CMSC 132: OBJECT-ORIENTED PROGRAMMING II

Algorithmic Complexity II

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Announcements

- It has been brought up to our attention that some students are:
  - Collaborating together on projects that are closed
  - Students are checking online for project solutions/hints or for help from others
- We want to remind you that this is not allowed and it represents academic dishonesty. Keep in mind that projects allow you to learn concepts we cover in quizzes and exams. It is in your best interest to do projects by yourself.

Analyzing Algorithms

- Goal
  - Find asymptotic complexity of algorithm
- Approach
  - Ignore less frequently executed parts of algorithm
  - Find critical section of algorithm
  - Determine how many times critical section is executed as function of problem size

Critical Section of Algorithm

- Heart of algorithm
- Dominates overall execution time
- Characteristics
  - Operation central to functioning of program
  - Usually contained inside deeply nested loops
- Sources
  - Loops
  - Recursion

Critical Section Example 1

- Code (for input size n)

```
1. A
2. for (int i = 0; i < n; i++) {
3.     B
4. }
5. C
```

critical section

- Code execution
  - A ⇒ once
  - B ⇒ n times
  - C ⇒ once
- Time ⇒ 1 + n + 1 = O(n)

Critical Section Example 2

- Code (for input size n)

```
1.  A
2.  for (int i = 0; i < n; i++) {
3.     B
4.     for (int j = 0; j < n; j++) {
5.        C
6.     }
7.  }
8.  D
```

critical section

- Code execution
  - A ⇒ once
  - B ⇒ n times
  - C ⇒ n^2 times
  - D ⇒ once
- Time ⇒ $1 + n + n^2 + 1 = O(n^2)$

Critical Section Example 3

- Code (for input size n)

```
1.  A
2.  for (int i = 0; i < n; i++) {
3.      for (int j = i+1; j < n; j++) {
4.          B
5.      }
6.  }
```

critical section

- Code execution
  - A ⇒ once
  - B ⇒ ½ n (n-1) times
- Time ⇒ $1 + \frac{1}{2} n^2 - \frac{1}{2} n = O(n^2)$

Critical Section Example 4

- Code (for input size n)

```
1.  A
2.  for (int i = 0; i < n; i++) {
3.      for (int j = 0; j < 10000; j++) {
4.          B
5.      }
6.  }
```

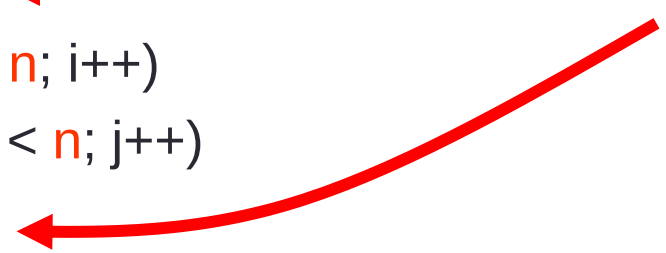
critical section

- Code execution
  - A ⇒ once
  - B ⇒ 10000 n times
- Time ⇒ 1 + 10000 n = O(n)

Critical Section Example 5

- Code (for input size n)

```
1.  for (int i = 0; i < n/2; i++) {
2.    for (int j = 0; j < n/2; j++)
3.      A
4.  for (int i = 0; i < n; i++)
5.    for (int j = 0; j < n; j++)
6.      B
```



critical sections

- Code execution
  - A $\Rightarrow$ n^2 times
  - B $\Rightarrow$ n^2 times
- Time $\Rightarrow$ $n^2 + n^2 = O(n^2)$

Critical Section Example 6

- Code (for input size n)

```
1.  i = 1
2.  while (i < n) {
3.    A
4.    i = 2 × i
      }
5.  B
```

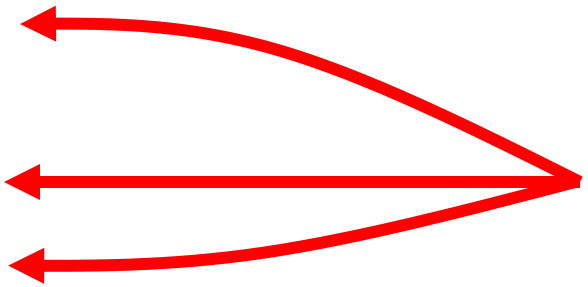
critical section

- Code execution
  - i = 1 ⇒ 1 times
  - A ⇒ log(n) times
  - B ⇒ 1 times
- Time ⇒ 1 + log(n) + 1 = O(log(n))

Critical Section Example 7 (Recursion)

- Code (for input size n)

```
1.  DoWork (int n)
2.    if (n == 1)
3.      A
4.    else {
5.      DoWork(n/2)
6.      DoWork(n/2)
7.    }
```



critical sections

- Code execution
  - A ⇒ 1 times
  - DoWork(n/2) ⇒ 2 times
- Time(1) ⇒ 1 Time(n) = 2 × Time(n/2) + 1

Comparing Complexity

- Compare two algorithms
  - f(n), g(n)
- Determine which increases at faster rate
  - As problem size n increases
- Can compare ratio
  - If ∞, f() is larger
  - If 0, g() is larger
  - If constant, then same complexity

$$\lim_{n \to \infty} \frac{f(n)}{g(n)}$$

- Example (**log(n) vs. n½**)

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} \rightarrow \lim_{n \to \infty} \frac{\log(n)}{n^{1/2}} \rightarrow 0$$

Additional Complexity Measures

- Upper bound
  - Big-O $\Rightarrow$ $O(\ldots)$
  - Represents upper bound on # steps
- Lower bound
  - Big-Omega $\Rightarrow$ $\Omega(\ldots)$
  - Represents lower bound on # steps

2D Matrix Multiplication Example

- Problem
  - C = A * B
- Lower bound
  - $\Omega(n^2)$ Required to examine 2D matrix
- Upper bounds
  - $O(n^3)$ Basic algorithm
  - $O(n^{2.807})$ Strassen's algorithm (1969)
  - $O(n^{2.376})$ Coppersmith & Winograd (1987)
- Improvements still possible (open problem)
  - Since upper & lower bounds do not match