Problem 1. Assume your machine has 32 bit words. Assume you can multiply two n word numbers in time $3n^2$ with a standard algorithm. Assume you can multiply two n word numbers in time $20n^{\lg 3}$ with a "fancy" algorithm.

(a) Approximately, how large does n have to be for the fancy algorithm to be better?

(b) How many bits is that?

(c) How many decimal digits is that?

Problem 2. Use the same assumptions as for problem (1), except assume you can multiply two n word numbers in time only $10n^{\lg 3}$ with a "fancy" algorithm.

(a) Approximately, how large does n have to be for the fancy algorithm to be better?

(b) How many bits is that?

(c) How many decimal digits is that?

Problem 3. Normally it takes nine atomic multiplies to multiply two three-digit numbers. How few atom multiplies do you need to do to obtain an algorithm that is asymptotically better than the $O(n^{\lg 3})$ algorithm. Justify.

Problem 4.

(a) Consider the recurrence:

$$T(n) \;=\; 3T(n-2) + 2^{(n-1)/2}, \quad T(1) = 10$$

Calculate $T(7)$.

(b) Consider the recurrence:

$$T(n) \;=\; 3T(n/2) + 2^{n/2}, \quad T(1) = 10$$

Calculate $T(8)$.