CMSC 132: Object-Oriented Programming II

Hash Tables

Key Value Map

- Red Black Tree: O(Log n)
- BST: O(n)
- 2-3-4 Tree: O(log n)

- Can we do better?

Hash Tables

- a data structure used to implement (a dictionary) an associative array, a structure that can map keys to values.

- O(1) best case. (Or average case).
- O(n) worst case. extremely unlikely to arise by chance, but a malicious adversary with knowledge of the hash function may be able to supply information to a hash that creates worst-case behavior by causing excessive collisions, resulting in very poor performance

Hash Table Example

- A hash table uses a magic hash function to compute an index into an array slots.
- An object can be found from the array using the index.

0	1	2	3	4	5	6	7	8	9	10	11	12

H(K) = K % 13

Grades: 85, 91, 66,96,80,88,95,87,77, 63, 93, 82

Average Search Times:

Hash Table Example

- A hash table uses a magic hash function to compute an index into an array slots.
- An object can be found from the array using the index.

0	1	2	3	4	5	6	7	8	9	10	11	12	13
91	66	80	93	95	96	82	85		87	88	63	77	

H(K) = K % 13

Grades: 85, 91, 66,96,80,88,95,87,77, 63, 93, 82

To find a number:
85, 91, 66,96,80,88,95,87,77, 63 need just 1 comparison,
93 needs 2 comparison, 82 needs 3 comparison

Average Search Times: (10 * 1 + 1 * 2 + 1 * 3)/ 12 = 1.25

Hash Function

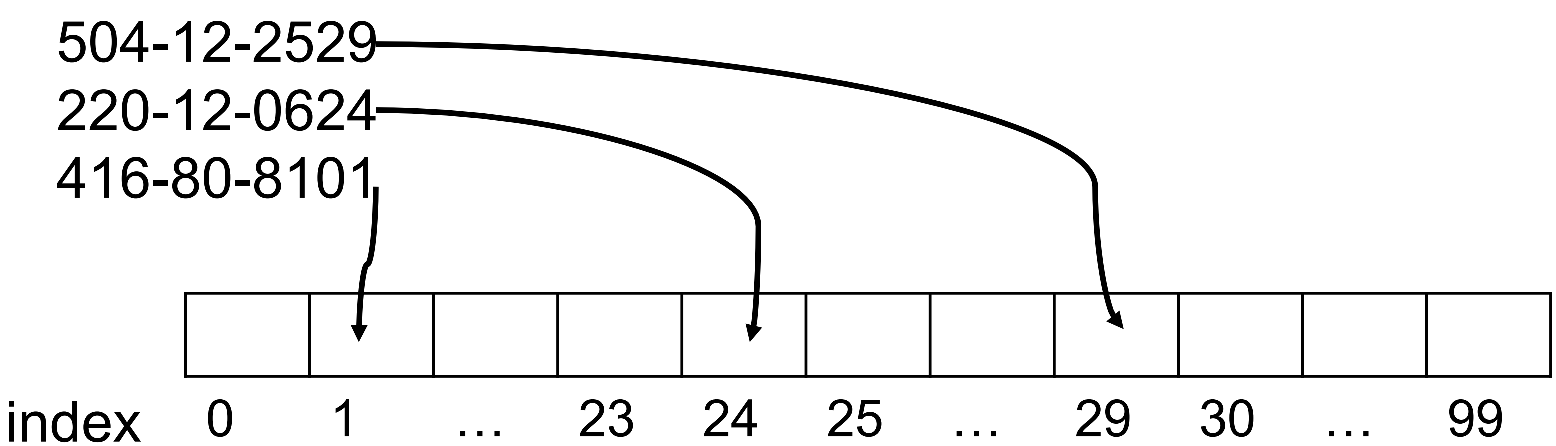
- a "magic function" that, given a value to search for, would tell us exactly where in the array to look
  - If it's in that location, it's in the array
  - If it's not in that location, it's not in the array
- When applied to an Object, returns a number
- When applied to *equal* Objects, returns the *same* number for each
- When applied to *unequal* Objects, is *very unlikely* to return the same number for each
- Hash functions is very important for searching.

Hash Functions

- Save items in a key-indexed table
- Array index is the function of the key

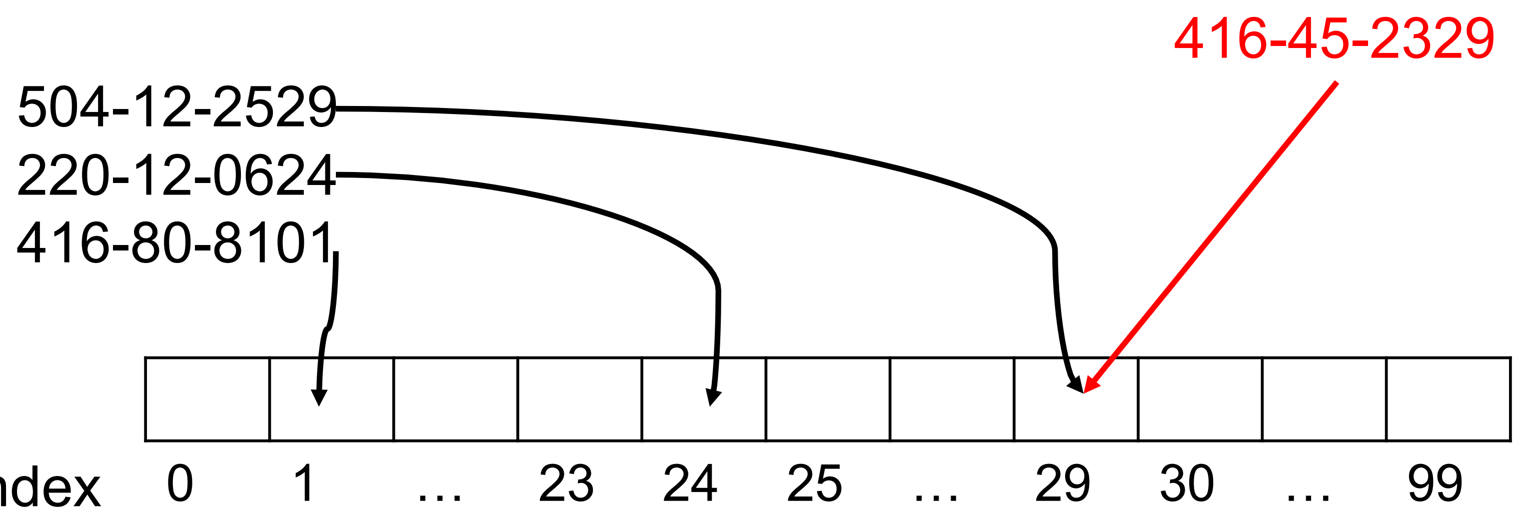
- Hash function: Method for computing array index from key

- Example:
  - Items are social security numbers
  - Hash function: `H(Key) = Key % 100`



Hash Functions

- Issues:
  - Computing the hash function
  - Equality test: Method for checking whether two keys are equal
  - Collison resolution: Algorithm and data structure to handle two keys that hash to the same index



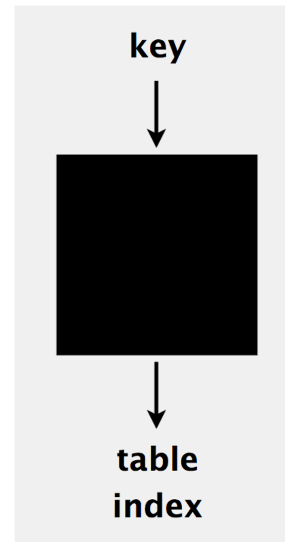
Hash Functions

- Classic space-time tradeoff
  - No space limit: trivial hash function with key as index
  - No time limit: trivial collision resolution with sequential search
  - Space and time limit: hashing in the real world

Computing the Hash Function

- Idealistic goal: Scramble the keys uniformly to produce a table index.
  - Efficiently computable.
  - Each table index equally likely for each key.

  - Ex 1. Phone numbers
    - Bad: first three digits.
    - Better: last three digits.
  - Ex 2. Social Security numbers.
    - Bad: first three digits.
    - Better: last three digits.
  - Practical challenge. Need different approach for each key type.



Java's hash code conventions

- Java classes inherit a method hashCode(), which returns a 32-bit int.

- Requirement:

```
If x.equals(y) then
 (x.hashCode() == y.hashCode()).
```

- Highly desirable:

```
If !x.equals(y) then
      (x.hashCode() != y.hashCode())
```

Java's Hash Code Conventions

- Java hashCode() Default implementation:
  - Memory address of x.

- Legal (but poor) implementation:
  - Always return 17.

- Customized implementations:
  - Integer, Double, String, File, URL, Date,

- User-defined types:
  - Users are on their own.

Hash code: integers, and doubles

```
public final class Integer{
   private final int value;
     ...
   public int hashCode(){
   return value;  }
}

final class Double{
  private final double value;
  ...
  public int hashCode(){
    long bits = doubleToLongBits(value);
    return (int) (bits ^ (bits >>> 32));
   }
}
```

Hash code: booleans

```
public final class Boolean{
    private final boolean value;
        ...
    public int hashCode() {
        if (value)
                return 1231;
        else
                return 1237;
    }
}
```

Implementing hash code: Strings

```
public final class String{
    private final char[] s;
    ...
    public int hashCode(){
        int hash = 0;
        for (int i = 0; i < length(); i++)
            hash = s[i] + (31 * hash);
        return hash;
    }
}
```

i^{th} character of s

char	Unicode
...	...
'a'	97
'b'	98
'c'	99
...	...

```
String s = "call";
int code = s.hashCode();
```

$$3045982 = 99 \cdot 31^3 + 97 \cdot 31^2 + 108 \cdot 31^1 + 108 \cdot 31^0$$
$$= 108 + 31 \cdot (108 + 31 \cdot (97 + 31 \cdot (99)))$$

(Horner's method)

Hash code: user-defined types

```
public final class Transaction implements Comparable<Transaction>{
   private final String  who;
   private final Date    when;
   private final double  amount;
   public Transaction(String who, Date when, double amount)
   {  /* as before */  }
       ...
   public boolean equals(Object y){/* as before */  }
   public int hashCode(){
          int hash = 17;
          hash = 31*hash + who.hashCode();
          hash = 31*hash + when.hashCode();
          hash = 31*hash + ((Double) amount).hashCode();
          return hash;
   }
}
```

Hash Code Design

- Standard” recipe for user-defined types:
  - Combine each significant field using the 31x + y rule.
  - If field is a primitive type, use wrapper type hashCode().
  - If field is null, return 0.
  - If field is a reference type, use hashCode().
  - If field is an array, apply to each entry.

- In practice:
  - Recipe works reasonably well; used in Java libraries.
  - In theory. Keys are bitstring; "universal" hash functions exist.

Modular Hashing

Hash code. An `int` between -2^{31} and $2^{31} - 1$.

Hash function. An `int` between `0` and `M - 1` (for use as array index).

typically a prime or power of 2

```
private int hash(Key key)
{  return key.hashCode() % M;  }
```

bug

```
private int hash(Key key)
{  return Math.abs(key.hashCode()) % M;  }
```

1-in-a-billion bug

hashCode() of "polygenelubricants" is -2^{31}

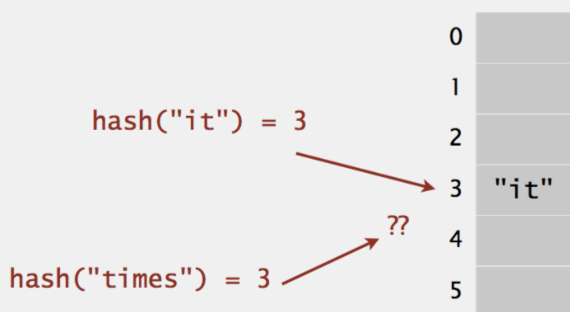
```
private int hash(Key key)
{  return (key.hashCode() & 0x7fffffff) % M;  }
```

correct

Collisions

Collision. Two distinct keys hashing to same index.

- Birthday problem $\Rightarrow$ can't avoid collisions unless you have a ridiculous (quadratic) amount of memory.
- Coupon collector + load balancing $\Rightarrow$ collisions are evenly distributed.



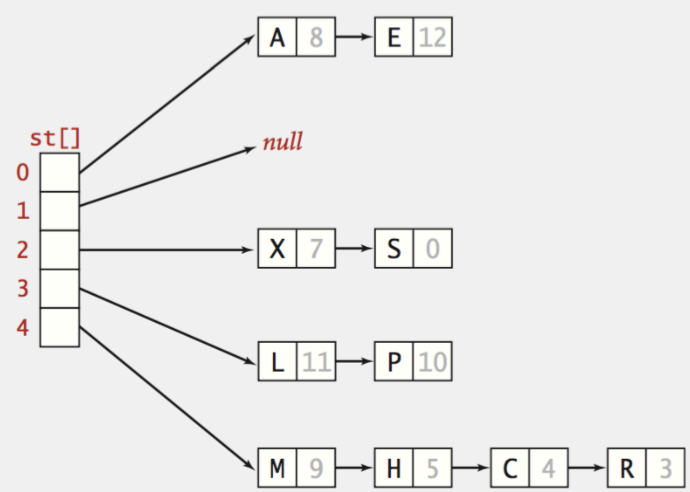
Challenge. Deal with collisions efficiently.

Separate chaining symbol table

Use an array of $M < N$ linked lists. [H. P. Luhn, IBM 1953]

- Hash: map key to integer i between 0 and $M - 1$.
- Insert: put at front of i^{th} chain (if not already there).
- Search: need to search only i^{th} chain.

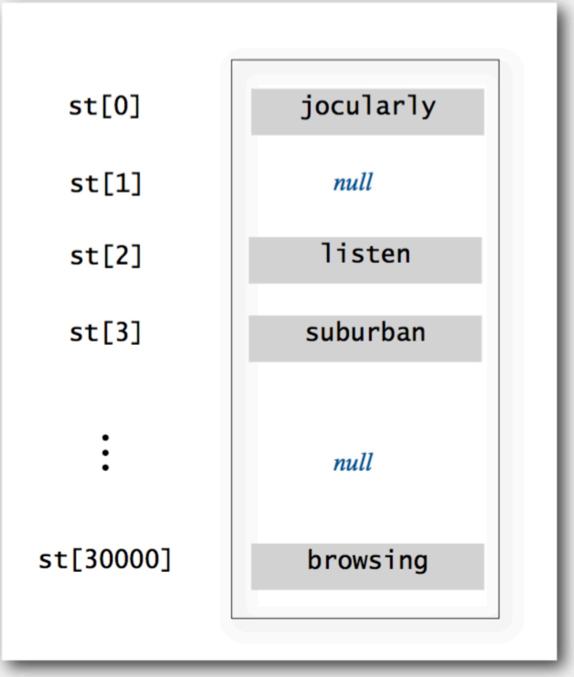
key	hash	value
S	2	0
E	0	1
A	0	2
R	4	3
C	4	4
H	4	5
E	0	6
X	2	7
A	0	8
M	4	9
P	3	10
L	3	11
E	0	12



Collision resolution: open addressing

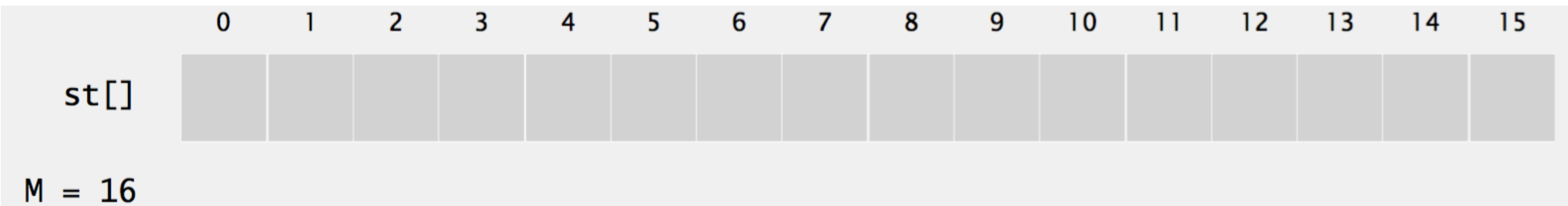
Open addressing. [Amdahl-Boehme-Rocherster-Samuel, IBM 1953]

When a new key collides, find next empty slot, and put it there.



linear probing (M = 30001, N = 15000)

Linear probing: Example



Clustering

- Cluster. A contiguous block of items.
- Observation. New keys likely to hash into middle of big clusters.
- Solutions:

Separate chaining vs. linear probing

- Separate chaining.
  - Easier to implement delete.
  - Performance degrades gracefully.
  - Clustering less sensitive to poorly-designed hash function.
- Linear probing.
  - Less wasted space.
  - Better cache performance.

Hash tables vs. balanced search trees

Hash tables.

- Simpler to code.
- No effective alternative for unordered keys.
- Faster for simple keys (a few arithmetic ops versus $\log N$ compares).
- Better system support in Java for strings (e.g., cached hash code).

Balanced search trees.

- Stronger performance guarantee.
- Support for ordered ST operations.
- Easier to implement `compareTo()` correctly than `equals()` and `hashCode()`.

Java system includes both.

- Red-black BSTs: `java.util.TreeMap`, `java.util.TreeSet`.
- Hash tables: `java.util.HashMap`, `java.util.IdentityHashMap`.

Quiz 1

What is the time complexity to retrieve from a hash table if there are no collisions?

A. $O(1)$
B. $O(n)$
C. $O(n^2)$
D. $O(\log n)$

Quiz 1

What is the time complexity to retrieve from a hash table if there are no collisions?

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Quiz 2

A hash function should have which properties?

A. Uniform distribution
B. Efficient hash code computation
C. Range is a subset of the integers
D. Equivalent objects produce equal hash codes

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Quiz 3

A hash table of length 10 uses open addressing with hash function h(k)=k mod 10, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below. Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

A. 46, 42, 34, 52, 23, 33
B. 34, 42, 23, 52, 33, 46
C. 46, 34, 42, 23, 52, 33
D. 42, 46, 33, 23, 34, 52

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0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

A. 46, 42, 34, 52, 23, 33
B. 34, 42, 23, 52, 33, 46
C. 46, 34, 42, 23, 52, 33
D. 42, 46, 33, 23, 34, 52

Quiz 4

The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function h(k) = k mod 10 and linear probing. What is the resultant hash table?

0	
1	
2	2
3	23
4	
5	15
6	
7	
8	18
9	

(A)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	

(B)

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

(C)

0	
1	
2	12, 2
3	13, 3, 23
4	
5	5, 15
6	
7	
8	18
9	

(D)

Quiz 4

The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function h(k) = k mod 10 and linear probing. What is the resultant hash table?

0	
1	
2	2
3	23
4	
5	15
6	
7	
8	18
9	

(A)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	

(B)

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

(C)

0	
1	
2	12, 2
3	13, 3, 23
4	
5	5, 15
6	
7	
8	18
9	

(D)

Quiz 5

Hash table of size seven, with starting index zero, and a hash function (3x + 4) mod 7. Keys 1, 3, 8, 10 are insrted into an empty table.

Which of the following is the contents of the table when?

Index	0	1	2	3	4	5	6
A	8	1				3	10
B	1	8	10				3
C	1	10	8				3
D	1	10	8				3

Quiz 5

Hash table of size seven, with starting index zero, and a hash function (3x + 4) mod 7. Keys 1, 3, 8, 10 are insrted into an empty table.

Which of the following is the contents of the table when?

Index	0	1	2	3	4	5	6
A	8	1				3	10
B	**1**	**8**	**10**				**3**
C	1	10	8				3
D	1	10	8				3

Quiz 6

Hash table keys are ordered.

A. True
B. False

Quiz 6

Hash table keys are ordered.

A. True
B. False

Quiz 7

What is the worst case time complexity to retrieve from a hash?

A. $O(1)$
B. $O(n)$
C. $O(n^2)$
D. $O(\log n)$

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