Big O Notation and Graphs

CMSC 132 - Week 9

Graphs API

- A graph is a pair (V, E), where
  - V is a set of nodes, called vertices
  - E is a collection of pairs of vertices, called edges
  - Vertices and edges can be objects that store some information.
- Example:
  - A vertex represents a
  - An edge represents a

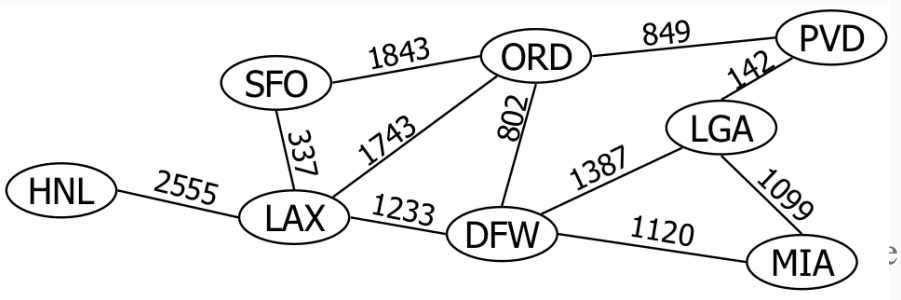
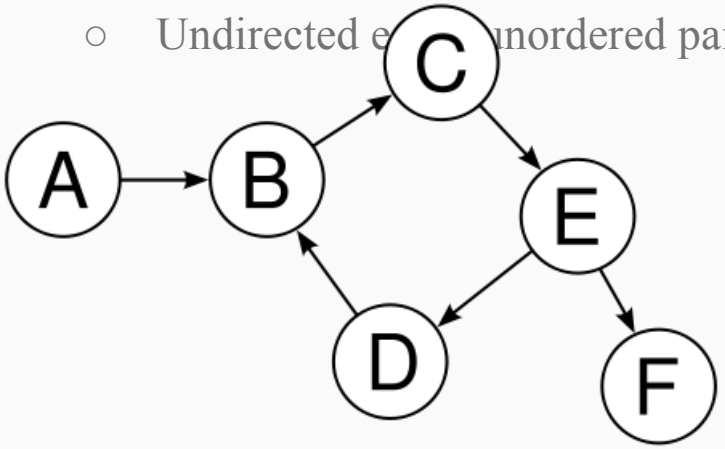


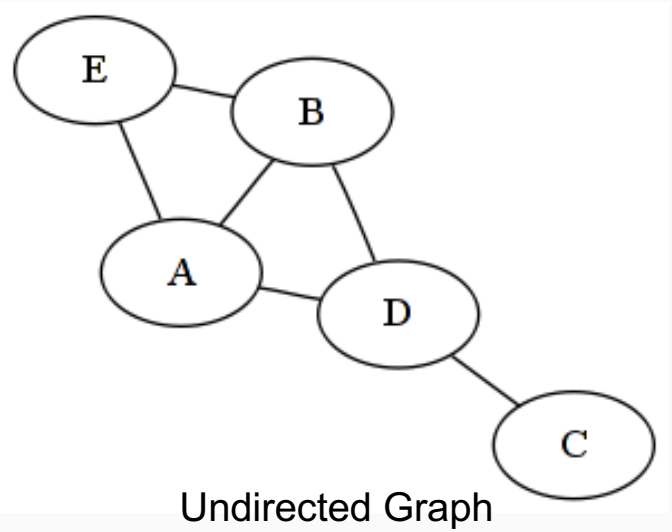
Figure from Dr. Noha Adly's lectures, Alexandria University

Graphs API

- Edge Types:
  - Directed edge: ordered pair of vertices (***u, v***), first vertex is the source
  - Undirected e nordered pair of vertic



Directed Graph



Undirected Graph

Terminology

- End vertices or endpoints of an edge (**u, v**)
- Degree of a vertex in an undirected graph (in-degree and out-degree for directed graphs)
- Parallel edges: ***h*** and ***i*** are parallel edges
- Self-loops: ***j*** is a self-loop

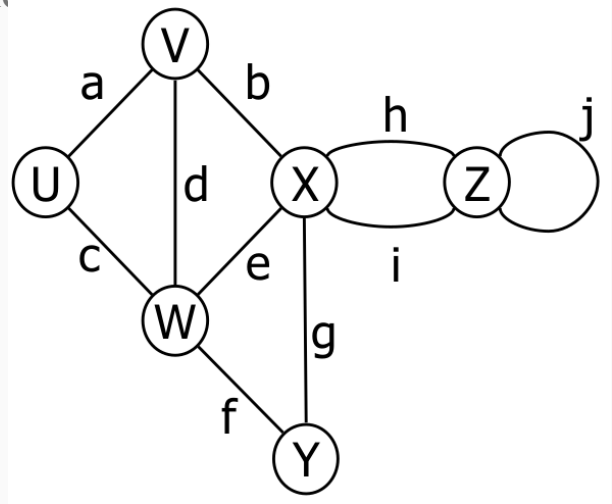
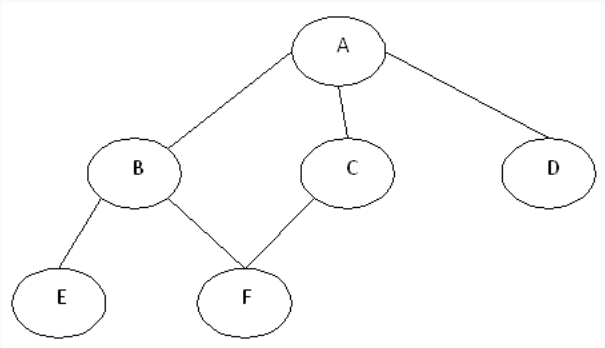
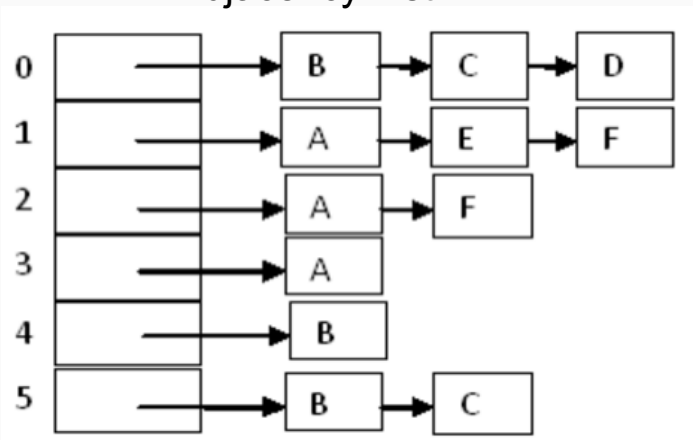


Figure from Dr. Noha Adly's lectures, Alexandria University

Graph Representation



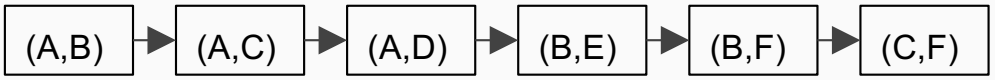
Adjacency List



Adjacency Matrix

	A	B	C	D	E	F
A	0	1	1	1	0	0
B	1	0	0	0	1	1
C	1	0	0	0	0	1
D	1	0	0	0	0	0
E	0	1	0	0	0	0
F	0	1	1	0	0	0

List of Edges



Graph Representation

representation	space	add edge	edge between v and w?	iterate over vertices adjacent to v?
list of edges	E	1	E	E
adjacency matrix	V^2	1 *	1	V
adjacency lists	E + V	1	degree(v)	degree(v)

* disallows parallel edges

Depth-first Search

```
public class DepthFirstPaths
{
   private boolean[] marked;
   private int[] edgeTo;
   private int s;

   public DepthFirstSearch(Graph G, int s)
   {
      ...
      dfs(G, s);
   }

   private void dfs(Graph G, int v)
   {
      marked[v] = true;
      for (int w : G.adj(v))
         if (!marked[w])
         {
            dfs(G, w);
            edgeTo[w] = v;
         }
   }
}
```

marked[v] = true if v connected to s

edgeTo[v] = previous vertex on path from s to v

initialize data structures

find vertices connected to s

recursive DFS does the work

Depth-first Search Recursive

```
for all vertices X
    X.tag= false
create a new queue
enqueue the start vertex in the queue
set the start vertex's tag to true
while (the queue is not empty)
    take a vertex X out of the queue
    process X
    for each neighbor Y of X
        if (Y.tag == false)
            enqueue Y in the queue
            Y.tag= true
```

Depth-first Search

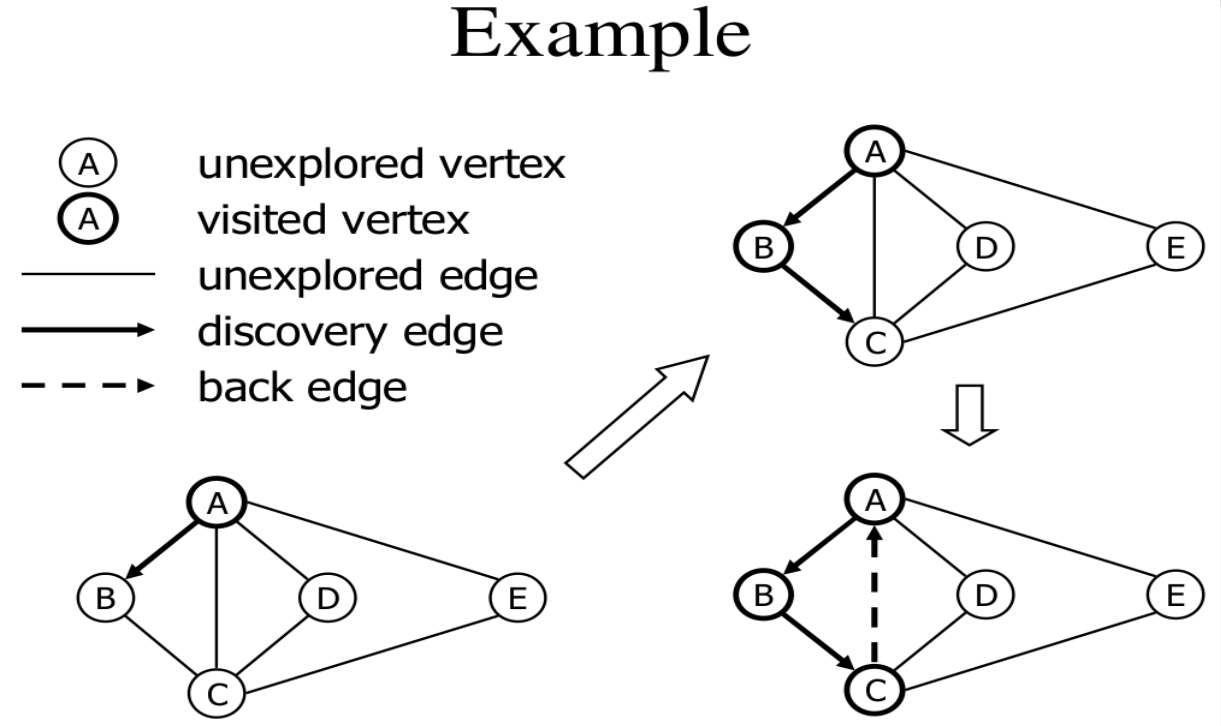


Figure from Dr. Noha Adly's lectures, Alexandria University

Depth-first Search

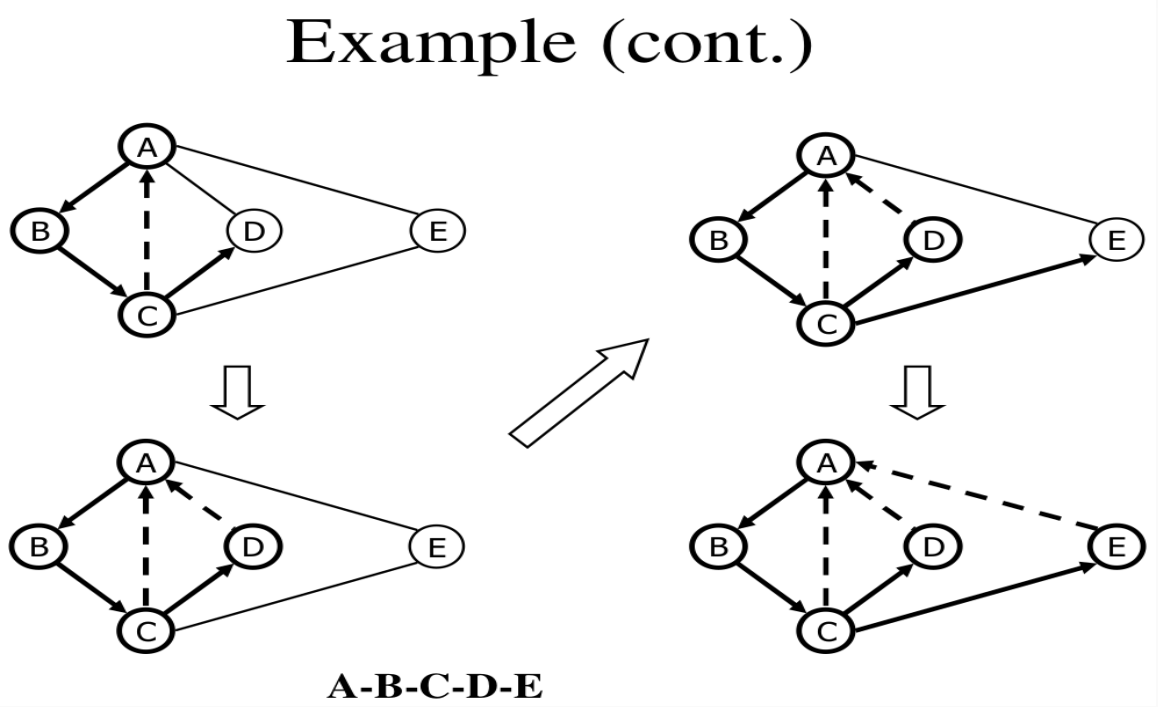
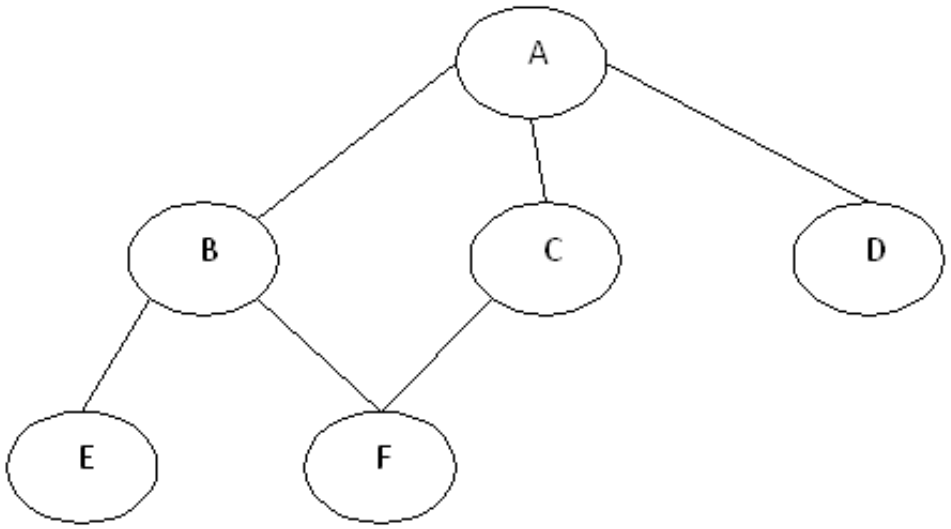


Figure from Dr. Noha Adly's lectures, Alexandria University

Depth-first Nonrecursive Pseudocode

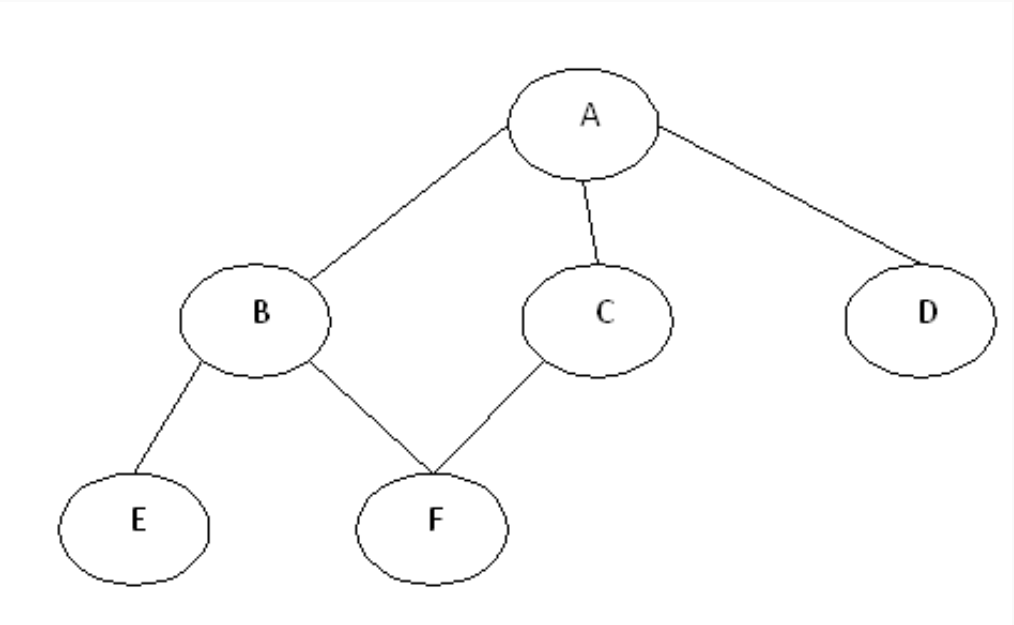
```
for all vertices X
    X.tag= false
create a new stack
push the start vertex onto the stack
set the start vertex's tag to true
while (the stack is not empty)
     pop a vertex X from the stack
     process X
     for each neighbor Y of X
         if (Y.tag == false)
              push Y onto the stack
              Y.tag= true
```

Example



What is the output of DFS traversal if we start at A and use the nonrecursive version?

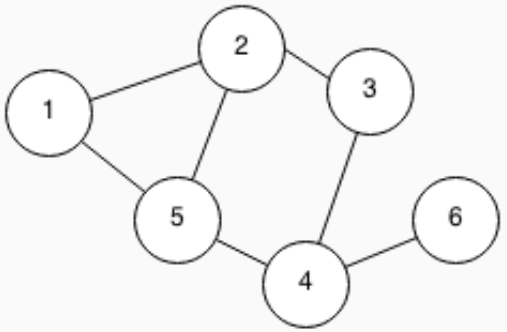
Example



Output: A D C F B E

Clicker Quiz

1. What is the traversal order of this graph using DFS-recursive version? The start point is 6.



A. 6, 4, 3, 2, 1, 5

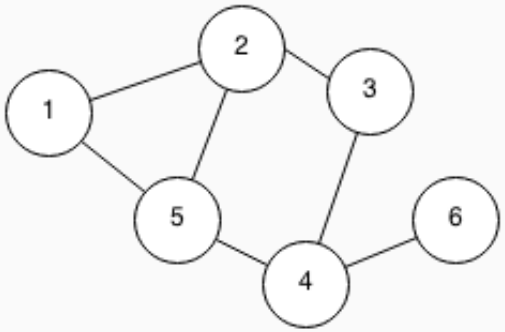
B. 6, 4, 5, 2, 3, 1

C. 6, 4, 5, 1, 2, 3

D. 6, 4, 3, 2, 5, 1

Clicker Quiz

1. What is the traversal order of this graph using DFS-recursive version? The start point is 6.



A. 6, 4, 3, 2, 1, 5

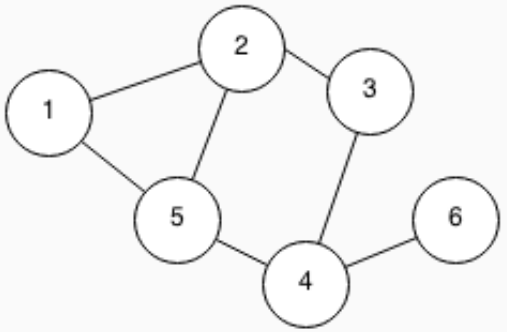
B. 6, 4, 5, 2, 3, 1

C. 6, 4, 5, 1, 2, 3

D. 6, 4, 3, 2, 5, 1

Clicker Quiz

2. What is the traversal order of this graph using DFS-nonrecursive version? The start point is 6.



A. 6, 4, 3, 2, 5, 1

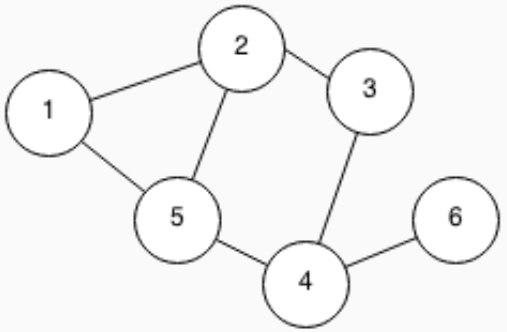
B. 6, 4, 5, 1, 2, 3

C. 6, 4, 5, 2, 1, 3

D. 6, 4, 3, 2, 1, 5

Clicker Quiz

2. What is the traversal order of this graph using DFS-nonrecursive version? The start point is 6.



A.6, 4, 3, 2, 5, 1

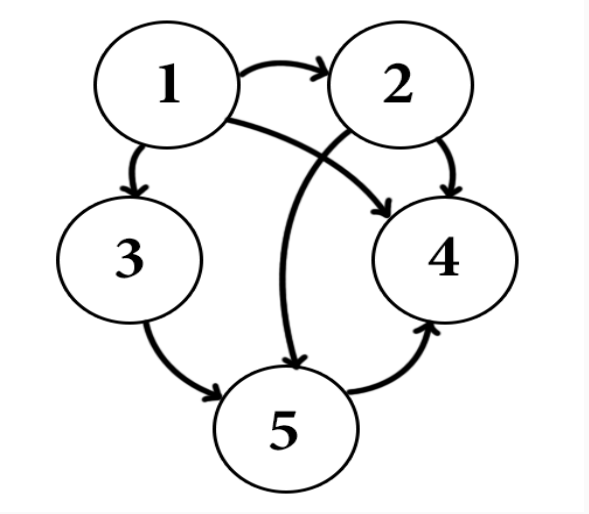
B.6, 4, 5, 1, 2, 3

C.6, 4, 5, 2, 1, 3

D.6, 4, 3, 2, 1, 5

Clicker Quiz

3. What is the traversal order of this graph using DFS-recursive version? The start point is 1.



A. 1, 3, 5, 4, 2

B. 1, 2, 4, 5, 3

C. 1, 2, 5, 4, 3

D. 1, 2, 3, 4, 5

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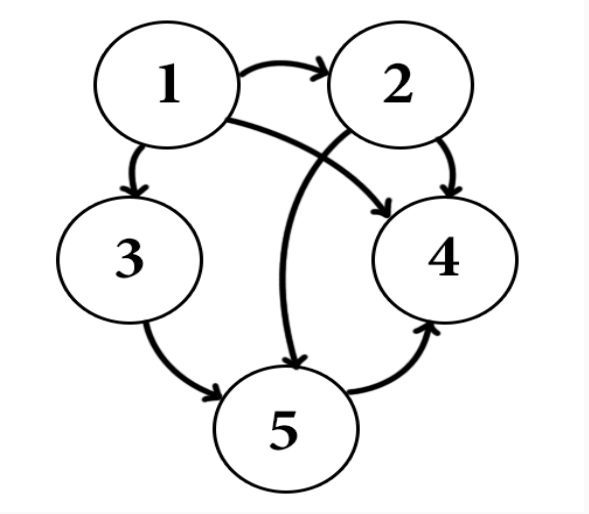
3. What is the traversal order of this graph using DFS-recursive version? The start point is 1.



A. 1, 3, 5, 4, 2

B. 1, 2, 4, 5, 3

C. 1, 2, 5, 4, 3

D. 1, 2, 3, 4, 5

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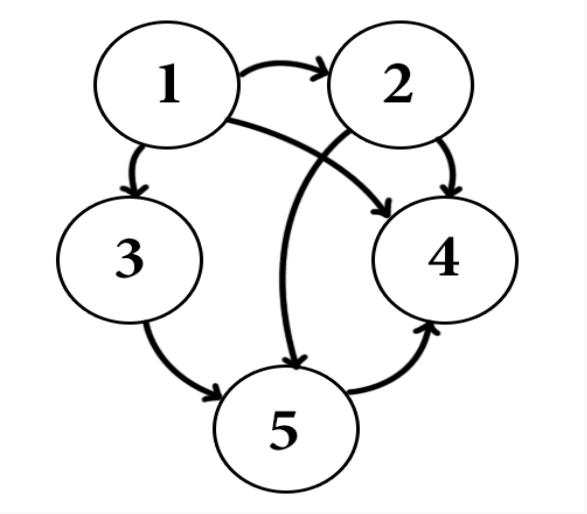
4. What is the traversal order of this graph using DFS-nonrecursive version? The start point is 1.



A. 1, 2, 3, 4, 5

B. 1, 2, 5, 4, 3

C. 1, 2, 4, 5, 3

D. 1, 4, 3, 5, 2

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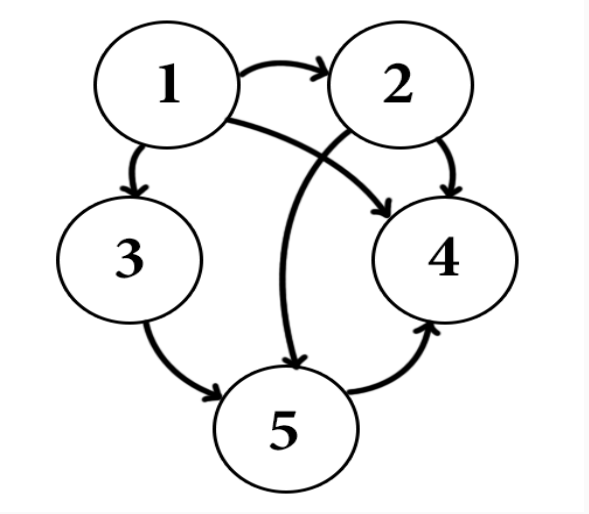
4. What is the traversal order of this graph using DFS-nonrecursive version? The start point is 1.



A. 1, 2, 3, 4, 5

B. 1, 2, 5, 4, 3

C. 1, 2, 4, 5, 3

D. 1, 4, 3, 5, 2

Big-O Notation Examples

```
void foo(int n) {

        int i,j,k;

        for(i = 1; i <= n; i++)

                for(j = 1; j <= i; j++)


        for(k=1; k <= 100; k++){


        print("good");

}
```

```
i = 1       i = 2       i = 3          i = n
j = 1       j = 2       j = 3    =>    j = n
k = 100     k = 200     k = 300        k = n * 100
```

total = 100 + 200 + 300 + 400 + 500 = 100 (1+2+3+..+n) = 100(n(n-1)/2) = O(n^2)

Big-O Notation Examples

```
void foo(int n) {

        int i,j,k;

        for(i = 1; i <= n; i++)

                for(j = 1; j <= i*i; j++)


        for(k=1; k <= n/2; k++){


        print("good");

}
```

i = 1	i = 2	i = 3	i = n
j = 1	j = 4	j = 9 =>	j = n^2
k = n/2	k = n/2 * 4	k = n/2 * 9	k = n/2 * n ^2

total = n/2 + n/2 * 4 + n/2 * 9 + … + n/2 * n^2 = n/2 (1 + 4 + 9 + n^2)
= n/2 * (n(n+1)(2n + 1)/6) = O (n^4)

Big-O Notation Examples

```
void foo(){
        int i,j,k;
        for(i= n/2, i <= n; i++)          //n/2
times
                for(j = 1; j <= n/2,
j++)          //n/2 times

        for(k=1; k <=n; k=k*2)          //
log_2^n times

        print("yes");
```

O(n/2 * n/2 * log_2 n) = O(n^2 log_2 n)

Clicker Quiz

5. What is the Big-O of the following code?

A. O (nlog_2 n)

B. O (n)

C. O (log_2 n)

D. O (log_2 n^2)

```
void foo(int n)
{
        for(int i = n; i > 0; i = i / 2)
                print("good");
}
```

Clicker Quiz

5. What is the Big-O of the following code?

A. O (nlog_2 n)

B. O (n)

C. O (log_2 n)

D. O (log_2 n^2)

```
void foo(int n)
{
        for(int i = n; i > 0; i = i / 2)
                print("good");
}
```

Clicker Quiz

6. What is the Big-O of the following code?

A. O (n)

B. O (log_2 n)

C. O (n^2 log_2 n)

D. O (log_2 n^2)

```
void foo()
{
        int i,j,k;
        for(i= n/2, i <= n; i++)
                for(j = 1; j <= n/2,
j++)
                        for(k=1;
k <=n; k=k*2)

        print("yes");
}
```

Clicker Quiz

6. What is the Big-O of the following code?

A. O (n)

B. O (log_2 n)

C. O (n^2 log_2 n)

D. O (log_2 n^2)

```
void foo()
{
        int i,j,k;
        for(i= n/2, i <= n; i++)
                for(j = 1; j <= n/2,
j++)
                        for(k=1;
k <=n; k=k*2)

        print("yes");
}
```

Clicker Quiz

7. What is the Big-O of the following code?

A. O (n^2 log_2 n)

B. O (n (log_2 n)^2)

C. O (n)

D. O (log_2 n^2)

```
void foo()
{
        int i,j,k;
        for(i = n/2; i <=n; i++)
                for(j = i; j <=n;j=j*2)

                for(k=1; k <=n;
k=k*2)
                print("good");

}
```

Clicker Quiz

7. What is the Big-O of the following code?

A. O (n^2 log_2 n)

B. O (n (log_2 n)^2)

C. O (n)

D. O (log_2 n^2)

```
void foo()
{
        int i,j,k;
        for(i = n/2; i <=n; i++)
                for(j = i; j <=n;j=j*2)

                for(k=1; k <=n;
k=k*2)
                print("good");

}
```

Clicker Quiz

8. What is the Big-O of the following code?

A. O (n)

B. O (n^2)

C. O (1)

D. O (log_2 n)

```
void (int n)
{
        for(i = 1; i<=n;  i++)

                for(j = 1; j <= i; j = j+i)

                print("good");
}
```

Clicker Quiz

8. What is the Big-O of the following code?

A. O (n)

B. O (n^2)

C. O (1)

D. O (log_2 n)

```
void (int n)
{
        for(i = 1; i<=n;  i++)

                for(j = 1; j <= i; j = j+i)

                print("good");
}
```

Clicker Quiz

9. What is the Big-O of the following code?

A. O (log_2 n)

B. O (1)

C. O (n^2)

D. O (n)

```
void foo(int n)
{
        for(int i = 1; i < n; i = i * 2)
                print("good");
}
```

Clicker Quiz

9. What is the Big-O of the following code?

A. O (log_2 n)

B. O (1)

C. O (n^2)

D. O (n)

```
void foo(int n)
{
        for(int i = 1; i < n; i = i * 2)
                print("good");
}
```

Clicker Quiz

10. What is the Big-O of the following code?

A. O (1)

B. O (log_2 n)

C. O (n)

D. O (n^2)

```
void (int n)
{
        for(i = 1; i<=n;  i++)
                for(j = 1; j <= n; j++)


        print("good");
}
```

Clicker Quiz

10. What is the Big-O of the following code?

A. O (1)

B. O (log_2 n)

C. O (n)

D. O (n^2)

```
void (int n)
{
        for(i = 1; i<=n;  i++)
                for(j = 1; j <= n; j++)


        print("good");
}
```