

CMSC 132: OBJECT-ORIENTED PROGRAMMING II



Algorithmic Complexity II

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Analyzing Algorithms

- **Goal**

- Find asymptotic complexity of algorithm

- **Approach**

- Ignore less frequently executed parts of algorithm
- Find **critical section** of algorithm
- Determine how many times critical section is executed as function of problem size

Critical Section of Algorithm

- Heart of algorithm
- Dominates overall execution time
- Characteristics
 - Operation central to functioning of program
 - Usually contained inside deeply nested loops
- Sources
 - Loops
 - Recursion

Critical Section Example 1

- Code (for input size n)

```
1.  A
2.  for (int i = 0; i <  $n$ ; i++) {
3.      B
4.  }
5.  C
```

**critical
section**



- Code execution
 - $A \Rightarrow$ once
 - $B \Rightarrow n$ times
 - $C \Rightarrow$ once
- Time $\Rightarrow 1 + n + 1 = O(n)$

Critical Section Example 2

- Code (for input size n)

```
1.  A
2.  for (int i = 0; i <  $n$ ; i++) {
3.      B
4.      for (int j = 0; j <  $n$ ; j++) {
5.          C
6.      }
7.  }
8.  D
```



**critical
section**

- Code execution
 - $A \Rightarrow$ once
 - $B \Rightarrow n$ times
 - $C \Rightarrow n^2$ times
 - $D \Rightarrow$ once
- Time $\Rightarrow 1 + n + n^2 + 1 = O(n^2)$

Critical Section Example 3

- Code (for input size n)

```
1.  A
2.  for (int i = 0; i < n; i++) {
3.      for (int j = i+1; j < n; j++) {
4.          B
5.      }
6.  }
```

**critical
section**

- Code execution
 - $A \Rightarrow$ once
 - $B \Rightarrow \frac{1}{2} n (n-1)$ times
- Time $\Rightarrow 1 + \frac{1}{2} n^2 - \frac{1}{2} n = O(n^2)$

Critical Section Example 4

- Code (for input size n)

```
1.  A
2.  for (int i = 0; i <  $n$ ; i++) {
3.      for (int j = 0; j < 10000; j++) {
4.          B
5.      }
6.  }
```

**critical
section**

- Code execution
 - $A \Rightarrow$ once
 - $B \Rightarrow$ 10000 n times
- Time $\Rightarrow 1 + 10000 n = O(n)$

Critical Section Example 5

- Code (for input size n)

```
1.  for (int i = 0; i <  $n/2$ ; i++)
2.      for (int j = 0; j <  $n/2$ ; j++)
3.          A
4.  for (int i = 0; i <  $n$ ; i++)
5.      for (int j = 0; j <  $n$ ; j++)
6.          B
```

**critical
sections**

- Code execution

- $A \Rightarrow n^2/4$ times
- $B \Rightarrow n^2$ times

- Time $\Rightarrow n^2/4 + n^2 = O(n^2)$

Critical Section Example 6

- Code (for input size n)

```
1.  i = 1
2.  while (i < n) {
3.      A
4.      i = 2 × i
    }
5.  B
```



**critical
section**

- Code execution

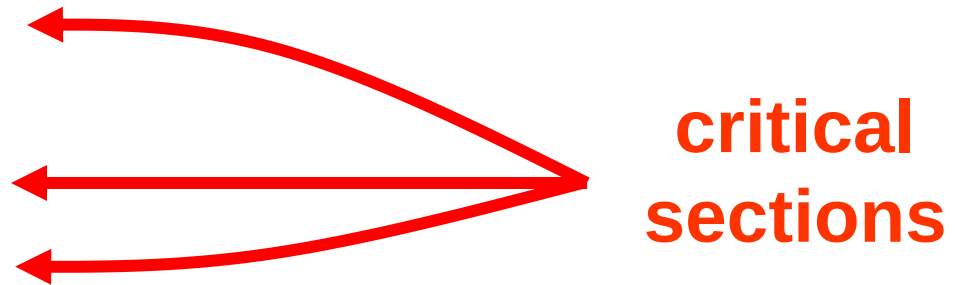
- $i = 1 \Rightarrow 1$ times
- $A \Rightarrow \log(n)$ times
- $B \Rightarrow 1$ times

- Time $\Rightarrow 1 + \log(n) + 1 = O(\log(n))$

Critical Section Example 7 (Recursion)

- Code (for input size **n**)

```
1. DoWork (int n)
2.   if (n == 1)
3.     A
4.   else {
5.     DoWork(n/2)
6.     DoWork(n/2)
7.   }
```



- Code execution

- $A \Rightarrow 1$ times
- $\text{DoWork}(n/2) \Rightarrow 2$ times

- $\text{Time}(1) \Rightarrow 1$ $\text{Time}(n) = 2 \times \text{Time}(n/2) + 1$

Comparing Complexity

- Compare two algorithms
 - $f(n)$, $g(n)$
- Determine which increases at faster rate
 - As problem size n increases
- Can compare ratio
 - If ∞ , $f()$ is larger
 - If 0, $g()$ is larger
 - If constant, then same complexity
- Example (**$\log(n)$ vs. $n^{1/2}$**)

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)}$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} \rightarrow \lim_{n \rightarrow \infty} \frac{\log(n)}{n^{1/2}} \rightarrow 0$$

Additional Complexity Measures

- Upper bound
 - Big-O $\Rightarrow O(\dots)$
 - Represents upper bound on # steps
- Lower bound
 - Big-Omega $\Rightarrow \Omega(\dots)$
 - Represents lower bound on # steps

2D Matrix Multiplication Example

- Problem
 - $C = A * B$
- Lower bound
 - $\Omega(n^2)$ Required to examine 2D matrix
- Upper bounds
 - $O(n^3)$ Basic algorithm
 - $O(n^{2.807})$ Strassen's algorithm (1969)
 - $O(n^{2.376})$ Coppersmith & Winograd (1987)
- Improvements still possible (open problem)
 - Since upper & lower bounds do not match