Summer 2013 **CMSC 351: Homework 7** Clyde Kruskal

Due at the start of class, Wednesday, June 26.

Problem 1. In class, we solved the selection problem by breaking the list into groups of 5 elements each.

(a) We used the fact that you can find the median of 5 numbers with 10 comparisons (by sorting). It turns out that you can find the median with only 6 comparisons.

  (i) Write down the recurrence for the running time using this new fact. (You can ignore floors and ceilings, as we did in class.)

  (ii) Solve the recurrence.

(b) (i) How may comparisons do you need to find the median of 3 elements? Justify your answer.

  (ii) Write down the recurrence for a selection algorithm based on columns with three elements each. (You can ignore floors and ceilings, as we did in class.)

  (iii) Solve the recurrence.

(c) You need to 10 comparisons to find the median of 7 elements?

  (i) Write down the recurrence for a selection algorithm based on columns with 7 elements each. (You can ignore floors and ceilings, as we did in class.)

  (ii) Solve the recurrence.

(d) What did you learn?

Problem 2. Assume we have n distinct elements $x_1, x_2, \ldots, x_n$ with non-negative weights $w_1, w_2, \ldots, w_n$, such that $\sum_{i=1}^{n} w_i = 1$. The *weighted middle element* is the element x_k satisfying

$$\sum_{x_i < x_k} w_i < \frac{1}{2} \qquad \text{and} \qquad \sum_{x_i > x_k} w_i \le \frac{1}{2}$$

(a) Show how to compute the weighted middle element in $\Theta(n \log n)$ time using sorting.

(b) Show how to compute the weighted middle element in $\Theta(n)$ time using a linear time selection algorithm.

Problem 3. Challenge problem. Show how to find the median of 5 numbers with only 6 comparisons.