

Problem 1. Consider the following recurrence, defined for n a power of 2:

$$T(n) = \begin{cases} 4 & \text{if } n = 1 \\ 5T(n/2) + 3n^2 & \text{otherwise} \end{cases}$$

- (a) Solve the recurrence exactly using the tree method. Simplify as much as possible.
- (b) Solve the recurrence exactly using the formulas derived in class. Simplify as much as possible.

Problem 2. Consider the following recurrence, defined for n a power of 4:

$$T(n) = \begin{cases} 3 & \text{if } n = 1 \\ 2T(n/4) + 4n + 1 & \text{otherwise} \end{cases}$$

Solve the recurrence exactly using the formulas derived in class. Simplify as much as possible.

Problem 3. Consider an array of size eight with the numbers in the following order 20, 40, 60, 80, 10, 30, 50, 70.

- (a) What is the array after heap formation? How many comparisons does the algorithm use?
- (b) Show the array after each element sifts down *after heap creation*. How many comparisons does the algorithm use for all of the sifts?