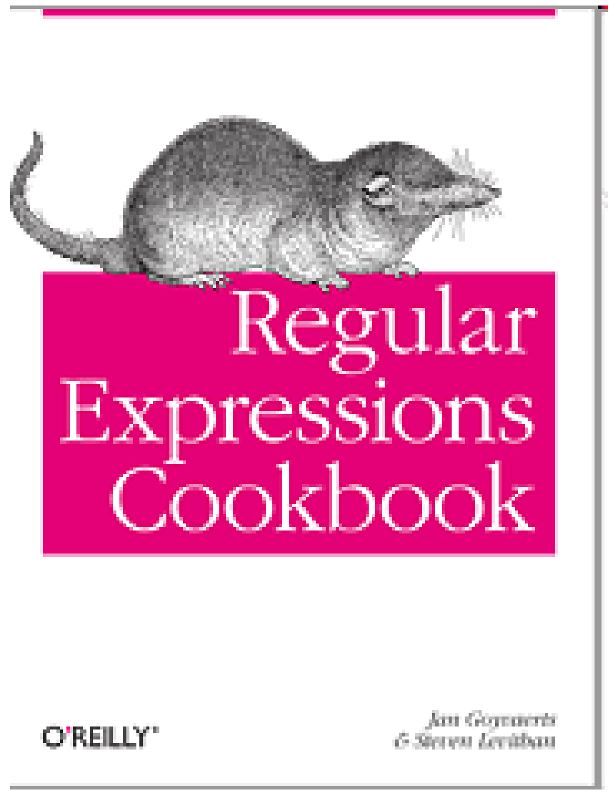
CMSC330

Regular Expressions

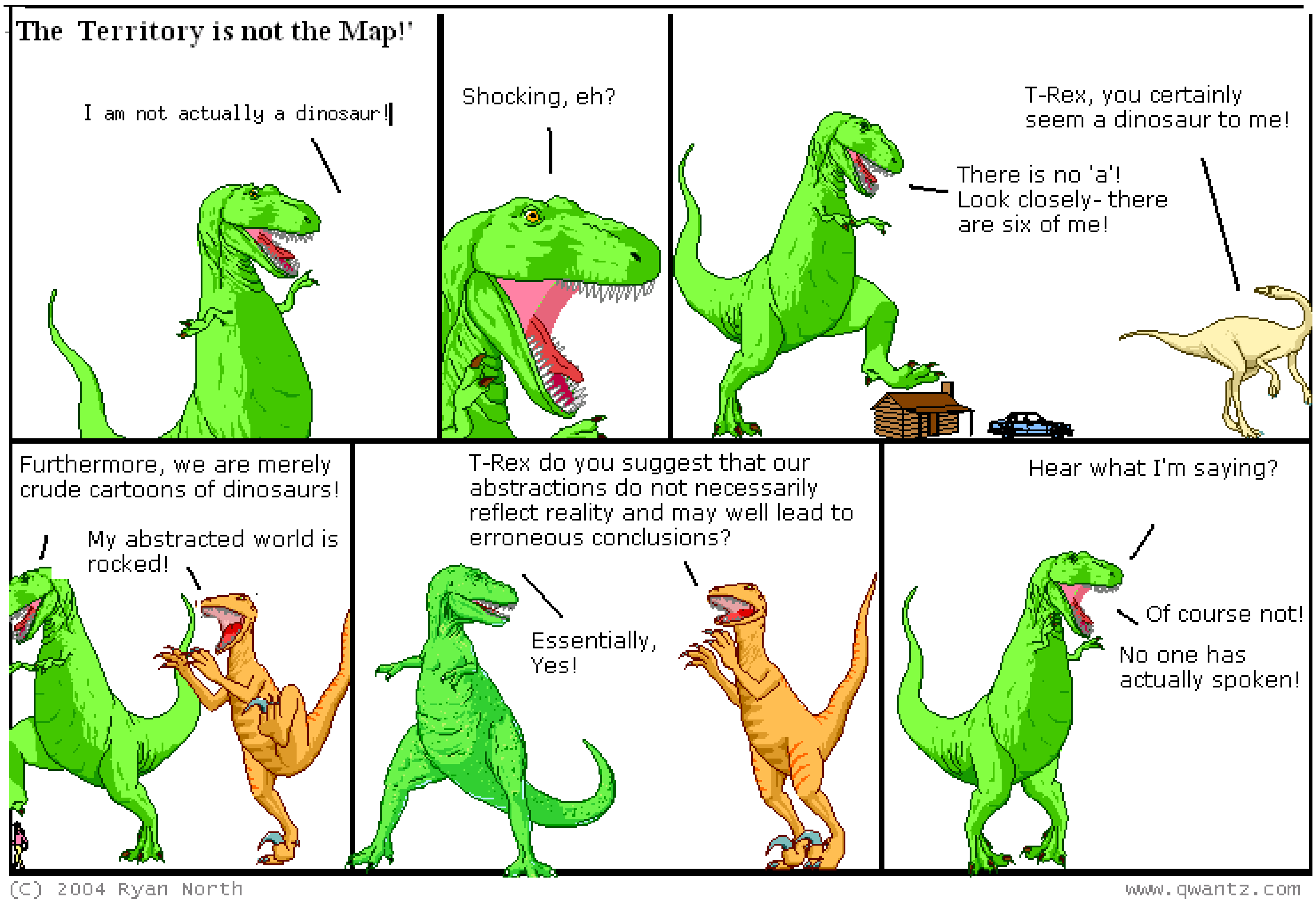
Last time

- Ruby language
  - Regular expressions
  - Arrays
  - Code blocks
  - Hash
  - File
  - Exceptions



This time

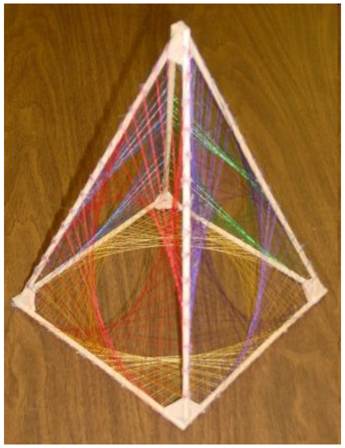
- Regular Expression Theory



Introduction

- That's it for the basics of Ruby
  - If you need other material for your project, come to office hours or check out the documentation
- Next up: How do regular expressions (REs) really work?
  - Mixture of a very practical tool (string matching with REs) and some nice theory
  - A great computer science result

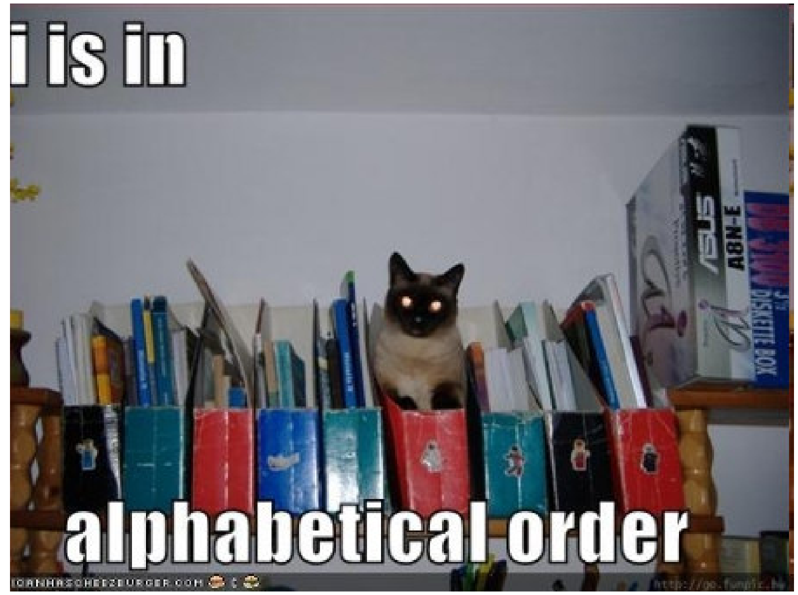
A Few Questions About REs



- What does a regular expression represent?
  - Just a set of strings
- What are the basic components of REs?
  - E.g., we saw that `e+` is the same as `ee*`
- How are REs implemented?
  - We'll see how to build a structure to parse REs

Definition: Alphabet

- An alphabet is a finite set of symbols
  - Usually denoted Σ
- Example alphabets:
  - Binary: $\Sigma = \{0,1\}$
  - Decimal: $\Sigma = \{0,1,2,3,4,5,6,7,8,9\}$
  - Alphanumeric: $\Sigma = \{0\text{-}9, a\text{-}z, A\text{-}Z\}$

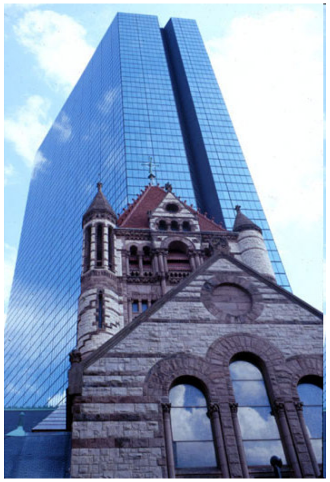


Definition: String

- A string is a finite sequence of symbols from Σ
  - ε is the empty string ("" in Ruby)
  - |s| is the length of string s
    - |Hello| = 5, |ε| = 0
  - Note: Ø is the empty set (with 0 elements); Ø ≠ { ε }
- Example strings:
  - 0101 is an element of Σ = {0,1} (binary), decimal, and alphanumeric alphabets

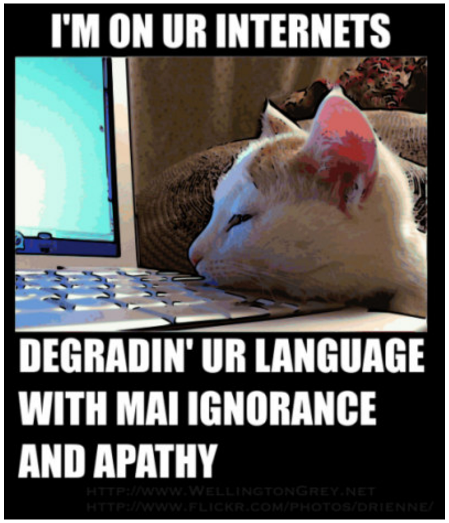
Definition: Concatenation

- Concatenation is indicated by juxtaposition
  - If s1 = super and s2 = hero, then s1s2 = superhero
  - Sometimes also written s1·s2
  - For any string s, we have $s\varepsilon = \varepsilon s = s$
  - You can concatenate strings from different alphabets, then the new alphabet is the union of the originals:
    - If s1 = super ∈ Σ1 = {s,u,p,e,r} and s2 = hero ∈ Σ2 = {h,e,r,o}, then s1s2 = superhero ∈ Σ3 = {e,h,o,p,r,s,u}



Definition: language

- A language is a set of strings over an alphabet
- Example: The set of phone numbers over the alphabet Σ = {0, 1, 2, 3, 4, 5, 6, 7, 9, (,), -}
  - Give an example element of this language
  - Are all strings over the alphabet in the language?
  - Is there a Ruby regular expression for this language?
- Example: The set of all strings over Σ
  - Often written Σ^*

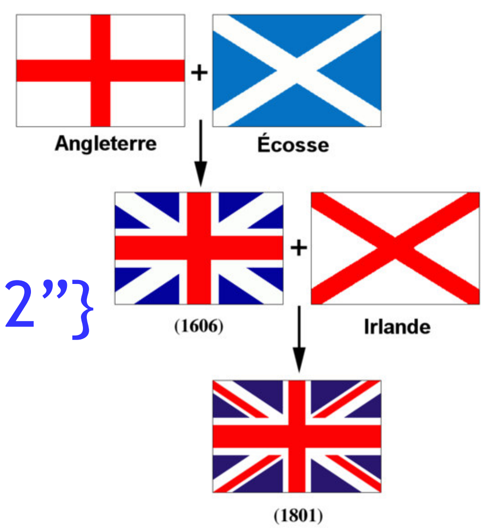


Definition: Language

- Example: The set of strings of length 0 over the alphabet $\Sigma = \{a, b, c\}$
  - $\{s \mid s \in \Sigma^* \text{ and } |s| = 0\} = \{\varepsilon\} \neq \emptyset$
- Example: The set of all valid Ruby programs
  - Is there a Ruby regular expression for this language?
- Can REs represent all possible languages?
  - The answer turns out to be no!
  - The languages represented by regular expressions are called, appropriately, the regular languages

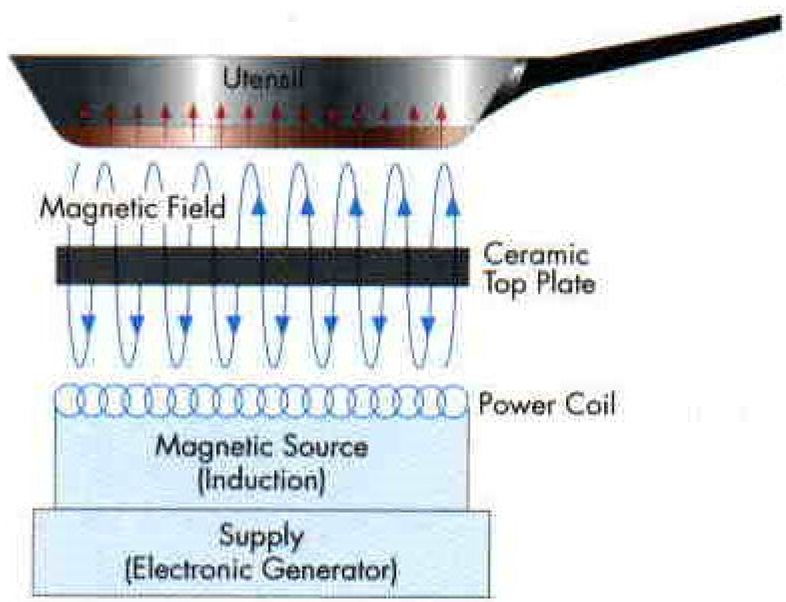
Operations on Languages

- Let Σ be an alphabet and let L, L_1, L_2 be languages over Σ
- Concatenation L_1L_2 is defined as
  - $L_1L_2 = \{xy \mid x \in L_2 \text{ and } y \in L_2\}$
  - Example: L_1 = {"hi", "bye"}, L_2 = {"1", "2"}
    - L_1L_2 = {"hi1", "hi2", "bye1", "bye2"}
- Union is defined as
  - $L_1 \cup L_2 = \{ x \mid x \, \varepsilon \, L_1 \text{ or } x \in L_2\}$
  - Example: L_1 = {"hi", "bye"}, L_2 = {"1", "2"}
    - $L_1 \cup L_2$ = {"hi", "bye", "1", "2"}



Operations on Languages

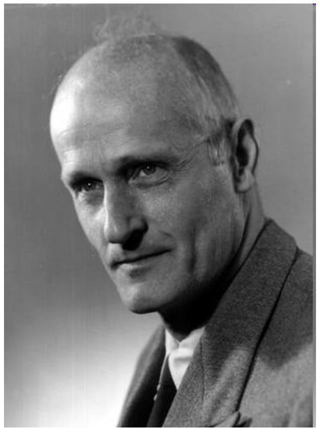
- Define L^n inductively as
  - $L^0 = \{\varepsilon\}$
  - $L^n = LL^{n-1}$ for $n > 0$
- In other words,
  - $L^1 = LL^0 = L\{\varepsilon\} = L$
  - $L^2 = LL^1 = LL$
  - $L^3 = LL^2 = LLL$



Examples of L^n

- Let $L = \{a, b, c\}$
- Then
  - $L^0 = \{\varepsilon\}$
  - $L^1 = \{a, b, c\}$
  - $L^2 = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$

Operations on Languages



- Kleene closure is defined as
  - $L^* = \bigcup_{i \in [0..\infty]} L^i$
- In other words...
  - L^* is the language (set of all strings) formed by concatenating together zero or more strings from L

Definition: Regular Expressions

- Given an alphabet Σ, the regular expressions over Σ are defined inductively as

regular expression	denotes language
$\emptyset$	$\emptyset$
ε	$\{\varepsilon\}$
each element $\sigma \in \Sigma$	$\{\sigma\}$

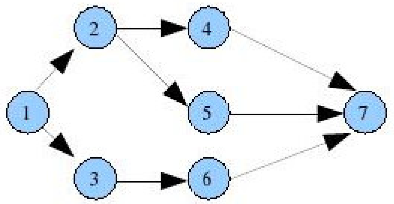
Definition: Regular Expressions

- Let A and B be regular expressions denoting languages L^A and L^B, respectively

regular expression	denotes language
AB	$L_A L_B$
(A|B)	$L_A \cup L_B$
A*	$L_A{}^*$

- There are no other regular expressions over Σ

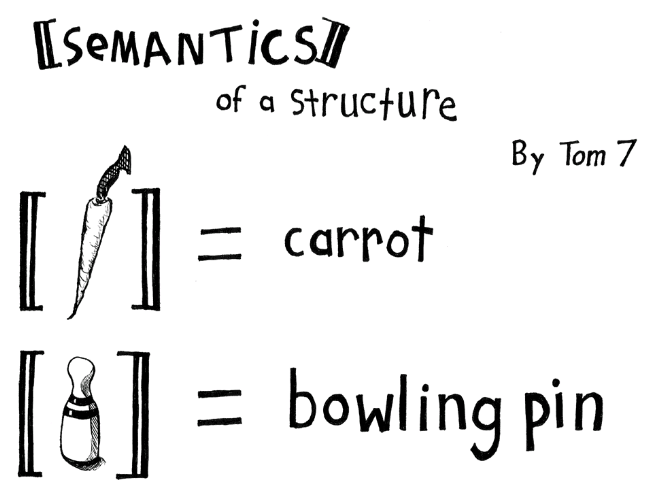
Precedence



- Order in which operators are applied
  - In arithmetic
    - Multiplication > addition
    - 2 × 3 + 4 = (2 × 3) + 4 = 10
  - In regular expressions
    - Kleene closure * > concatenation > union
    - ab|c = (a b) | c = {“ab”, “c”}
    - ab* = a (b*) = {“a”, “ab”, “abb”...}
    - a|b* = a | (b*) = {“a”, “”, “b”, “bb”, “bbb”...}
  - Can change order using parentheses ( )
    - E.g., a(b|c), (ab)*, (a|b)*

The Language Denoted by an RE

- For a regular expression e, we will write [[e]] to mean the language denoted by e
  - [[a]] = {a}
  - [[(a|b)]] = {a, b}
- If s ∈ [[RE]], we say that RE *accepts*, *describes*, or *recognizes* s



Example 1

- All strings over $\Sigma = \{a, b, c\}$ such that all the a's are first, the b's are next, and the c's last
  - Example: aaabbbbccc but not abcb
- Regexp:
  - This is a valid regexp because:
    - a is a regexp ([[a]] = {a})
    - a* is a regexp ([[a*]] = {ε, a, aa, ...})
    - Similarly for b* and c*
    - So a*b*c* is a regular expression
    - (Remember that we need to check this way because regular expressions are defined inductively.)

Which Strings Does a*b*c* Recognize?

- aabbbcc
  - Yes; aa ∈ [[a*]], bbb ∈ [[b*]], and cc ∈ [[c*]], so entire string is in [[a*b*c*]]
- abb
  - Yes, abb = abbε, and ε ∈ [[c*]]
- ac
  - Yes
- ε
  - Yes
- aacbc
  - No
- abcd
  - No -- outside the language

Example 2

- All strings over $\Sigma = \{a, b, c\}$
- Regexp: (a|b|c)*
- Other regular expressions for the same language?
  - (c|b|a)*
  - (a*|b*|c*)*
  - (a*b*c*)*
  - ((a|b|c)*|abc)
  - etc.

Example 3

- All whole numbers containing the substring 330
  – Regular expression: (0|1|…|9)*330(0|1|…|9)*
- What if we want to get rid of leading 0's?
  – ((1|…|9)(0|1|…|9)*330(0|1|…|9)* | 330(0|1|…|9)*)
- Any other solutions?
- Challenge: What about all whole numbers not containing the substring 330?
  – Is it recognized by a regexp?
  – Yes. We'll see how to find it later…

Example 4

- What is the English description for the language that (10|0)*(10|1)* denotes?
  - (10|0)*
    - 0 may appear anywhere
    - 1 must always be followed by 0
  - (10|1)*
    - 1 may appear anywhere
    - 0 must always be preceded by 1
  - Put together, all strings of 0's and 1's where every pair of adjacent 0's precedes any pair of adjacent 1's
    - i.e., no 00 may appear after 11

What Strings are in (10|0)*(10|1)* ?

- 00101000 110111101
  - First part in [[(10|0)*]]
  - Second part in [[(10|1)*]]
  - Notice that 0010 also in [[(10|0)*]]
    - But remainder of string is not in [[(10|1)*]]
- 0010101
  - Yes
- 101
  - Yes
- 011001
  - No

Example 5

- What language does this regular expression recognize?
  - ((1|ε)(0|1|…|9) | (2(0|1|2|3))) : (0|1|…|5)(0|1|…|9)
    - All valid times written in 24-hour format
  - 10:17
  - 23:59
  - 0:45
  - 8:30

Two more examples

- (000|00|1)*
  - Any string of 0's and 1's with no single 0's
- (00|0000)*
  - Strings with an even number of 0's
  - Notice that some strings can be accepted more than one way
    - 000000 = 00·00·00 = 00·0000 = 0000·00
  - How else could we express this language?
    - (00)*
    - (00|000000)*
    - (00|0000|000000)*
    - etc

Regular Languages

- The languages that can be described using regular expressions are the regular languages or regular sets
- Not all languages are regular
  - Examples (without proof):
    - The set of palindromes over Σ
  - reads the same backward or forward
    - $\{a^n b^n \mid n > 0\}$ (a^n = sequence of n a's)
- Almost all programming languages are not regular
  - But aspects of them sometimes are (e.g., identifiers)
  - Regular expressions are commonly used in parsing tools

Ruby Regular Expressions

- Almost all of the features we've seen for Ruby REs can be reduced to this formal definition
  - **/Ruby/** – concatenation of single-character REs
  - **/(Ruby|Regular)/** – union
  - **/(Ruby)*/** – Kleene closure
  - **/(Ruby)+/** – same as (Ruby)(Ruby)*
  - **/(Ruby)?/** – same as (ε|(Ruby)) (// is ε)
  - **/[a-z]/** – same as (a|b|c|...|z)
  - **/[^0-9]/** – same as (a|b|c|...) ∈ for a,b,c,... ∈ Σ - {0..9}
  - **^**, **$** – correspond to extra characters in alphabet

Summary

- Languages
  - Sets of strings
  - Operations on languages
- Regular expressions
  - Constants
  - Operators
  - Precedence