

Lecture 13

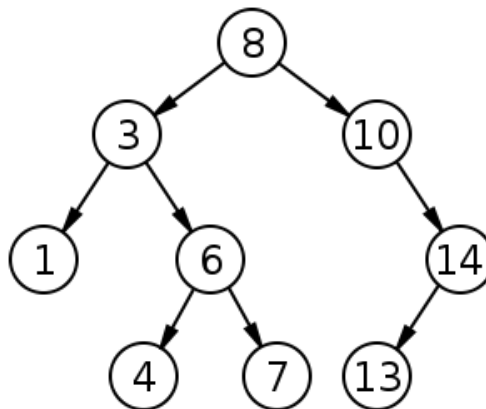
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13.1 Binary Search Tree

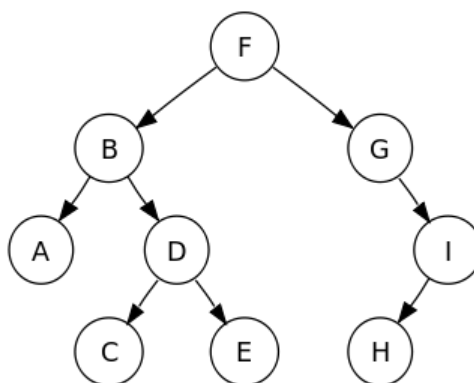
A binary search tree is a rooted binary tree, in which key in each node must be greater than all keys stored in the left sub-tree, and smaller than all keys in right sub-tree. InOrder traversal of a binary search tree sorts all keys.

Figure 13.1: Binary Search Tree



InOrder traversal of the binary search tree shown in Figure 13.1: 1,3,4,6,7,8,10,13,14

Figure 13.2: Binary Search Tree



InOrder traversal of the binary search tree shown in Figure 13.1: A,B,C,D,E,F,G,H,I

13.1.1 Binary Search Tree Implementation

Listing 1: Binary Search Tree

```

1 public class BST<Key extends Comparable<Key>, Value> {
2     private Node root;
3     private class Node{
4         private Key key;
5         private Value value;
6         private Node left, right;
7         public Node(Key k, Value v){
8             key = k;
9             value = v;
10        }
11    }
12    BST(){
13        root = null;
14    }
15    BST(Node r){
16        this.root = r;
17    }
18
19
20    public void put(Key key, Value val){
21        if(val == null) {
22            delete(key);
23            return;
24        }
25        root = put(root, key, val);
26        //dumpToGraphViz();
27    }
28
29    private Node put(Node x, Key key, Value val){
30        if(x == null) { return new Node(key,val);}
31        int cmp = key.compareTo(x.key);
32        if(cmp < 0) {x.left = put(x.left, key, val);}
33        else if (cmp > 0) {x.right = put(x.right, key, val);}
34        else {x.value = val;}
35        return x;
36    }
37
38
39    public Value get(Key key){
40        return get(root, key);
41    }
42    private Value get(Node x, Key key){
43        if(x == null) return null;
44        int cmp = key.compareTo(x.key);
45        if(cmp < 0) return get(x.left, key);
46        else if (cmp > 0) return get(x.right, key);
47        else return x.value;
48    }
49
50
51    public void delete(Key key){
52        root = delete(root, key);
53    }
54
55    public Value value(){
56        return root.value;
57    }
58    public String info(){
59        return root.key + "/" + root.value;
60    }

```

```

61
62 public BST<Key, Value> getLeft() {
63     return new BST(root.left);
64 }
65 public BST<Key, Value> getRight() {
66     return new BST(root.right);
67 }
68
69 private Node delete(Node x, Key k) {
70     if (x == null) return null;
71     int cmp = k.compareTo(x.key);
72     if (cmp < 0) x.left = delete(x.left, k);
73     else if (cmp > 0) x.right = delete(x.right, k);
74     else {
75         if (x.right == null) return x.left;
76         if (x.left == null) return x.right;
77         Node t = x;
78         x = min(t.right);
79         x.right = deleteMin(t.right);
80         x.left = t.left;
81     }
82     return x;
83 }
84
85 public void deleteMin() {
86     deleteMin(root);
87 }
88
89 private Node deleteMin(Node x) {
90     if (x.left == null) return x.right;
91     x.left = deleteMin(x.left);
92     //System.out.println("Key=" + x.key + "\tValue=" + x.value);
93     return x;
94 }
95 public void deleteMax() {
96     root = deleteMax(root);
97 }
98
99 private Node deleteMax(Node x) {
100     if (x.right == null) return x.left;
101     x.right = deleteMax(x.right);
102     return x;
103 }
104
105 public Key min() {
106     if (isEmpty()) return null;
107     return min(root).key;
108 }
109
110 private Node min(Node x) {
111     if (x.left == null) return x;
112     return min(x.left);
113 }
114
115 public Key max() {
116     if (isEmpty()) return null;
117     return max(root).key;
118 }
119
120 private Node max(Node x) {
121     if (x.right == null) return x;
122     return max(x.right);
123 }
124 public boolean isEmpty() {
125     return size() == 0;
126 }
127
128 public int size() {

```

```

129         return size(root);
130     }
131
132     private int size(Node x){
133         if(x == null) return 0;
134         else return 1 + size(x.left) + size(x.right);
135     }
136
137     private int height(Node r){
138         if(r == null) return -1;
139         return 1 + Math.max(height(r.left), height(r.right));
140     }
141     public int height(){
142
143         return height(root);
144     }
145     public boolean contains(Key key){
146         return get(key) == null;
147     }
148     public void preOrder(){
149         preOrder(root);
150     }
151     private void preOrder(Node r){
152         if(r == null) return;
153         System.out.print(r.key+", ");
154         preOrder(r.left);
155         preOrder(r.right);
156     }
157
158     public void postOrder(){
159         postOrder(root);
160     }
161     private void postOrder(Node r){
162         if(r == null) return;
163         postOrder(r.left);
164         postOrder(r.right);
165         System.out.print(r.key+", ");
166     }
167
168     public void inOrder(){
169         inOrder(root);
170     }
171     private void inOrder(Node r){
172         if(r == null) return;
173         inOrder(r.left);
174         System.out.print(r.key+", ");
175         inOrder(r.right);
176     }
177
178     public void levelOrder(){
179         levelOrder(root);
180     }
181     private void levelOrder(Node r){
182         if( r == null) return;
183         Queue<Node> q = new LinkedList();
184         q.offer(r);
185         while(!q.isEmpty()){
186             Node t = q.poll();//pop
187             System.out.print("<"+t.key+", "+t.value+">");
188             if(t.left != null) q.offer(t.left);
189             if(t.right != null) q.offer(t.right);
190         }
191     }
192     public static void main(String[] args) {
193         BST<Integer, String> bst = new BST();
194         bst.put(100, "Alice");
195         bst.put(200, "Bob");
196     }

```

197 | }

13.1.2 Non Recursive Implementation

Listing 2: Binary Search Tree

```

1
2 /*****
3  * Copyright  2002?2010, Robert Sedgewick and Kevin Wayne.
4  * Compilation:  javac NonrecursiveBST.java
5  * Execution:   java  NonrecursiveBST < input.txt
6  *
7  * A symbol table implemented with a binary search tree using
8  * iteration instead of recursion for put(), get(), and keys().
9  *
10 *****/
11
12 public class NonrecursiveBST<Key extends Comparable<Key>, Value> {
13
14     // root of BST
15     private Node root;
16
17     private class Node {
18         private Key key;           // sorted by key
19         private Value val;         // associated value
20         private Node left, right;  // left and right subtrees
21
22         public Node(Key key, Value val) {
23             this.key = key;
24             this.val = val;
25         }
26     }
27
28
29 /*****
30  * Insert key-value pair into symbol table (nonrecursive version)
31  *****/
32     public void put(Key key, Value val) {
33         Node z = new Node(key, val);
34         if (root == null) {
35             root = z;
36             return;
37         }
38
39         Node parent = null, x = root;
40         while (x != null) {
41             parent = x;
42             int cmp = key.compareTo(x.key);
43             if (cmp < 0) x = x.left;
44             else if (cmp > 0) x = x.right;
45             else {
46                 x.val = val;
47                 return;
48             }
49         }
50         int cmp = key.compareTo(parent.key);
51         if (cmp < 0) parent.left = z;
52         else parent.right = z;
53     }
54
55
56 /*****
57  * Search BST for given key, nonrecursive version
58  *****/
59     Value get(Key key) {

```

```

60     Node x = root;
61     while (x != null) {
62         int cmp = key.compareTo(x.key);
63         if (cmp < 0) x = x.left;
64         else if (cmp > 0) x = x.right;
65         else return x.val;
66     }
67     return null;
68 }
69
70 /*****
71  * Level-order traversal - need to make nonrecursive.
72  *****/
73 public Iterable<Key> keys() {
74     Stack<Node> stack = new Stack<Node>();
75     Queue<Key> queue = new Queue<Key>();
76     Node x = root;
77     while (x != null || !stack.isEmpty()) {
78         if (x != null) {
79             stack.push(x);
80             x = x.left;
81         }
82         else {
83             x = stack.pop();
84             queue.enqueue(x.key);
85             x = x.right;
86         }
87     }
88     return queue;
89 }
90
91 /*****
92  * Test client
93  *****/
94 public static void main(String[] args) {
95     String[] a = StdIn.readAllStrings();
96     int N = a.length;
97     NonrecursiveBST<String, Integer> st = new NonrecursiveBST<String, Integer>();
98     for (int i = 0; i < N; i++)
99         st.put(a[i], i);
100     for (String s : st.keys())
101         StdOut.println(s + "_" + st.get(s));
102 }
103
104 }
105

```

13.2 BST exercises

- check if a given binary tree is a BST
- Find the minimum key
- Find the maximum key
- Build a BST from a preOrder traversal