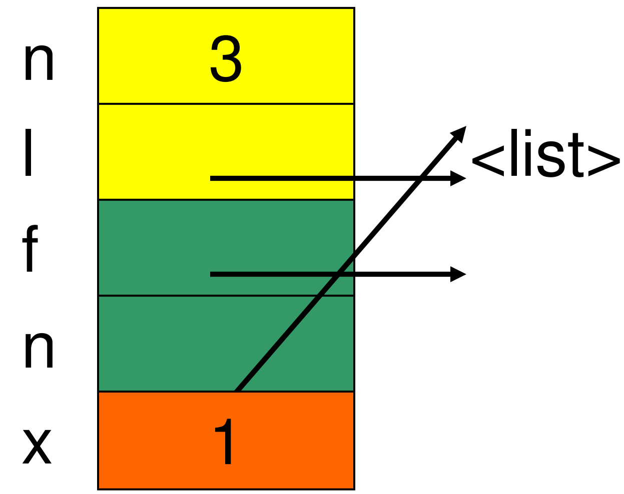
Accessing variable from outer scope

– (Equivalent to...)

```
let addN (n, l) =
  map ((fun x -> n + x), l)
```

Consider the call stack again

```
let map (f, n) = match n with
   [] -> []
| (h::t) -> (f h)::(map (f, t))
let addN (n, l) =
 let add x = n + x in
   map (add, l)
```




```
addN (3, [1; 2; 3])
```

- Uh oh...how does `add` know the value of `n`?
  - Dynamic scoping: it reads it off the stack
    - The language could do this, but can be confusing (see above)
  - OCaml uses static scoping like C, C++, Java, and Ruby

Static scoping

- In static or lexical scoping, (nonlocal) names refer to their nearest binding in the program text
  - Going from inner to outer scope
  - In our example, **`add`** refers to **`addN`**'s **`n`**
  - C example:

Refers to the **`x`** at file scope – that's the nearest **`x`** going from inner scope to outer scope in the source code

```
int x;
void f() { x = 3; }
void g() { char *x = "hello"; f(); }
```

Summary

- Tuples
- Recursion
- Higher order functions

Discussion tomorrow

- Return Quizzes
- Ocaml examples (posted on schedule)