CMSC 132: Object-Oriented Programming II

Recursion

Recursion

- When one function calls itself directly or indirectly
- a method where the solution to a problem depends on solutions to smaller instances of the same problem.

Factorial

Definition:

$$n! = \begin{cases} 1 & \text{if } n = 0, \\ (n-1)! \times n & \text{if } n > 0 \end{cases}$$

5! =?

5! = 5 * 4!

4 = 4 * 3!

3! = 3 * 2!

2! = 2 * 1!

1! = 1

5 * 24 = 120

4 * 6

3 * 2

2 * 1

Greatest Common Divisor

- gcd: Find largest integer d that evenly divides into p and q **Euclid's algorithm. [300 BCE]**

$$\gcd(p, q) = \begin{cases} p & \text{if } q = 0 \\ \gcd(q, p \,\%\, q) & \text{otherwise} \end{cases}$$

← base case

← reduction step, converges to base case

```
gcd(4032, 1272)   = gcd(1272, 216)
                  = gcd(216, 192)
                  = gcd(192, 24)
                  = gcd(24, 0)
                  = 24.
```

4032 = 3 × 1272 + 216

Greatest Common Divisor

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← base case

← reduction step, converges to base case

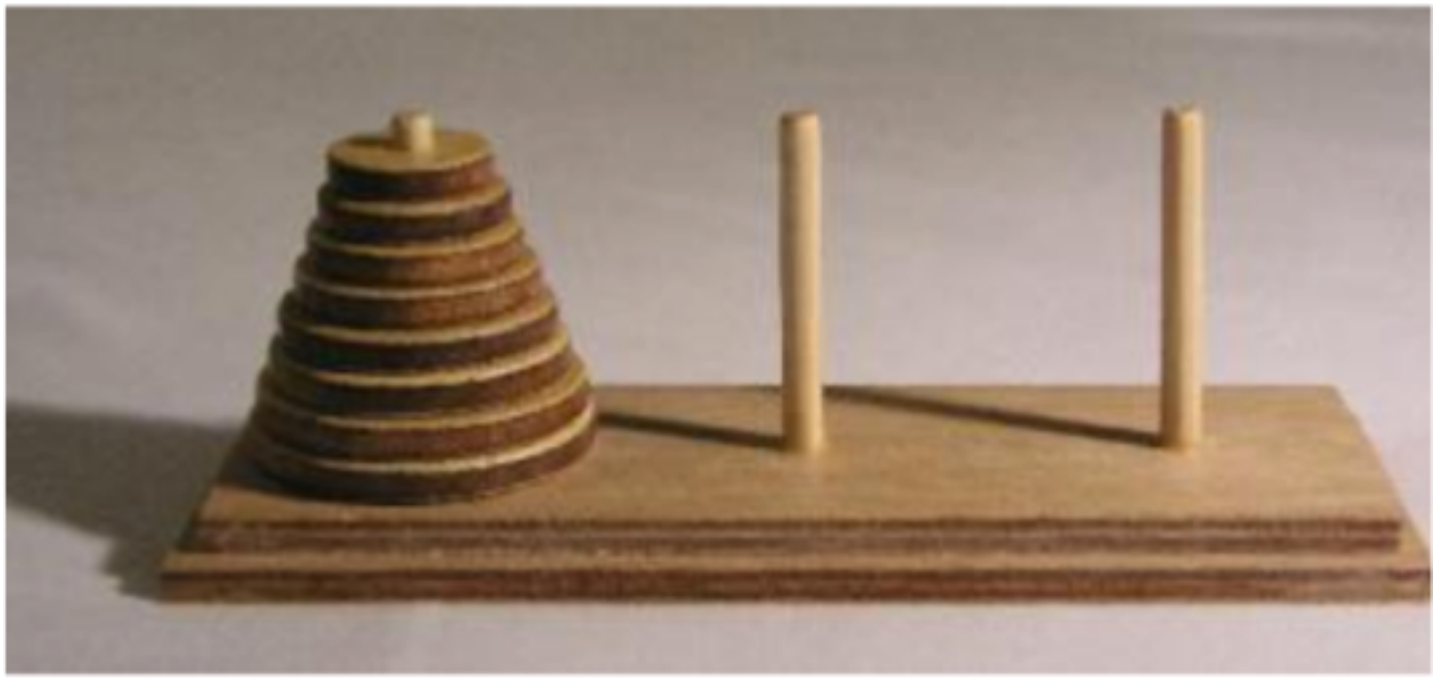
Java implementation.

```
public static int gcd(int p, int q) {
   if (q == 0) return p;
   else return gcd(q, p % q);
}
```

← base case

← reduction step

Towers of Hanoi



http://en.wikipedia.org/wiki/Image:Hanoiklein.jpg

Towers of Hanoi

- Move all the discs from the leftmost peg to the rightmost one.
  - Only one disc may be moved at a time.
  - A disc can be placed either on empty peg or on top of a larger disc

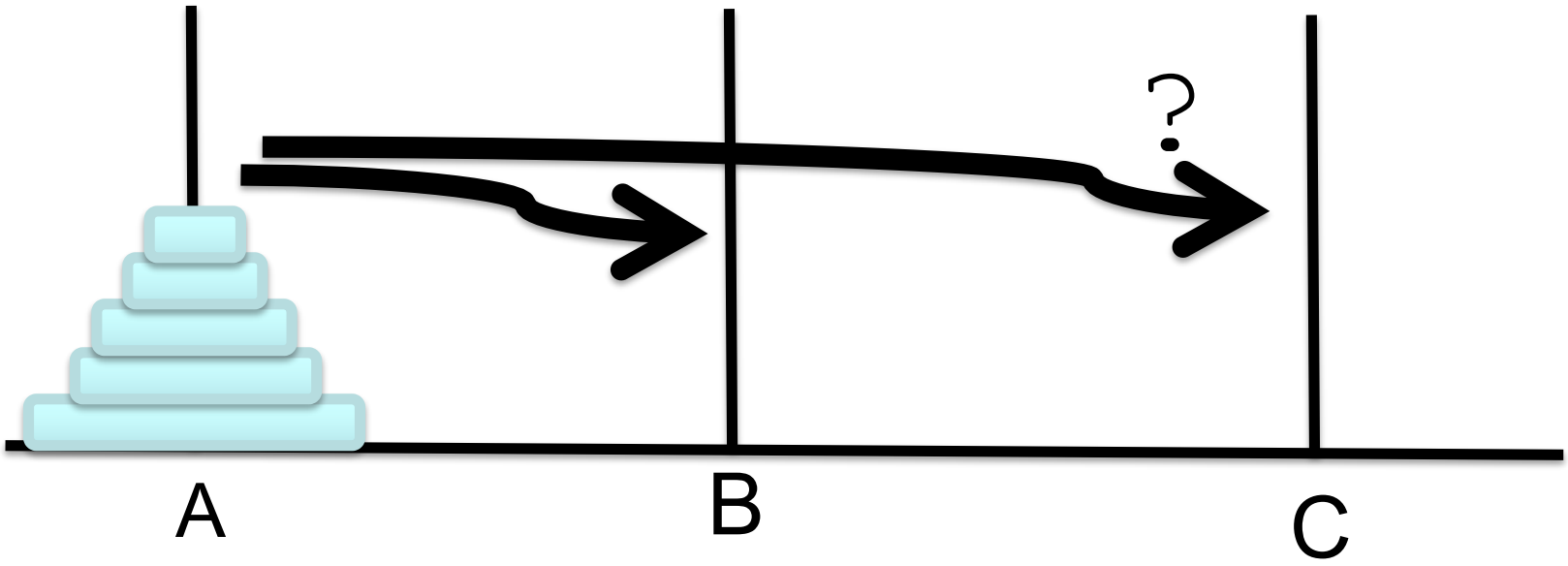
start

finish

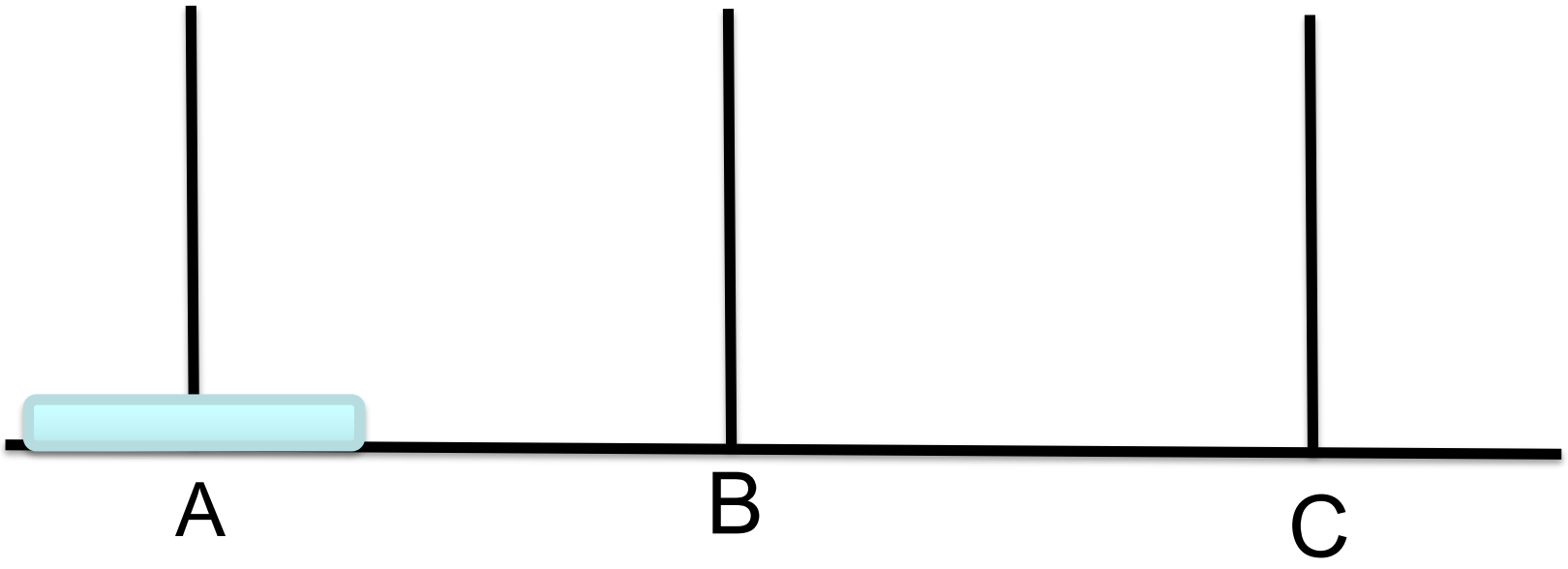
Towers of Hanoi Legend

- Q. Is world going to end (according to legend)?
  - 64 golden discs on 3 diamond pegs.
  - World ends when certain group of monks accomplish task.
- Q. Will computer algorithms help

Let's move disks

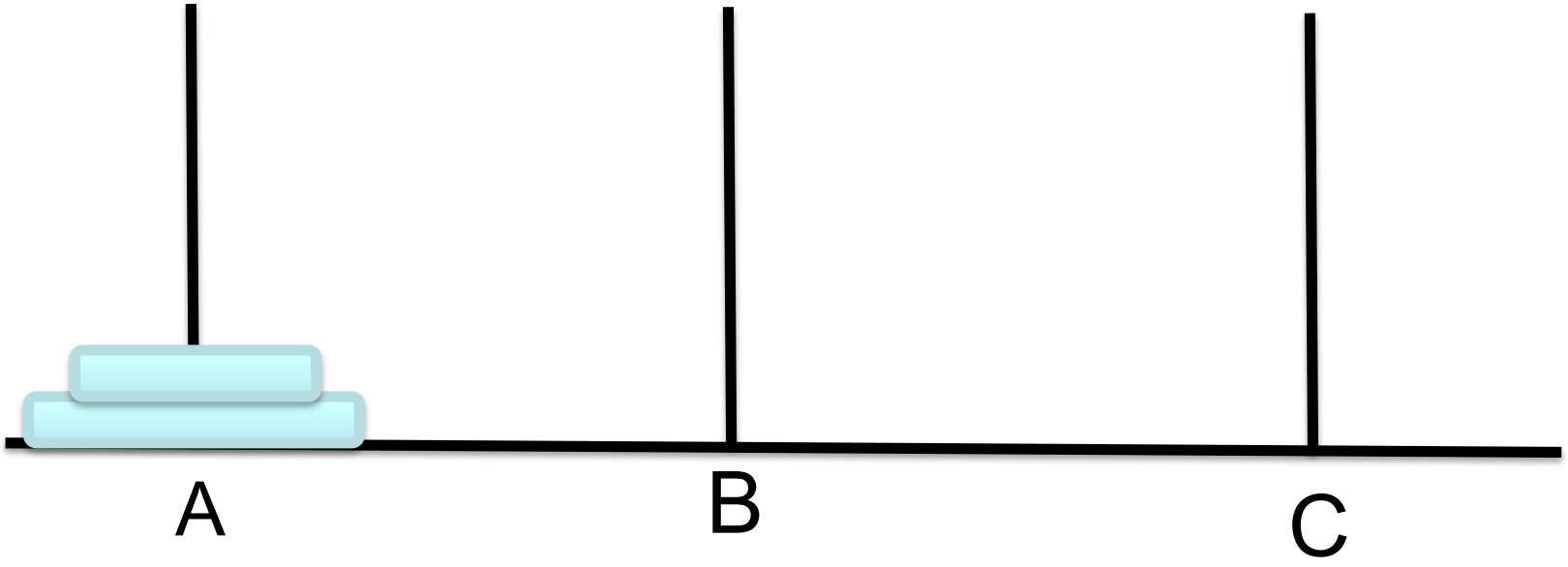


Move one disk



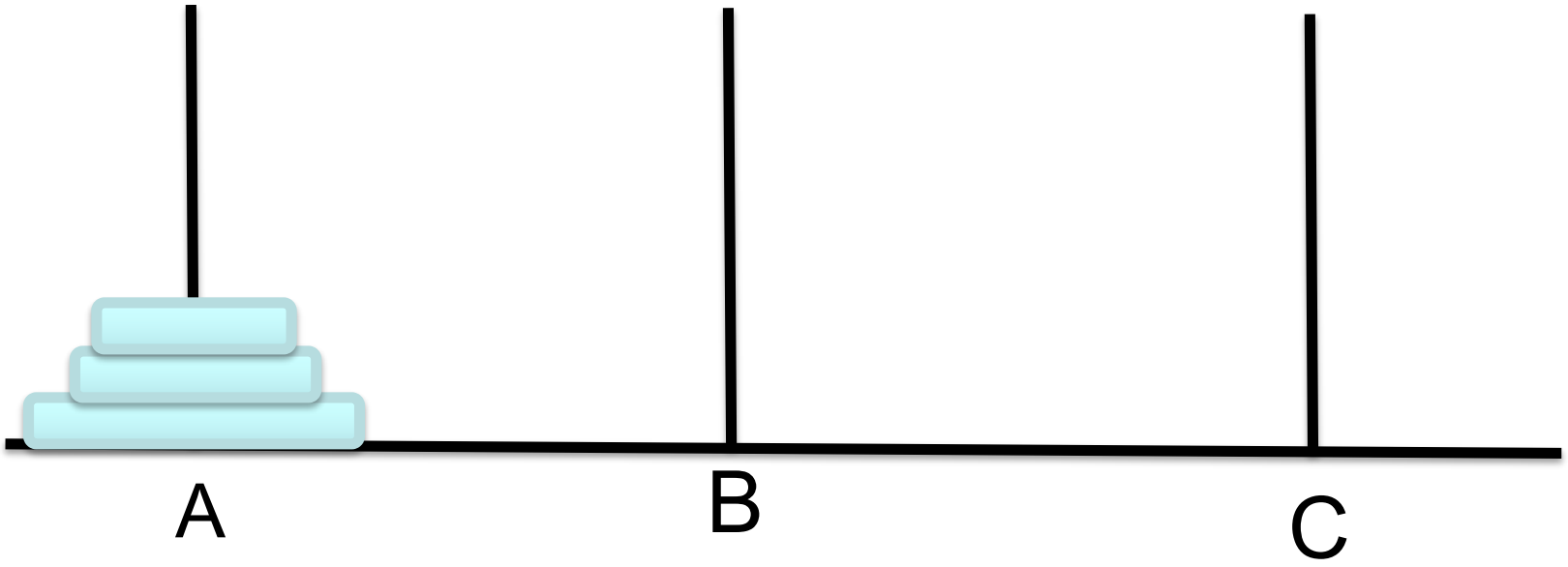

```
void move1();
```

Move two disks



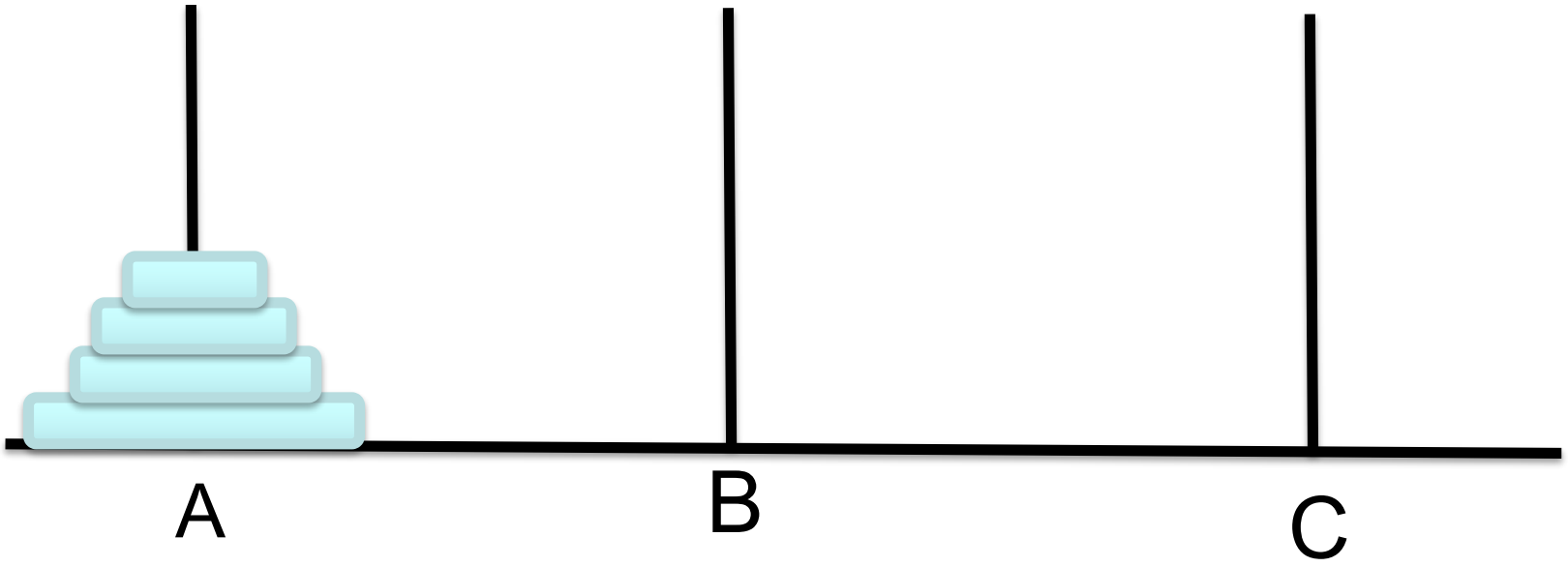

```
void move2(){
       move1();
       move1();
       move1();
}
```

Move three disks



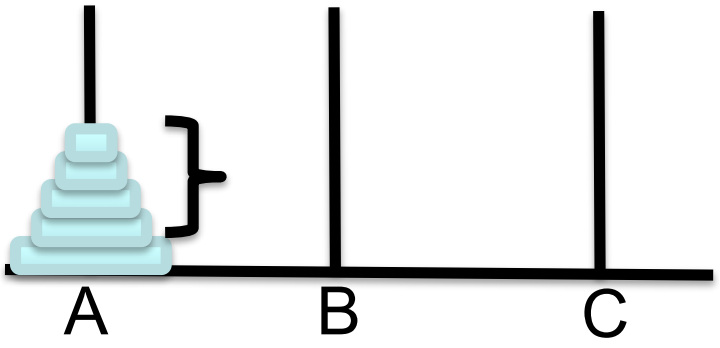

```
void move3(){
       move2();
       move1();
       move2();
}
```

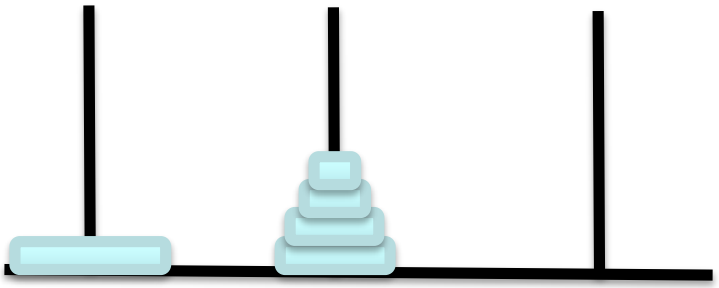
Move four disks



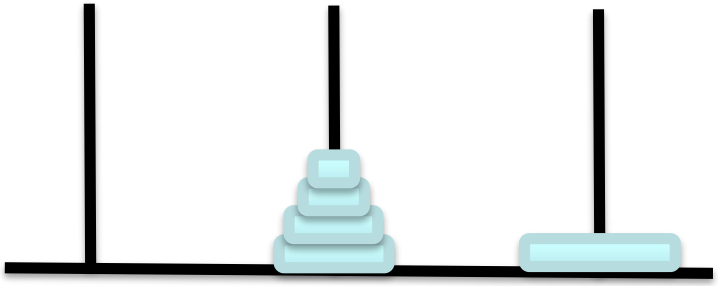

```
void move4(){
        move3();
        move1();
        move3();
}
```

Move n disks

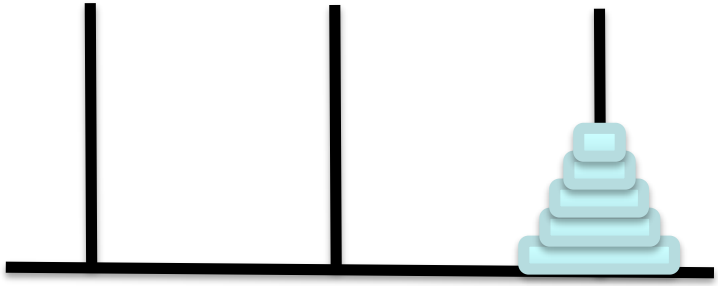




1. Move n-1 disks
2. from A->B



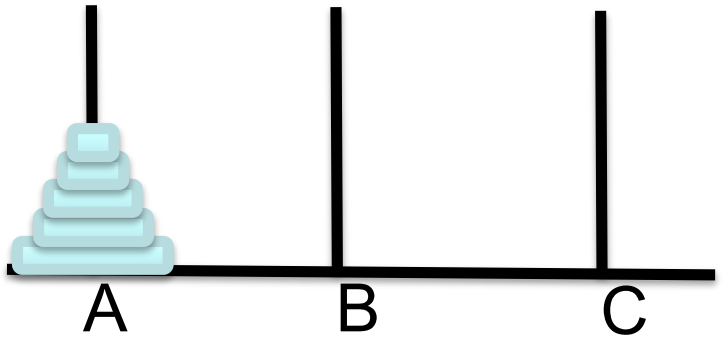
2. Move the largest disk to C



3. Move n-1 disks from B->C

Towers of Hanoi: Recursive Solution

```
public class TowersOfHanoi {
    public static void solve(int n, String A, String B, String C) {
        if (n == 1) {
            System.out.println(A + " -> " + C);
        } else {
            solve(n - 1, A, C, B);
            System.out.println(A + " -> " + C);
            solve(n - 1, B, A, C);
        }
    }
    public static void main(String[] args) {
         int discs = 3;
         solve(discs, "A", "B", "C");
    }
}
```



Remarkable properties of recursive solution

- Takes 2^n steps to solve n disc problem.
- Takes 585 billion years for n = 64(at rate of 1 disc per second).
- Reassuring fact: any solution takes at least this long

Quiz 1

It takes _____ steps to solve 8 disk problem.

A. 64
B. 128
C. 256
D. 512

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B. 128
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Quiz 2:

What is the output of `fun(2)`?

```
int fun(int n){
  if (n == 4)
    return n;
  else
    return 2*fun(n+1);
}
```

A. 4
B. 8
C. 16
D. Runtime Erro

Quiz 2:

What is the output of `fun(2)`?

```
int fun(int n){
  if (n == 4)
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  else
    return 2*fun(n+1);
}
```

A. 4
B. 8
C. 16
D. Runtime Erro

Quiz 3:

What is the output of fun(25)?

```
void fun(int n){
  if (n == 0)
    return;
   print(n%2);
   fun(n/2);
}
```

A. 11001
B. 10011
C. 11111
D. 00000

Quiz 3:

What is the output of fun(25)?

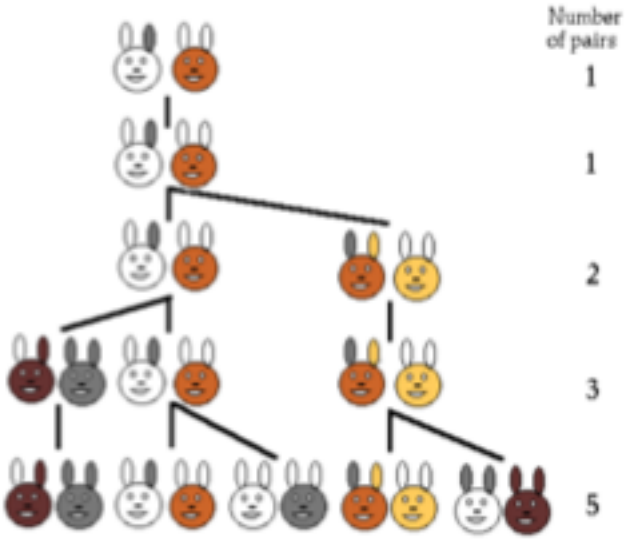
```
void fun(int n){
  if (n == 0)
    return;
   print(n%2);
   fun(n/2);
}
```

A. 11001
B. 10011
C. 11111
D. 00000

Fibonacci Number

Fibonacci numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$



Fibonacci rabbits

L. P. Fibonacci
(1170 - 1250)

Fibonacci Number

Fibonacci numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34

A natural for recursion?

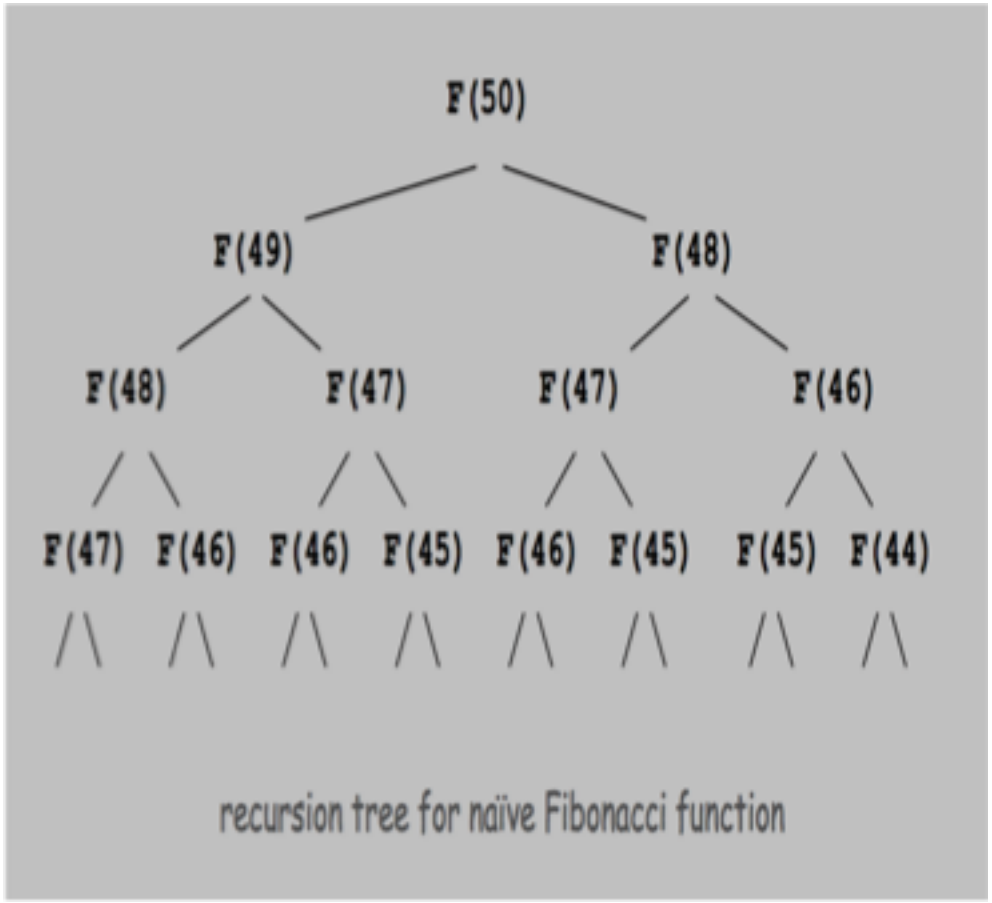
```
public static long F(int n) {
   if (n == 0) return 0;
   if (n == 1) return 1;
   return F(n-1) + F(n-2);
}
```

spectacularly inefficient code

Observation. It takes a really long time to compute `F(50)`.

Inefficient Recursion

Fibonacci numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34



recursion tree for naïve Fibonacci function

F(50) is called once.

F(49) is called once.

F(48) is called 2 times.

F(47) is called 3 times.

F(46) is called 5 times.

F(45) is called 8 times.

...

F(1) is called 12,586,269,025 times.

F(50)

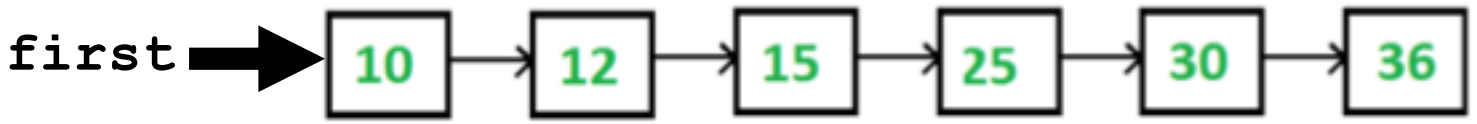
Memoized Version of the Fibonacci

```
public Map<Integer, Integer> fibo;
public int fib(int n){
  int f1=0,f2=0;
  if( (n == 1)|| (n == 2)) return 1;
  else{
       if(fibo.containsKey(n-1)) f1 = fibo.get(n-1);
        else{
           f1 = fib(n-1);
           fibo.put(n-1,f1);
        }
        if(fibo.containsKey(n-2)) f2 = fibo.get(n-2);
       else{
           f2 = fib(n-2);
           fibo.put(n-2,f2);
        }
  return f2+f1;
  }
}
```

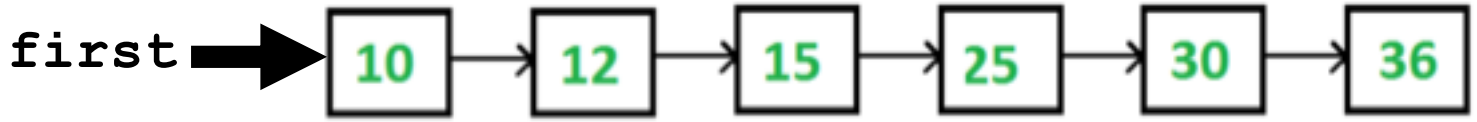
Recursive Print a Linked List

```
public void print(){
  print(first);
  System.out.println("");
}

private void print(Node r){
  if(r == null) return;
  System.out.print(r.data+",");
  print(r.next);
}
```



Insert a Node into a Sorted List




```
public Node insert(Node r, E item){
  if(r == null) return new Node(item);
  if(r.data.compareTo(item)<0){
    r.next = insert(r.next, item);
    return r;
  }else{
    Node t = new Node(item);
    t.next = r;
    return t;
  }
}
```

Contains Method

Iterative:

```
public boolean contains(E item){
  Node<E> current = first;
  while(current != null){
    if(current.data.equals(item)){ return true; }
      current = current.next;
    }
  return false;
}
```

Recursive:

```
public boolean contains_rec(Node r, E item){
  if(r == null) return false;
  return (r.data.equals(item))
         || contains_rec(r.next, item);
}
```

Merge 2 Sorted Lists

```
public Node merge(Node list1, Node list2) {
  if (list1 == null) return list2;
  if (list2 == null) return list1;
  if (list1.data < list2.data) {
    list1.next = merge(list1.next, list2);
    return list1;
  } else {
    list2.next = merge(list2.next, list1);
    return list2;
  }
}
```

List1: 1->2->4->9
List2: 1->3->4->7->8->10
After merge
1->1->2->3->4->4->7->8->9->10