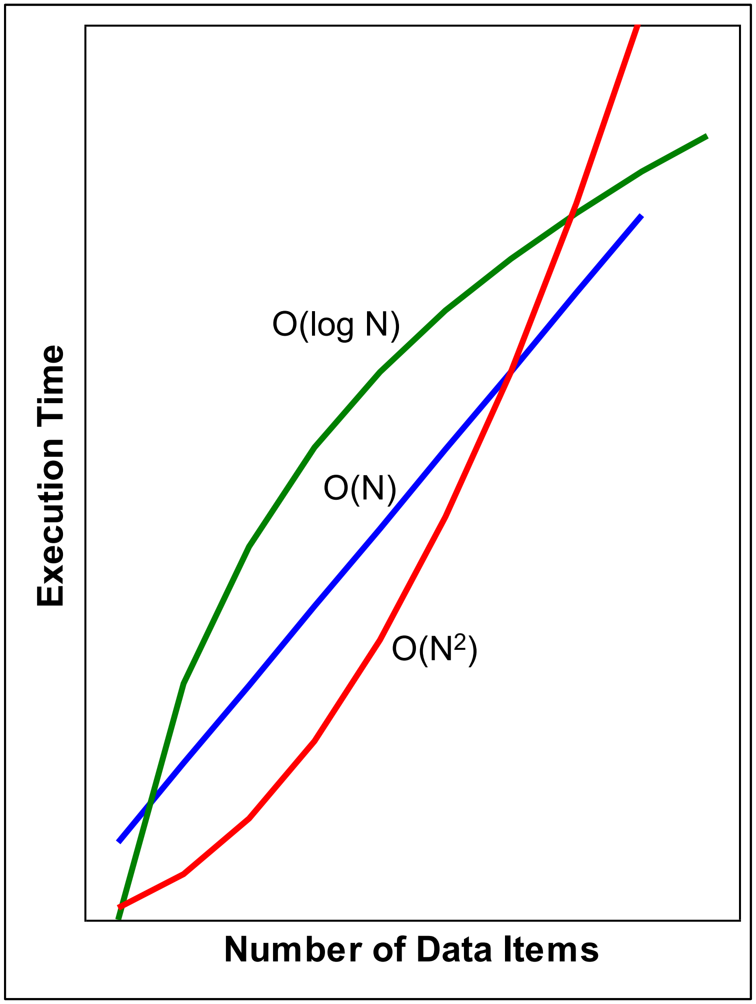
CMSC 132: Object-Oriented Programming II

Big-O Performance Analysis

Execution Time Factors

- Computer:
  - CPU speed, amount of memory, etc.
- Compiler:
  - Efficiency of code generation.
- Data:
  - Number of items to be processed.
  - Initial ordering (e.g., random, sorted, reversed)
- Algorithm:
  - E.g., linear vs. binary search.

Are Algorithms Important?



- The fastest algorithm for 100 items may <u>not</u> be the fastest for 10,000 items!
- Algorithm choice is more important than any other factor!

Counting the instructions

```
public void SelectionSort ( int [ ] num ){
     int i, j, first, temp;
     for ( i = num.length - 1; i > 0; i - - )
     {
          first = 0;   //initialize to subscript of first element
          for(j = 1; j <= i; j ++)   //locate smallest element between positions 1 and i.
          {
               if( num[ j ] < num[ first ] )
                 first = j;
          }
          temp = num[ first ];   //swap smallest found with element in position i.
          num[ first ] = num[ i ];
          num[ i ] = temp;
     }
}
```

i times

1 time

n times

$4 + 2*(n-1) + 4 + 2 * (n-2)+ \ldots 4 + 2*1 =$

$4(n-1) + 2((n-1)+(n-2)+(n-3)\ldots1) = 4(n-1) * 2\ n(n-1)/2$

$=4(n-1) + n^2 - n= n^2 + 3n - 4$

What is Big-O?

- Big-O characterizes algorithm performance.
- Big-O describes how execution time grows as the number of data items increase.
- Big-O is a function with parameter N, where N represents the number of items.

Predicting Execution Time

- If a program takes 10ms to process one item, how long will it take for 1000 items?

- (time for 1 item) x (Big-O( ) time complexity of N items)

$\log_{10} N$	3 x 10ms	.03 sec
N	10^3 x 10ms	10 sec
$N \log_{10} N$	10^3 x 3 x 10ms	30 sec
N^2	10^6 x 10ms	16 min
N^3	10^9 x 10ms	12 days

Complexity

- In general, we are not so much interested in the time and space complexity for small inputs.
- For example, while the difference in time complexity between linear and binary search is meaningless for a sequence with $n = 10$, it is gigantic for $n = 2^{30}$.

Complexity

- For example, let us assume two algorithms A and B that solve the same class of problems.
- The time complexity of A is 5,000n, the one for B is $\lceil 1.1^n \rceil$ for an input with n elements.
- For n = 10, A requires 50,000 steps, but B only 3, so B seems to be superior to A.
- For n = 1000, however, A requires 5,000,000 steps, while B requires $2.5 \cdot 10^{41}$ steps.

Complexity

- This means that algorithm B cannot be used for large inputs, while algorithm A is still feasible.
- So what is important is the **growth** of the complexity functions.
- The growth of time and space complexity with increasing input size **`n`** is a suitable measure for the comparison of algorithms.

Complexity

- Comparison: time complexity of algorithms A and B

Input Size	Algorithm A	Algorithm B
n	5,000n	1.1^n
10	50,000	3
100	500,000	13,781
1,000	5,000,000	$2.5*10^{41}$
1,000,000	$5*10^9$	$4.8*10^{41392}$

The Growth of Functions

- The growth of functions is usually described using the **big-O notation**.

- **Definition:** Let f and g be functions from the integers or the real numbers to the real numbers.
- We say that f(x) is O(g(x)) if there are constants C and k such that

- $|f(x)| \leq C|g(x)|$

- whenever $x > k$.

The Growth of Functions

- When we analyze the growth of **complexity functions**, f(x) and g(x) are always positive.
- Therefore, we can simplify the big-O requirement to
- $f(x) \le C \cdot g(x)$ whenever $x > k$.
- If we want to show that f(x) is O(g(x)), we only need to find **one** pair (C, k) (which is never unique).

The Growth of Functions

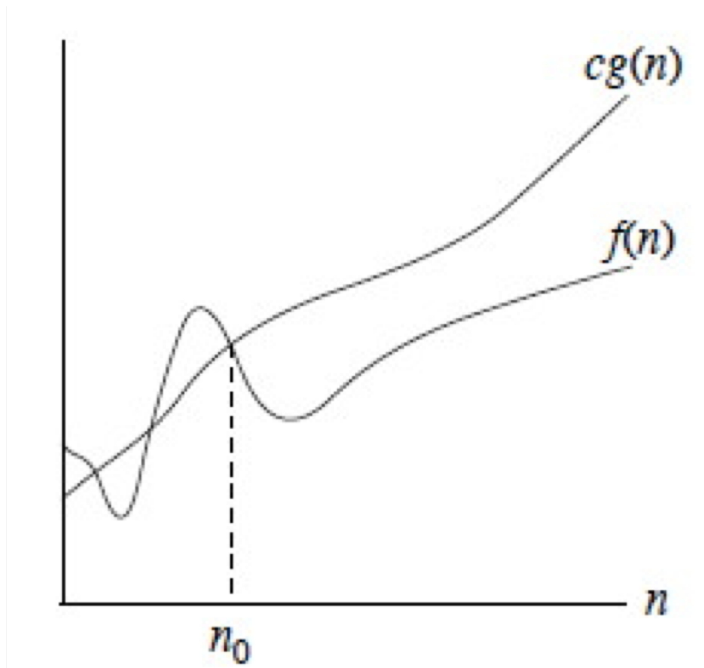
- The idea behind the big-O notation is to establish an **upper boundary** for the growth of a function f(x) for large x.

- This boundary is specified by a function g(x) that is usually much **simpler** than f(x).
- We accept the constant C in the requirement
- $f(x) \leq C \cdot g(x)$ whenever $x > k$,

- because **C does not grow with x**.
- We are only interested in large x, so it is OK if $f(x) > C \cdot g(x)$ for $x \leq k$.

What is Big-O

$f(n) = O(g(n))$ iff $\exists$ positive constants c and n_0 such that $0 \leq f(n) \leq cg(n)\ \forall\ n \geq n_0$.



Big-O Example

$f(x) = 6x^4 - 2x^3 + 5$

Prove $\mathrm{f(x)=O(n^4)}$

$$\begin{aligned} |6x^4 - 2x^3 + 5| &\leq 6x^4 + |2x^3| + 5 \\ &\leq 6x^4 + 2x^4 + 5x^4 \\ &= 13x^4 \end{aligned}$$

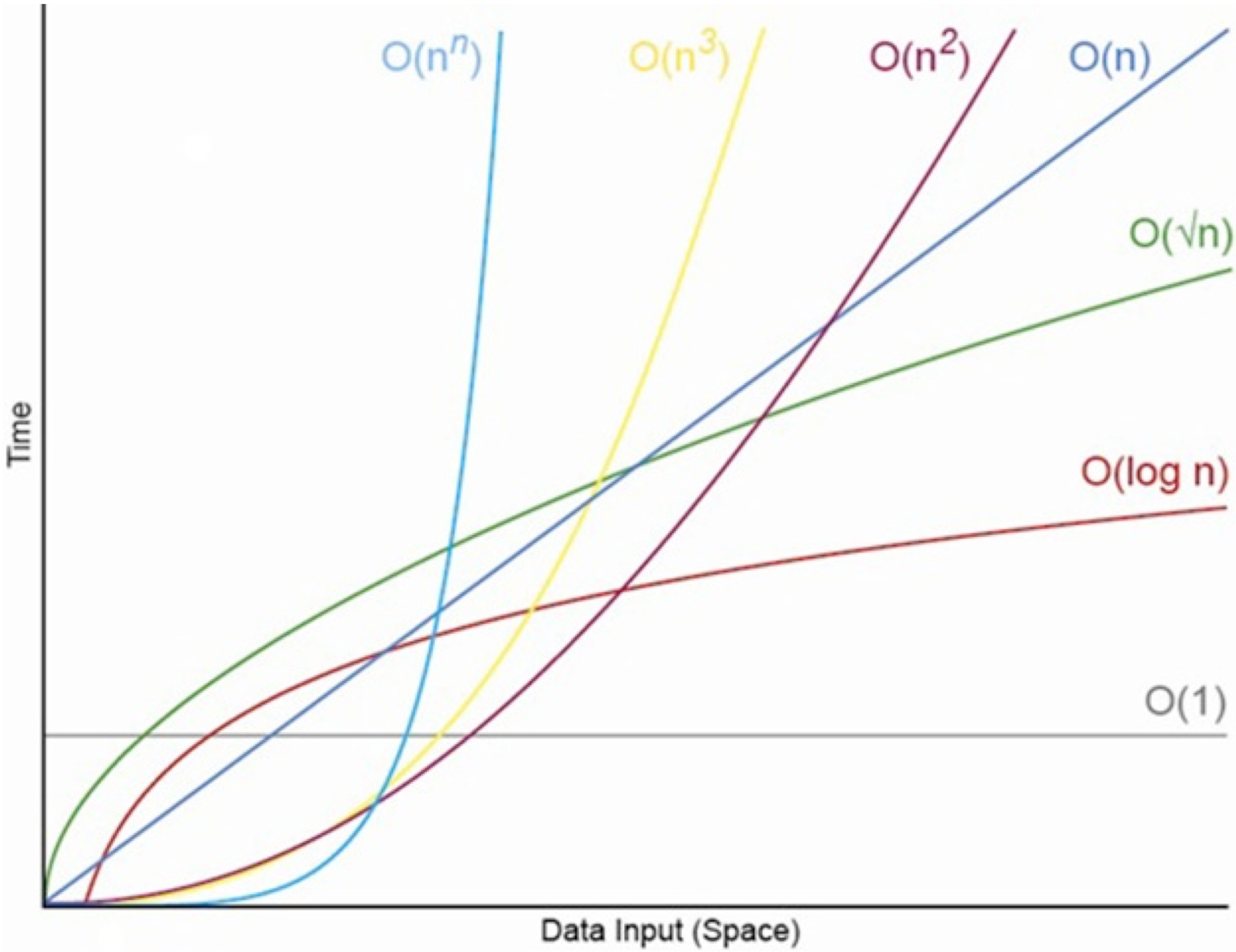
The Growth of Functions

- Example:
- Show that $f(x) = x^2 + 2x + 1$ is O(x²).
- For x > 1 we have:
- $\mathbf{x^2 + 2x + 1 \ \leq \ x^2 + 2x^2 + x^2}$
- $\Rightarrow \mathbf{x^2 + 2x + 1 \leq 4x^2}$
- Therefore, for C = 4 and k = 1:
- $f(x) \leq Cx^2$ whenever x > k.
- $\Rightarrow f(x) \text{ is } O(x^2)$.

Common Growth Rates

Big-O Characterization		**Example**
$O(1)$	*constant*	Adding to the front of a linked list
$O(\log N)$	*log*	Binary search
$O(N)$	*linear*	Linear search
$O(N \log N)$	*n-log-n*	Binary merge sort
$O(N^2)$	*quadratic*	Bubble Sort
$O(N^3)$	*cubic*	Simultaneous linear equations
$O(2^N)$	*exponential*	The Towers of Hanoi problem

Common Growth Rates



The Growth of Functions

- Question: If f(x) is $O(x^2)$, is it also $O(x^3)$?

- **Yes.** x^3 grows faster than x^2, so x^3 grows also faster than f(x).

- Therefore, we always have to find the **smallest** simple function g(x) for which f(x) is O(g(x)).

The Growth of Functions

- "Popular" functions g(n) are
  - n, log n, 1, 2^n, n^2, n!, n, n^3, log n
- Listed from slowest to fastest growth:
- 1
- log n
- n
- n log n
- n^2
- n^3
- 2^n
- n!

The Growth of Functions

- A problem that can be solved with polynomial worst-case complexity is called **tractable**.
- Problems of higher complexity are called **intractable.**
- Problems that no algorithm can solve are called **unsolvable**.

Determining Big-O: Repetition

executed
n times

```
for (i = 1; i <= n; i++)
{
    m = m + 2 ;
}
```

constant time

Total time = (a constant c) * n = cn = **O(*N*)**

Ignore multiplicative constants (e.g., "c").

Determining Big-O: Repetition

outer loop executed *n* times

```
for (i = 1; i <= n; i++)
{
   for (j = 1; j <= n; j++)
   {
      k = k+1 ;
   }
}
```

inner loop executed *n* times

constant time

Total time = c * n * n * = cn^2 = **O(*N*^{2})**

Determining Big-O: Repetition

outer loop
executed
n times

```
for (i = 1; i <= n; i++)
{
   for (j = 1; j <= 100; j++)
   {
      k = k+1 ;
   }
}
```

inner loop
executed
100 times

constant time

Total time = c * 100 * n * = 100cn = **O(*N*)**

Determining Big-O: Sequence

constant time (c_0) → `x = x +1;`

```
x = x +1;
for (i=1; i<=n; i++)
{
   m = m + 2;
}
for (i=1; i<=n; i++)
{
   for (j=1; j<=n; j++)
   {
      k = k+1;
   }
}
```

constant time (c_1) → `m = m + 2;`

executed *n* times

outer loop executed *n* times

inner loop executed *n* times

constant time (c_2) → `k = k+1;`

Total time = $c_0 + c_1 n + c_2 n^2$ = $\mathbf{O(}\boldsymbol{N}^{\mathbf{2}}\mathbf{)}$

Only dominant term is used

Determining Big-O: Selection

test + worst-case(then, else)

test: constant (c_0)

```
if (depth( ) != otherStack.depth( ) )
{
  return false;
}
else
{
  for (int n = 0; n < depth( ); n++)
  {
    if (!list[n].equals(otherStack.list[n]))
      return false;
  }
}
```

then part: constant (c_1)

else part: $(c_2 + c_3) * n$

another if : test (c_2) + then (c_3)

Total time = c_0 + Worst-Case(c_1, $(c_2 + c_3) * n$) = **O(*N*)**

Total time = c_0 + Worst-Case(then, else)

Total time = c_0 + Worst-Case(c_1, else)

Quiz 1

What is the Big-O of the following code?

```
void foo(int n){
  int i;
  for(int i = 1; i < n; n++);
  print("good");
}
```

A. $O(n^2)$
B. $O(\log n)$
C. $O(n)$
D. $O(1)$

Quiz 1

What is the Big-O of the following code?

```
void foo(int n){
  int i;
  for(int i = 1; i < n; n++);
  print("good");
}
```

A. $O(n^2)$
B. O(log n)
C. O(n)
D. O(1)

Quiz 2

What is the Big-O of the following code?

```
void foo(int n){
  int i;
  for(int i = 1; i < n; i++)
    for(int j = 1; j < n; j++)
       print("good");
}
```

A. $O(n^2)$
B. O(log n)
C. O(n)
D. O(1)

Quiz 2

What is the Big-O of the following code?

```
void foo(int n){
  int i;
  for(int i = 1; i < n; i++);
    for(int j = 1; j < n; j++);
       print("good");
}
```

A. **$O(n^2)$**
B. O(log n)
C. O(n)
D. O(1)

Quiz 3

What is the Big-O of the following code?

```
void foo(int n){
   int i = 1;
   int s = 1;
   while(s <= n){
    i++;
    s = s + i;
    print("work");
   }
}
```

A. $O(n^2)$
B. $O(\log n)$
C. $O(n)$
D. $O(\sqrt{n})$

Quiz 3

What is the Big-O of the following code?

```
void foo(int n){
    int i = 1;
    int s = 1;
    while(s <= n){
     i++;
     s = s + i;
     print("work");
    }
}
```

A. $O(n^2)$
B. O(log n)
C. O(n)
D. $O(\sqrt{n})$

S = 1
1+2
1+2+3
S_k= 1+2+3+k+(k+1) after k iteration
S_k = 2(k+1) k <= n
k < sqrt(n)

Quiz 4

What is the Big-O of the following code?

```
void foo(int n){
  int i;
  for(i = 1; i*i <= n; i++)
  print("hello");
}
```

A. $O(n^2)$
B. $O(\log n)$
C. $O(n)$
D. $O(\sqrt{n})$

Quiz 4

What is the Big-O of the following code?

```
void foo(int n){
  int i;
  for(i = 1; i*i <= n; i++)
  print("hello");
}
```

A. $O(n^2)$
B. O(log n)
C. O(n)
D. $O(\sqrt{n})$

Quiz 5

What is the Big-O of the following code?

```
void foo(int n){
  int i,j,k;
  for(i = 1; i <= n; i++)
    for(j = 1; j <= i; j++)
     for(k=1; k <= 100; k++)
      print("good");
}
```

A. $O(n^2)$
B. O(log n)
C. O(n)
D. $O(\sqrt{n})$

Quiz 5

What is the Big-O of the following code?

```
void foo(int n){
  int i,j,k;
  for(i = 1; i <= n; i++)
    for(j = 1; j <= i; j++)
     for(k=1; k <= 100; k++)
      print("good");
}
```

A. $O(n^2)$
B. O(log n)
C. O(n)
D. $O(\sqrt{n})$

total = 100 + 200 + 300 + 400 + 500 = 100 (1+2+3+..+n) = 100(n(n-1)/2) = O(n^2)

Quiz 6

What is the Big-O of the following code?

```
void foo(int n){
  for(int i = 1; i < n; i = i * 2)
        print("good");
}
```

A. $O(n^2)$
B. $O(\log n)$
C. $O(n)$
D. $O(\sqrt{n})$

Quiz 6

What is the Big-O of the following code?

```
void foo(int n){
  for(int i = 1; i < n; i = i * 2)
      print("good");
}
```

A. $O(n^2)$
B. $O(\log n)$
C. $O(n)$
D. $O(\sqrt{n})$