

# CMSC 330: Organization of Programming Languages

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## Logic Programming with Prolog Lists

# Review: Execution = Search

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- ▶ Prolog execution: Goal-directed search
  - Query = predicate you wish to prove is true
- ▶ Key feature: unification
  - Two terms unify if they are **identical**, or they can be made identical by **substituting variables**
    - `is_bigger(X, gnat) = is_bigger(horse, gnat)` when `X=horse`
    - execution goal is often to discover such `X`
- ▶ Attempt to unify goal with head of a rule
  - If succeeds, clauses in body become subgoals
  - Continue until all subgoals satisfied
    - If search fails, backtrack and try untried subgoals

# Review: Equality

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- ▶ Not all forms of equality are the same!
  - $p = q$  iff  $p$  unifies with  $q$
  - $p \text{ is } q$  iff  $p$  unifies with  $q'$  where  $q'$  is  $q$  evaluated
    - Meaning that  $q'$  is treated as an arithmetic expression, and run as such
  - $p ::= q$  iff  $p'$  unifies with  $q'$  where  $q'$  is  $q$  evaluated and  $p'$  is  $p$  evaluated
  - $p == q$  iff  $p$  and  $q$  are identical
    - No substitutions or evaluations permitted

# Warmup: What is the query result?

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john(C, E, N, A) :-

C = N,

E = A,

C = 2 + 3.

?- john(5, 1, 5, 1).

A. true

B. false

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# Lists In Prolog

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- ▶ `[a, b, 1, 'hi', [X, 2]]`
- ▶ But really represented as compound terms
  - `[]` is an atom
  - `[a, b, c]` is represented as `.(a, .(b, .(c, [])))`
- ▶ Matching over lists
  - `?- [X, 1, Z] = [a, _, 17]`
  - `X = a,`
  - `Z = 17.`

# List Deconstruction

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- ▶ Syntactically similar to Ocaml:  $[H|T]$  like  $h::t$

?- [Head | Tail] = [a,b,c].

Head = a,

Tail = [b, c].

?- [1,2,3,4] = [\_, X | \_].

X = 2

- ▶ This is sufficient for defining complex predicates
- ▶ Let's define `concat(L1, L2, C)`

?- concat([a,b,c], [d,e,f], X).

X = [a,b,c,d,e,f].

# Example: Concatenating Lists

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- ▶ To program this, we define the “rules” of concatenation

- If L1 is empty, then  $C = L2$

`concat( [], L2, L2 ).`

- Prepending a new element to L1 prepends it to C, so long as C is the concatenation of L1 with some L2

`concat( [E | L1], L2, [E | C] ) :-  
concat(L1, L2, C).`

- ▶ ... and we're done



# Why Is The Return Value An Argument?

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- ▶ Now we can ask **what inputs lead to an output**

?- concat(X, Y, [a,b,c]).

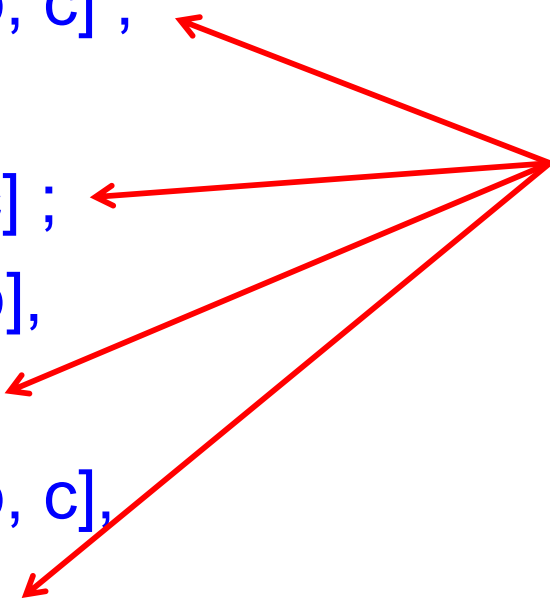
{ X = [],  
Y = [a, b, c] ;

{ X = [a],  
Y = [b, c] ;

{ X = [a, b],  
Y = [c] ;

{ X = [a, b, c],  
Y = [] ;

User types ; to request  
additional answers



# Quiz 1: T/F: This is a Valid Prolog List

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[3, 4, 'papaya', blueberry]

- A. True
- B. False

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## Quiz 2: What does this query return?

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?- [a|T] = [a,b,c,[d,a],[1,2],list].

- A. T = [b, c, [d, a], [1, 2], list].
- B. false
- C. T = [d, a]
- D. T = list

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- B. false
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## Quiz 3: What does `mystery(A,L)` do?

---

`mystery(X, [H|T]) :- X = H.`

`mystery(X, [H|T]) :- mystery(X,T).`

- A. Evaluates to false if A is contained in list L
- B. Evaluates to true if A is contained in list L
- C. Assigns the last element in L to A
- D. Assigns the first element in L to A

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## Quiz 4: What's result of `mystery(A,B)` ?

---

`mystery(L1,L2) :-`

`L1 = [H|T1],`

`L2 = [H,H|T2].`

- A. true if A and B have equal lengths
- B. true if the first element in A is equal to the first and the last element in B.
- C. true if the first element in A is equal to the first and the second element in B.
- D. true if the first element in A is equal to the last element in B.



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# Built-in List Predicates

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- ▶ `length(List,Length)`
  - ?- `length([a, b, [1,2,3] ], Length).`  
`Length = 3.`
- ▶ `member(Elem,List)`
  - ?- `member(duey, [huey, duey, luey]).`  
`true.`
  - ?- `member(X, [huey, duey, luey]).`  
`X = huey; X = duey; X = luey.`
- ▶ `append(List1,List2,Result)`
  - ?- `append([duey], [huey, duey, luey], X).`  
`X = [duey, huey, duey, luey].`

# Built-in Predicates

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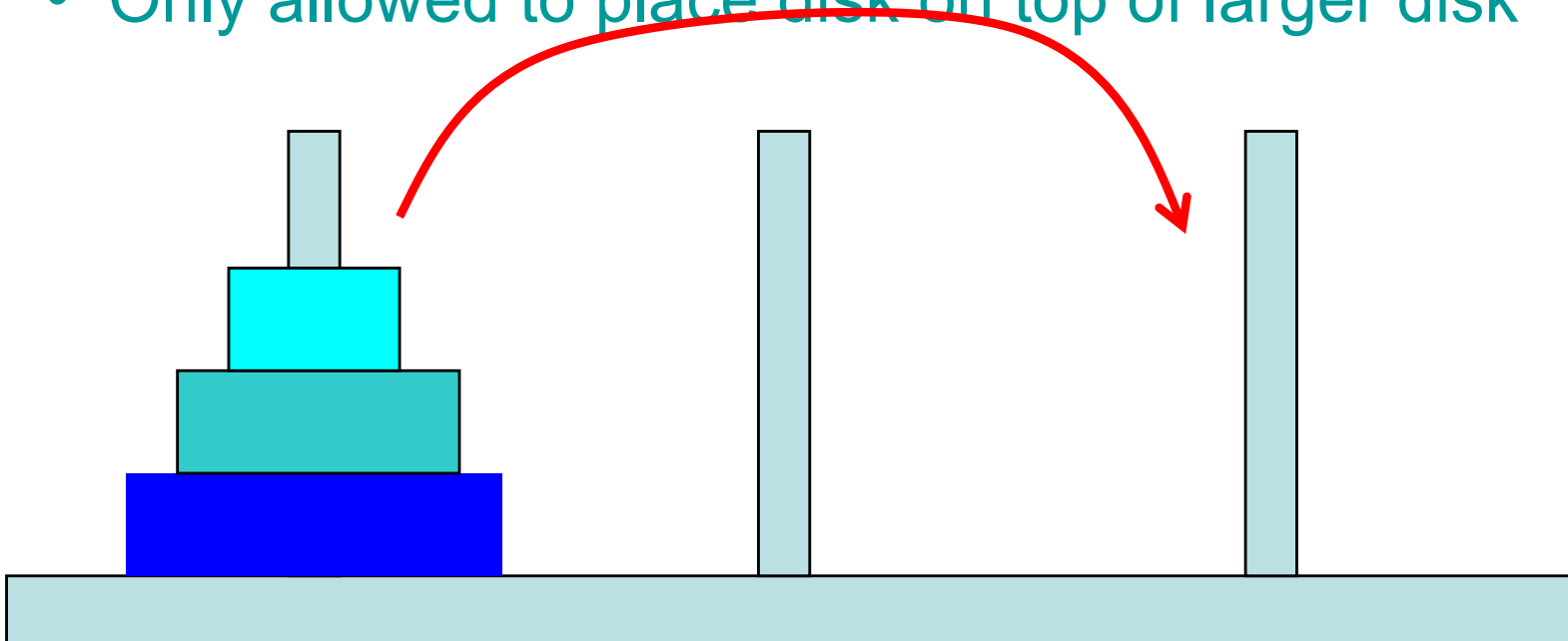
- ▶ `sort(List, SortedList)`
  - ?- `sort([2,1,3], R).`  
`R= [1,2,3].`
- ▶ `findall(Elem, Predicate, ResultList)`
  - ?- `findall(E, member(E, [huey, duey, luey]), R).`  
`R=[huey, duey, luey].`
- ▶ `setof(Elem, Predicate, ResultSortedList)`
  - ?- `setof(E, member(E, [huey, duey, luey]), R).`  
`R=[duey, huey, luey].`
- ▶ See documentation for more
  - <http://www.swi-prolog.org/pldoc/man?section=builtin>

# Example – Towers of Hanoi

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## ► Problem

- Move stack of disks between pegs
- Can only move top disk in stack
- Only allowed to place disk on top of larger disk



# Example – Towers of Hanoi

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- ▶ To move a stack of  $n$  disks from peg X to Y
  - Base case
    - If  $n = 1$ , move disk from X to Y
  - Recursive step
    1. Move top  $n-1$  disks from X to 3<sup>rd</sup> peg (Z)
    2. Move bottom disk from X to Y
    3. Move top  $n-1$  disks from 3<sup>rd</sup> peg (Z) to Y

Iterative algorithm would take much longer to describe!

# Towers of Hanoi

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- ▶ Code

```
move(1,X,Y,_) :-
```

```
    write('Move top disk from '), write(X),  
    write(' to '), write(Y), nl.
```

```
move(N,X,Y,Z) :-
```

```
    N>1,
```

```
    M is N-1,
```

```
    move(M,X,Z,Y),
```

```
    move(1,X,Y,_),
```

```
    move(M,Z,Y,X).
```

# Prolog Terminology

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- ▶ A query, goal, or term where variables do not occur is called **ground**; else it's **nonground**
  - $\text{foo}(a,b)$  is ground;  $\text{bar}(X)$  is nonground
- ▶ A **substitution**  $\theta$  is a partial map from variables to terms where  $\text{domain}(\theta) \cap \text{range}(\theta) = \emptyset$ 
  - Variables are terms, so a substitution can map variables to other variables, but not to themselves
- ▶  $A$  is an **instance** of  $B$  if there is a substitution such that  $A = B\theta$  ← The substitution  $\theta$  applied to  $B$
- ▶  $C$  is a **common instance** of  $A$  and  $B$  if it is an instance of  $A$  and an instance of  $B$

# Prolog's Algorithm Solve()

Starts as empty

**Solve**(goal  $G$ , program  $P$ , substitution  $\theta$ ) =

- ▶ Suppose  $G$  is  $A_1, \dots, A_n$ . Choose goal  $A_1$ .
- ▶ For each clause  $A :- B_1, B_2, \dots, B_k$  in  $P$ ,
  - if  $\theta_1$  is the **mgu** of  $A$  and  $A_1\theta$  then
    - ▶ If **Solve**( $\{B_1, \dots, B_k, A_2, \dots, A_n\}, P, \theta \cdot \theta_1$ ) = some  $\theta'$  then **return**  $\theta'$
    - ▶ (else it has failed, so we continue the for loop)
  - (else unification has failed, so try another rule)
- ▶ If loop exits return **fail**
- ▶ **Output**:  $\theta$  s.t.  $G\theta$  can be deduced from  $P$ , or fail

Chooses goals in order

Most  
General  
Unifier

Implements backtracking