

Information CMSC251

Asymptotic Notations.

$\Theta(g(n)) = \{f(n): \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0\}.$

$O(g(n)) = \{f(n): \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq c g(n) \text{ for all } n \geq n_0\}.$

$\Omega(g(n)) = \{f(n): \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq c g(n) \leq f(n) \text{ for all } n \geq n_0\}.$

$f(n) = o(g(n))$ if $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0.$

$f(n) = \omega(g(n))$ if $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty.$

$f(n) \sim g(n)$ if $f(n) = g(n) + o(g(n)).$

Logarithms.

$$\begin{array}{llll} a = b^{\log_b a} & \log_c(ab) = \log_c a + \log_c b & \log_b a^n = n \log_b a \\ \log_b a = \frac{\log_c a}{\log_c b} & \log_b(1/a) = -\log_b a & \log_b a = \frac{1}{\log_a b} & a^{\log_b n} = n^{\log_b a} \end{array}$$

Stirling's Formula.

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

Recurrences "Master Theorem":

$$T(n) = \begin{cases} aT(n/b) + cn^d & n > 1 \\ f & n = 1 \end{cases}$$

implies

$$T(n) = \begin{cases} \left(f + \frac{c}{ab^{-d}-1}\right) n^{\log_b a} - \frac{cn^d}{ab^{-d}-1} & \begin{cases} \Theta(n^{\log_b a}) & a > b^d \\ \Theta(n^d) & a < b^d \end{cases} \\ n^d(f + c \log_b n) = \Theta(n^d \log_b n) & a = b^d \end{cases}.$$

Summing solutions: If

$$T(n) = \begin{cases} aT(n/b) + \sum c_i n^{d_i} & n > 1 \\ f & n = 1 \end{cases}$$

then we can just sum the solutions of each recurrence:

$$T_i(n) = \begin{cases} aT_i(n/b) + c_i n^{d_i} & n > 1 \\ 0 & n = 1 \end{cases}$$

and add in $f n^{\log_b a}$ for the contribution from the leaves.

Summations.

Distribution law:

$$\left(\sum_{i=1}^m a_i\right) \left(\sum_{j=1}^n b_j\right) = \sum_{i=1}^m \left(\sum_{j=1}^n a_i b_j\right)$$

Interchanging order of summation:

$$\sum_{i=1}^m \sum_{j=1}^n a_{ij} = \sum_{j=1}^n \sum_{i=1}^m a_{ij}$$

Splitting range:

$$\sum_{k=1}^n a_k = \sum_{k=1}^r a_k + \sum_{k=r+1}^n a_k$$

Arithmetic series:

$$\sum_{k=1}^n k = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Geometric series:

$$\sum_{k=0}^n x^k = 1 + x + x^2 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1} \quad x \neq 1$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad x < 1$$

Harmonic series:

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} = \sum_{k=1}^n \frac{1}{k} = \ln n + O(1)$$

Telescoping series:

$$\sum_{k=1}^n (a_k - a_{k-1}) = a_n - a_0$$

Products:

$$\prod_{k=1}^n a_k = a_1 a_2 \cdots a_n \quad \log \prod_{k=1}^n a_k = \sum_{k=1}^n \log a_k$$

Approximation by integrals:

$$\int_{m-1}^n f(x) dx \leq \sum_{k=m}^n f(k) \leq \int_m^{n+1} f(x) dx \quad \text{for } f(x) \text{ monotonically increasing}$$

$$\int_m^{n+1} f(x) dx \leq \sum_{k=m}^n f(k) \leq \int_{m-1}^n f(x) dx \quad \text{for } f(x) \text{ monotonically decreasing}$$

Quadratic Formula.

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Probability.

$$\mathbb{E}[X] = \sum_x x \Pr\{X = x\}, \quad \text{Var}[X] = \mathbb{E}[(X - \mathbb{E}(X))^2] = \mathbb{E}[X^2] - \mathbb{E}^2[X], \quad \sigma[X] = \sqrt{\text{Var}[X]}.$$