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21.1 Sorting

21.1.1 Insertion Sort

Insertion sort sorts the input by inserting each item into a sorted list. Each iteration, insertion sort removes one element from the input data, finds the location it belongs within the sorted list, and inserts it there. It repeats until no input elements remain.

Figure 21.1: Insertion Sort

Listing 1: Insertion Sort

```java
import java.util.Comparator;

/** *
 * @author Robert Sedgewick
 * @author Kevin Wayne
```
Lecture 21:

```java
public class Insertion {
    // This class should not be instantiated.
    private Insertion() {
    }

    /**
     * Rearranges the array in ascending order, using the natural order.
     * @param a the array to be sorted
     */
    public static void sort(Comparable[] a) {
        int N = a.length;
        for (int i = 0; i < N; i++) {
            for (int j = i; j > 0 && less(a[j], a[j - 1]); j--) {
                exch(a, j, j - 1);
            }
        }
    }

    /**
     * Rearranges the array in ascending order, using a comparator.
     * @param a the array
     * @param c the comparator specifying the order
     */
    public static void sort(Object[] a, Comparator c) {
        int N = a.length;
        for (int i = 0; i < N; i++) {
            for (int j = i; j > 0 && less(c, a[j], a[j - 1]); j--) {
                exch(a, j, j - 1);
            }
        }
    }

    /**************************************************************
     * Helper sorting functions
     ***************************************************************/

    // is v < w ?
    private static boolean less(Comparable v, Comparable w) {
        return (v.compareTo(w) < 0);
    }

    // is v < w ?
    private static boolean less(Comparator c, Object v, Object w) {
        return (c.compare(v, w) < 0);
    }

    // exchange a[i] and a[j]
    private static void exch(Object[] a, int i, int j) {
        Object swap = a[i];
        a[i] = a[j];
        a[j] = swap;
    }
```
21.1.2 Selection Sort

Selection sort divides the list into two parts: sorted list (shown as red in Figure 21.2) and unsorted list (shown as black in Figure 21.2). Initially the sorted list is empty. In each iteration, the algorithm finds the smallest (or largest, depending on sorting order) element in the unsorted list, exchanges it with the leftmost unsorted element (putting it in sorted order), and moves the unsorted list boundary one element to the right. It will repeat until the unsorted list becomes empty.
The sort method of selection is shown in Listing 2. “less” and “exch” methods are same as in Insertion Sort code.

Listing 2: Selection Sort

```java
public static void sort(Comparable[] a) {
    int N = a.length;
    for (int i = 0; i < N; i++) {
        int min = i;
        for (int j = i + 1; j < N; j++) {
            if (less(a[j], a[min])) min = j;
        }
        exch(a, i, min);
    }
}
```

### 21.1.3 Merge Sort

Merge sort is a divide and conquer algorithm and it works as follows:
• Divide the unsorted list into \( n \) sublists, each containing 1 element (a list of 1 element is considered sorted) as shown in Figure 21.3.

• Repeatedly merge sublists to produce new sorted sublists until there is only 1 sublist remaining. This will be the sorted list, as show in Figure 21.4.
public class Merge {

    // This class should not be instantiated.
    private Merge() {

    }

    // stably merge a[lo .. mid] with a[mid+1 .. hi] using aux[lo .. hi]
    private static void merge(Comparable[] a, Comparable[] aux, int lo, int mid, int hi) {
        // precondition: a[lo .. mid] and a[mid+1 .. hi] are sorted subarrays
        // copy to aux[]
        for (int k = lo; k <= hi; k++) {
            aux[k] = a[k];
        }

        // merge back to a[]
        int i = lo, j = mid+1;
        for (int k = lo; k <= hi; k++) {
            if (i > mid) a[k] = aux[j++];
            else if (j > hi) a[k] = aux[i++];
            else if (less(aux[j], aux[i])) a[k] = aux[j++];
            else a[k] = aux[i++];
        }
    }

    // mergesort a[lo..hi] using auxiliary array aux[lo..hi]
    private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi) {
        if (hi <= lo) return;
        int mid = lo + (hi - lo) / 2;
        sort(a, aux, lo, mid);
        sort(a, aux, mid + 1, hi);
        merge(a, aux, lo, mid, hi);
    }

    /**
     * Rearranges the array in ascending order, using the natural order.
     * @param a the array to be sorted
     */
    public static void sort(Comparable[] a) {
        Comparable[] aux = new Comparable[a.length];
        sort(a, aux, 0, a.length - 1);
    }

    /******************************************************************************/
    * Helper sorting functions
    /******************************************************************************/

    // is v < w ?
    private static boolean less(Comparable v, Comparable w) {
        return (v.compareTo(w) < 0);
    }
Lecture 21:

```java
private static void exch(Object[] a, int i, int j) {
    Object swap = a[i];
    a[i] = a[j];
    a[j] = swap;
}
```

```java
public static void main(String[] args) {
    String[] a = new String[] {"Alice", "David", "Bob", "Cathy", "Harry");
    Merge.sort(a);
    for (String s : a) {System.out.print(s + "");}
    System.out.println();
}
```

### 21.1.4 Quick Sort

### 21.1.5 Heap Sort

### 21.1.6 Sorting Algorithm Comparison

<table>
<thead>
<tr>
<th>Name</th>
<th>Best</th>
<th>Average</th>
<th>Worst</th>
<th>Memory</th>
<th>Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble Sort</td>
<td>$n$</td>
<td>$n^2$</td>
<td>$n^2$</td>
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<td>yes</td>
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<tr>
<td>Selection Sort</td>
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<td>$n^2$</td>
<td>n2</td>
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<tr>
<td>Insertion Sort</td>
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<td>yes</td>
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<tr>
<td>Merge Sort</td>
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<td>nlogn</td>
<td>nlogn</td>
<td>n</td>
<td>yes</td>
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<tr>
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<td>$n^2$</td>
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<td>Heap Sort</td>
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<td>nlogn</td>
<td>nlogn</td>
<td>1</td>
<td>no</td>
</tr>
</tbody>
</table>

### References

[Algorithms, 4/E] PRINCETON UNIVERSITY, ROBERT SEDGEWICK and KEVIN WAYNE