Problem 1. Consider the Boolean Formula

$$A \equiv (B \wedge C)\ .$$

Write it in Conjunctive Normal Form (CNF). Explain how you found this formula.

Problem 2. A formula is in Disjunctive Normal Form (DNF) if it consists of the disjunction of a set of clauses, where each clause is the conjunction of literals. For example:

$$(A \wedge \bar{B} \wedge C) \vee (B \wedge \bar{C} \wedge D \wedge E) \vee (\bar{A} \wedge C \wedge \bar{D}) \vee (A \wedge \bar{B} \wedge D \wedge \bar{E})\ .$$

The DNF SAT problem is: Given a Boolean Formula in Disjunctive Normal Form (DNF), is it satisfiable.

Is DNF SAT NP-complete? Justify.

Problem 3. A *simple cycle* is a cycle in a graph such that no vertex (on the cycle) is visited more than once. A *Hamiltonian Cycle* is a (simple) cycle that visits every vertex exactly once. The *(Directed) Hamiltonian Cycle problem* is: Given a directed graph $G = (V, E)$ does it have a Hamiltonian Cycle? Show that the Hamiltonian Cycle problem is in NP. Make sure to state what the certificate is.

The Hamiltonian cycle problem is known to be NP-complete, but do not use this fact in solving this problem.

Problem 4. For this problem assume that the graph $G = (V, E)$ is directed?

(a) What is the *optimization* version of the *Longest Simple Cycle Problem*.

(b) What is the *decision* version of the *Longest Simple Cycle Problem*?

(c) Show that the decision version of the Longest Simple Cycle Problem is in NP. Make sure to state what the certificate is.

(d) Show that the decision version of the Longest Simple Cycle Problem is Hard for NP (i.e., that it is NP-complete). [NOTE: There will not be a problem like this (4d) on the final exam.]

(e) Show that if you can solve the optimization version version of longest cycle in polynomial time you can solve the decision in polynomial time.

(f) Show that if you can solve the decision version of longest cycle in polynomial time you can solve the optimization version in polynomial time.