CMSC 132:
OBJECT-ORIENTED PROGRAMMING II

Graph Implementation

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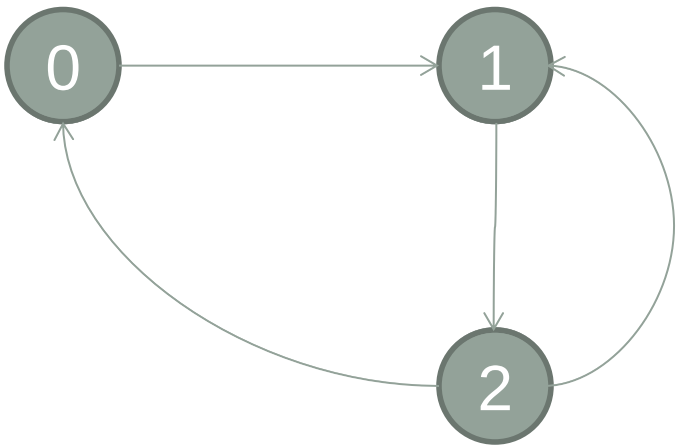
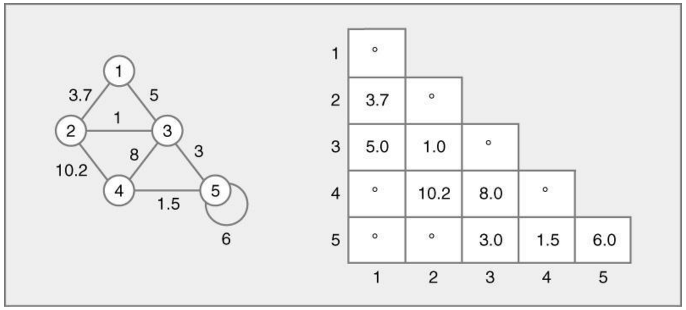
Graph Implementation

- How do we represent nodes/edges?
  - Adjacency matrix
    - 2D array of neighbors
  - Adjacency list/set/map
    - List/set/map of neighbors
- Important for very large graphs
  - Affects efficiency / storage

Adjacency Matrix

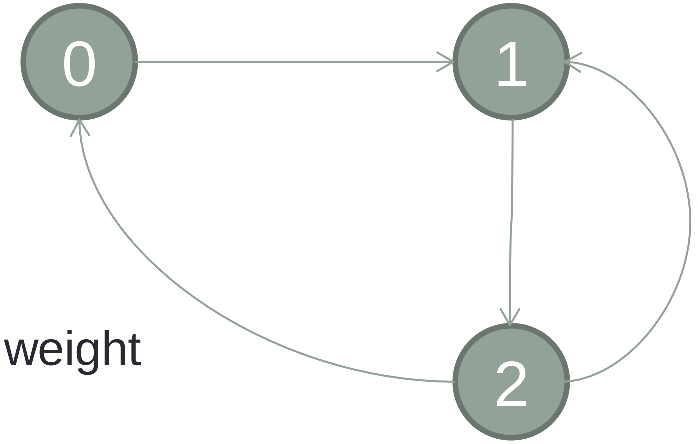
- Single **two-dimensional array** for entire graph
- Directed Graph
  - **Unweighted graph**
    - Matrix elements $\Rightarrow$ boolean
  - **Weighted graph**
    - Matrix elements $\Rightarrow$ values
  - Let's see an example of each
- Undirected Graph
  - Let's see an example for unweighted graph
  - Let's see an example for weighted graph
- For Undirected graph
  - Only upper / lower triangle matrix needed
  - Since n_j, n_k implies n_k, n_j





Adjacency List/Set/Map

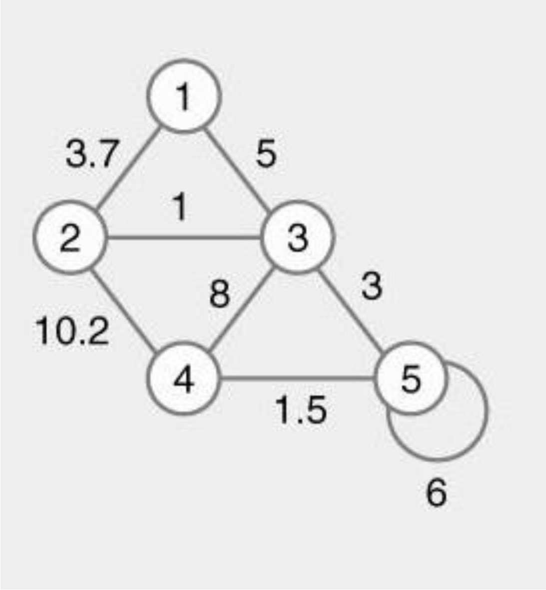
- For each node, **store neighbor information in a list, set, or map**
- The main structure can be a list, set, or map
- Directed Graph
  - **Unweighted graph**
    - List or set of neighbors
  - **Weighted graph**
    - Each entry keeps track of neighbor and weight
    - Easy to implement with maps
      - Maps of Maps (using HashMaps for efficiency)
  - Let's see an example of each
- Undirected Graph
  - Let's see an example for unweighted graph
  - Let's see an example for weighted graph



Additional Examples

- Examples
  - **Unweighted graph**

```
node 1: {2, 3}
node 2: {1, 3, 4}
node 3: {1, 2, 4, 5}
node 4: {2, 3, 5}
node 5: {3, 4, 5}
```



  - **Weighted graph**

```
node 1: {2=3.7, 3=5}
node 2: {1=3.7, 3=1, 4=10.2}
node 3: {1=5, 2=1, 4=8, 5=3}
node 4: {2=10.2, 3=8, 5=1.5}
node 5: {3=3, 4=1.5, 5=6}
```

Graph Properties

- Graph density
  - Ratio edges to nodes (dense vs. sparse)
  - For adjacency matrix many empty entries for large, sparse graph
- Adjacency matrix
  - Can find individual edge (a,b) quickly
  - Examine entry in array Edge[a,b]
    - Constant time operation
- Adjacency list / set / map
  - Can find all edges for node (a) quickly
  - Iterate through collection of edges for a
    - On average E / N edges per node

Complexity

- Average Complexity of operations
  - For graph with N nodes, E edges

Operation	Adj Matrix	Adj List	Adj Set/Map
Find edge	**O(1)**	**O(E/N)**	**O(1)**
Insert edge	**O(1)**	**O(E/N)**	**O(1)**
Delete edge	**O(1)**	**O(E/N)**	**O(1)**
Enumerate edges for node	**O(N)**	**O(E/N)**	**O(E/N)**

Choosing Graph Implementations

- Factors to consider
  - **Graph density**
  - **Graph algorithm**
    - Neighbor based

```
For each node X in graph
    For each neighbor Y of X          // adj list faster if sparse
        doWork( )
```

    - Connection based

```
For each node X in …
    For each node Y in …
        if (X,Y) is an edge  // adj matrix faster if dense
            doWork( )
```