CMSC 330: Organization of Programming Languages

DFAs, and NFAs, and Regexps (Oh my!)

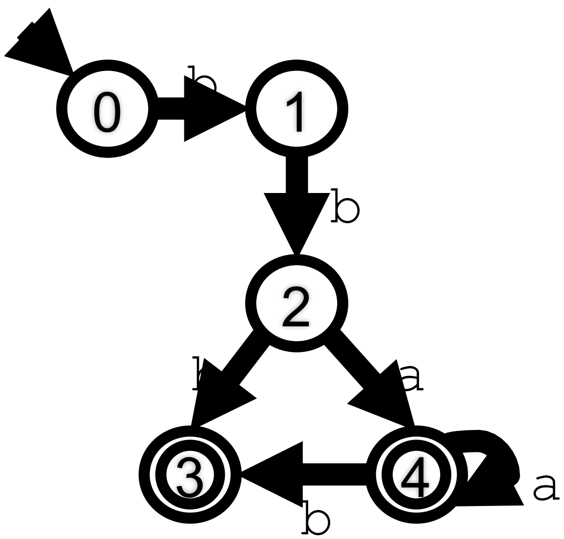
Types of Finite Automata

- Deterministic Finite Automata (DFA)
  - Exactly one sequence of steps for each string
  - All examples so far
- Nondeterministic Finite Automata (NFA)
  - May have many sequences of steps for each string
  - Accepts if any path ends in final state at end of string
  - More compact than DFA
    - But more expensive to test whether a string matches

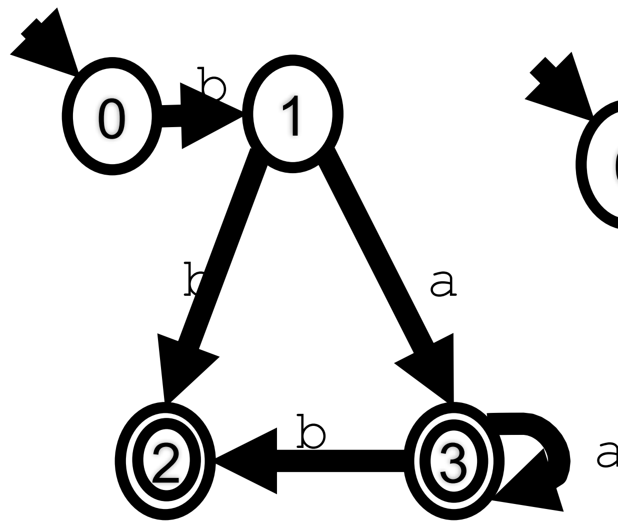
Quiz 1: Which DFA matches this regexp?

`b(b|a+b?)`

A.



B.



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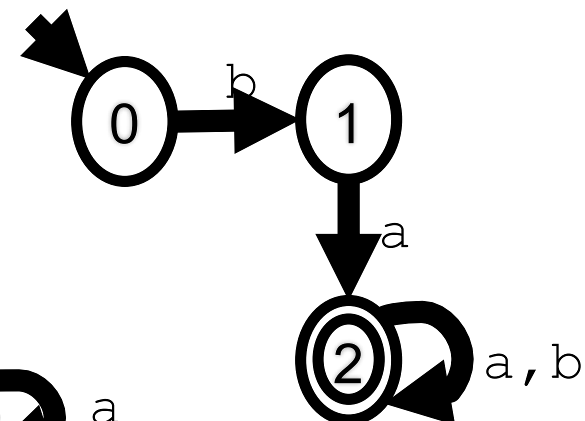

D. None of the above

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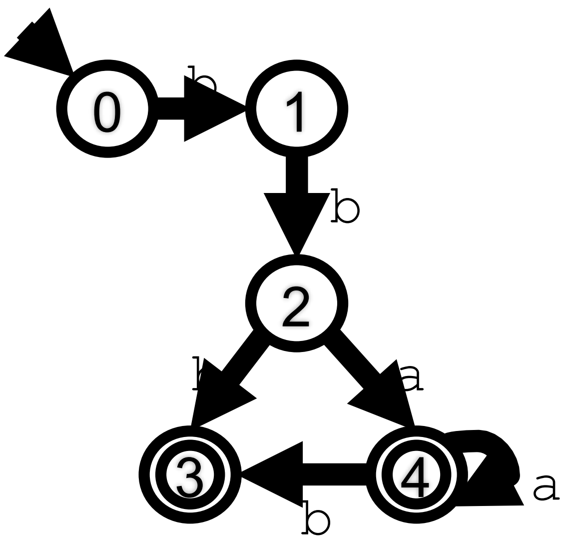

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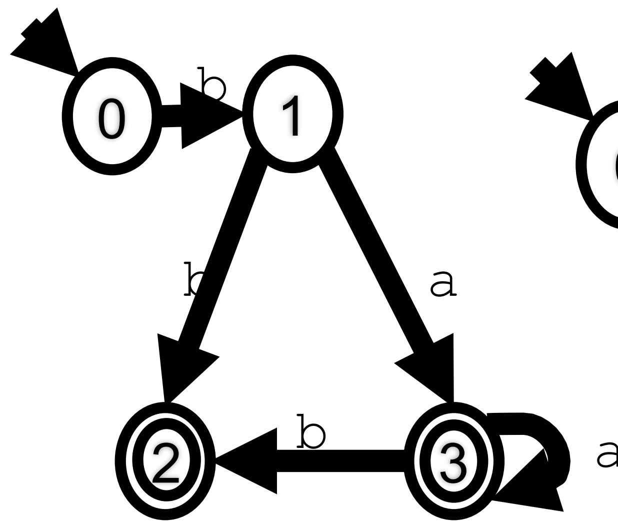

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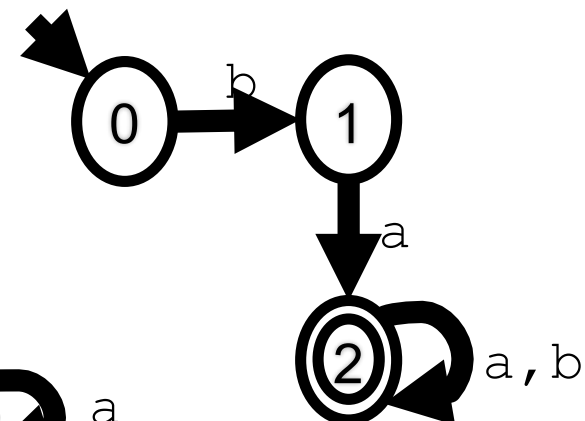
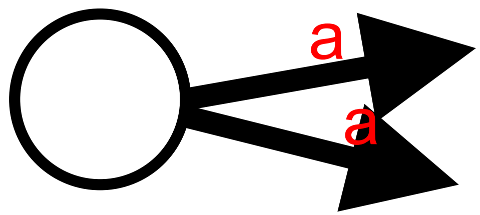


D. None of the above

Comparing DFAs and NFAs

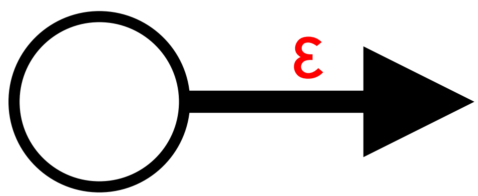
- NFAs can have more than one transition leaving a state on the same symbol



- DFAs allow only one transition per symbol
  - I.e., transition function must be a valid function
  - DFA is a special case of NFA

Comparing DFAs and NFAs (cont.)

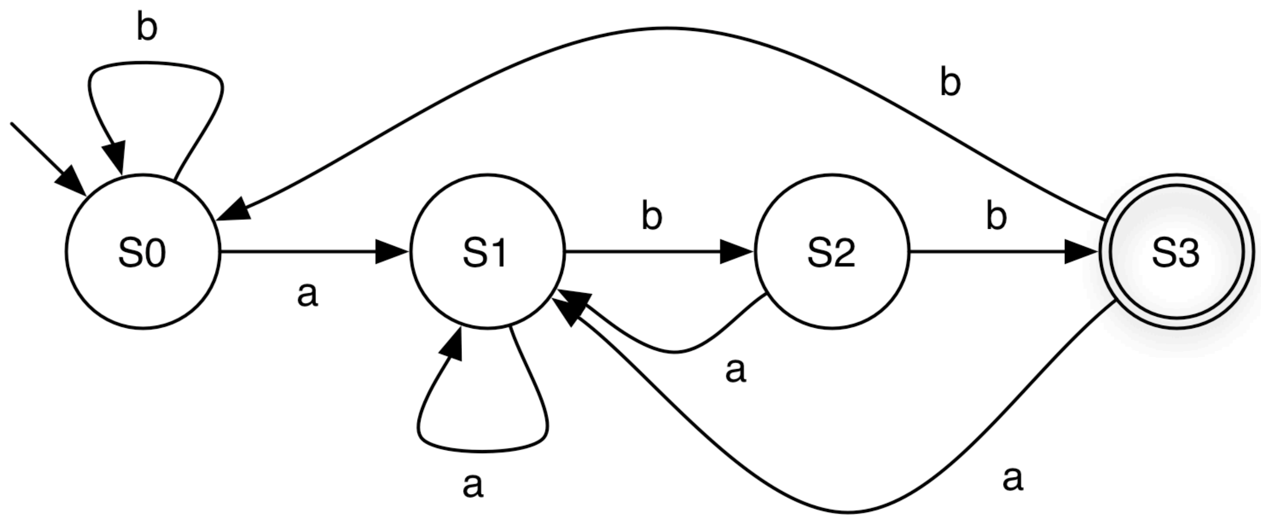
- NFAs may have transitions with empty string label
  - May move to new state without consuming character



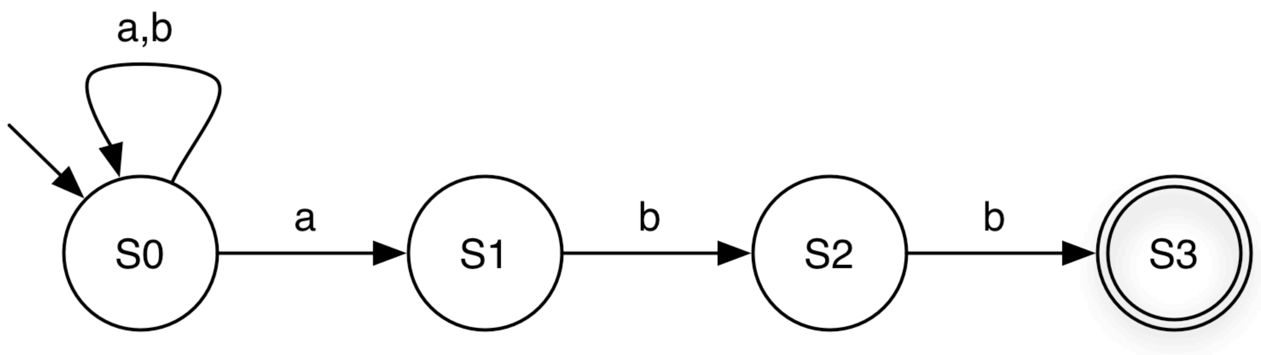
ε-transition

- DFA transition must be labeled with symbol
  - DFA is a special case of NFA

DFA for (a|b)*abb

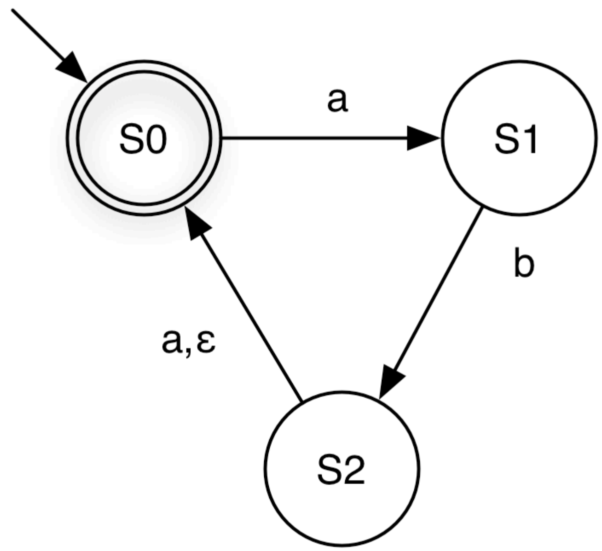


NFA for (a|b)*abb



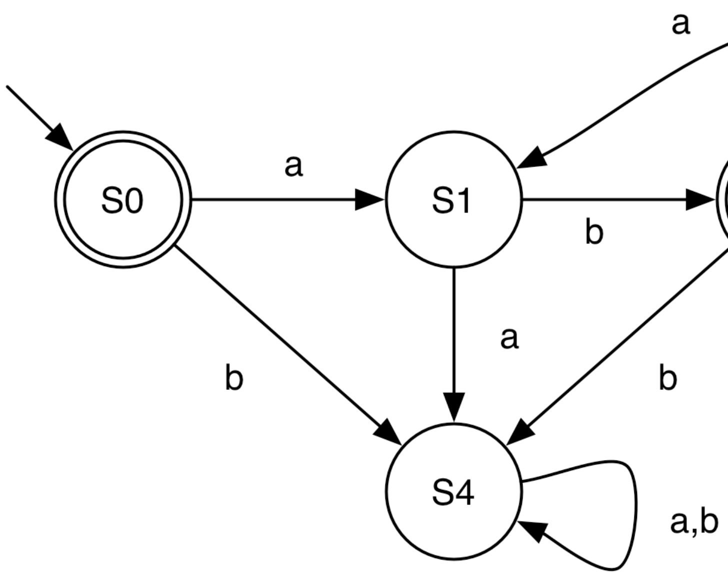
- ba
  - Has paths to either S0 or S1
  - Neither is final, so rejected
- babaabb
  - Has paths to different states
  - One path leads to S3, so accepts string

NFA for (ab|aba)*

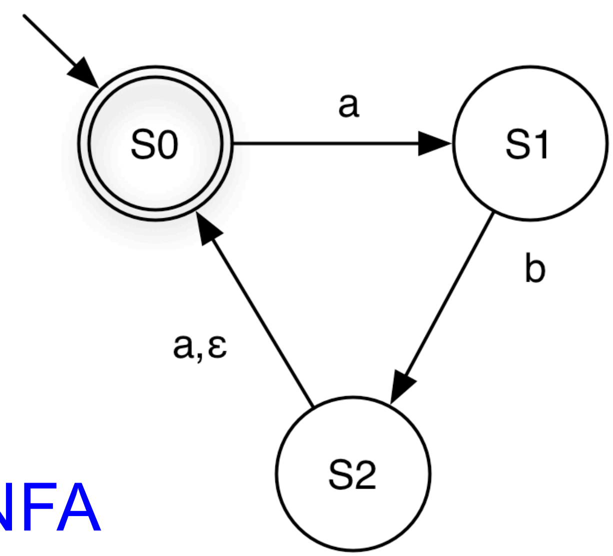


- aba
  - Has paths to states S0, S1
- ababa
  - Has paths to S0, S1
  - Need to use ε-transition

Comparing NFA and DFA for (ab|aba)*



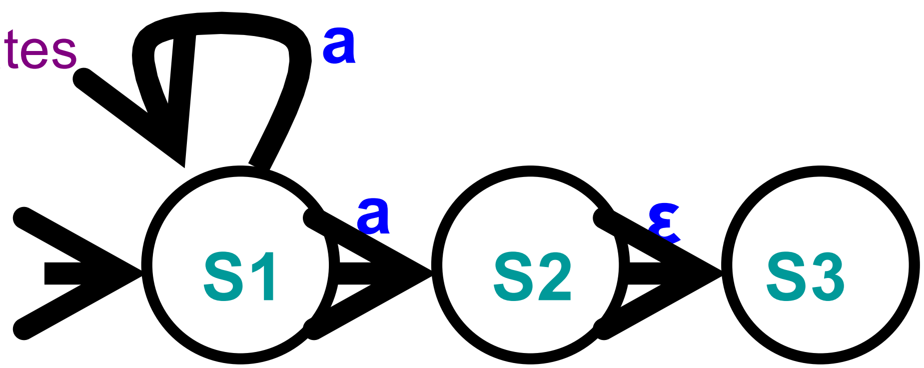
DFA



NFA

NFA Acceptance Algorithm Sketch

- When NFA processes a string s
  - NFA must keep track of several “current states”
    - Due to multiple transitions with same label
    - ε-transitions
  - If any current state is final when done then accept s
- Example
  - After processing “a”
    - NFA may be in states
      S1
      S2
      S3



Formal Definition

- A deterministic finite automaton *(DFA)* is a 5-tuple $(\Sigma, Q, q_0, F, \delta)$ where
  - Σ is an alphabet
  - Q is a nonempty set of states
  - $q_0 \in Q$ is the start state
  - $F \subseteq Q$ is the set of final states
  - $\delta : Q \times \Sigma \rightarrow Q$ specifies the DFA's transitions
    - What's this definition saying that δ is?
- A DFA accepts s if it stops at a final state on s

Formal Definition: Example

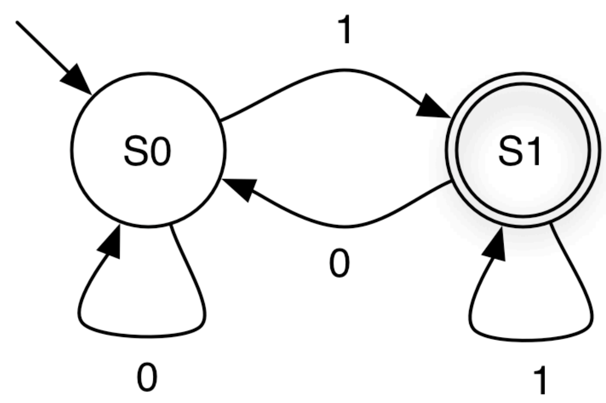
- $\Sigma = \{0, 1\}$
- $Q = \{S0, S1\}$
- $q_0 = S0$
- $F = \{S1\}$
-

symbol

δ	0	1
S0	S0	S1
S1	S0	S1

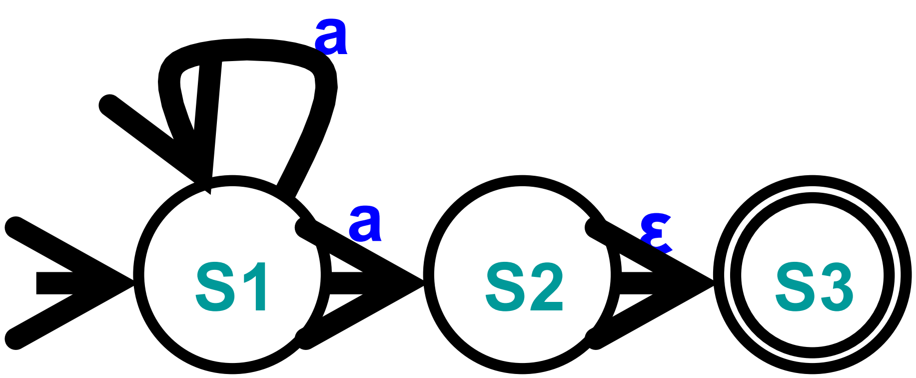
input state

or as { (S0,0,S0),(S0,1,S1),(S1,0,S0),(S1,1,S1) }



Nondeterministic Finite Automata (NFA)

- An *NFA* is a 5-tuple $(\Sigma, Q, q_0, F, \delta)$ where
  - Σ, Q, $q0$, F as with DFAs
  - $\delta \subseteq Q \times (\Sigma \cup \{\varepsilon\}) \times Q$ specifies the NFA's transitions



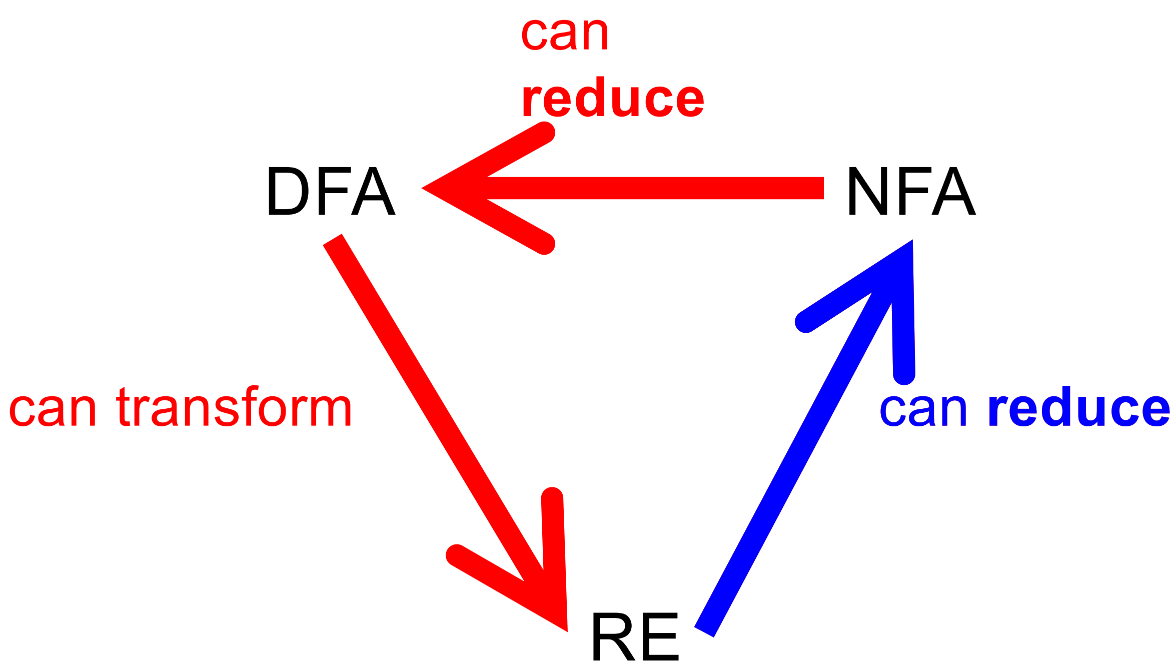
Example

- $\Sigma = \{a\}$
- $Q = \{S1, S2, S3\}$
- $q_0 = S1$
- $F = \{S3\}$
- $\delta = \{ (S1,a,S1), (S1,a,S2), (S2,\varepsilon,S3) \}$

- An NFA accepts s if there is at least one path via s from the NFA's start state to a final state

Relating REs to DFAs and NFAs

- Regular expressions, NFAs, and DFAs accept the same languages!



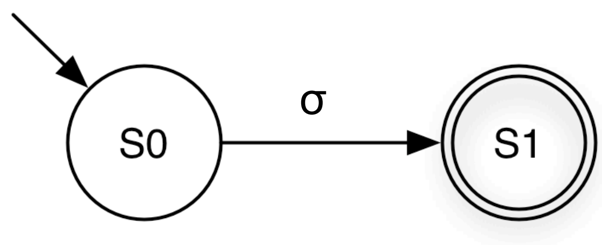
Reducing Regular Expressions to NFAs

- Goal: Given regular expression *A*, construct NFA: <*A*> $= (\Sigma, Q, q_0, F, \delta)$
  - Remember regular expressions are defined recursively from primitive RE languages
  - Invariant: $|F| = 1$ in our NFAs
    - Recall F = set of final states

- Will define <*A*> for base cases: σ , ε , $\emptyset$
  - Where σ is a symbol in Σ
- And for inductive cases: AB, $A|B$, A^*

Reducing Regular Expressions to NFAs

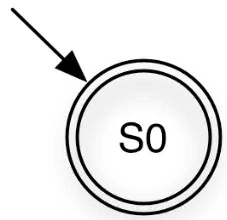
- Base case: σ



<σ> = ({σ}, {S0, S1}, S0, {S1}, {(S0, σ, S1)})

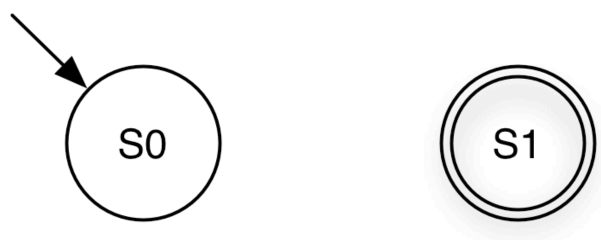
Reduction

- Base case: ε



<ε> = (∅, {S0}, S0, {S0}, ∅)

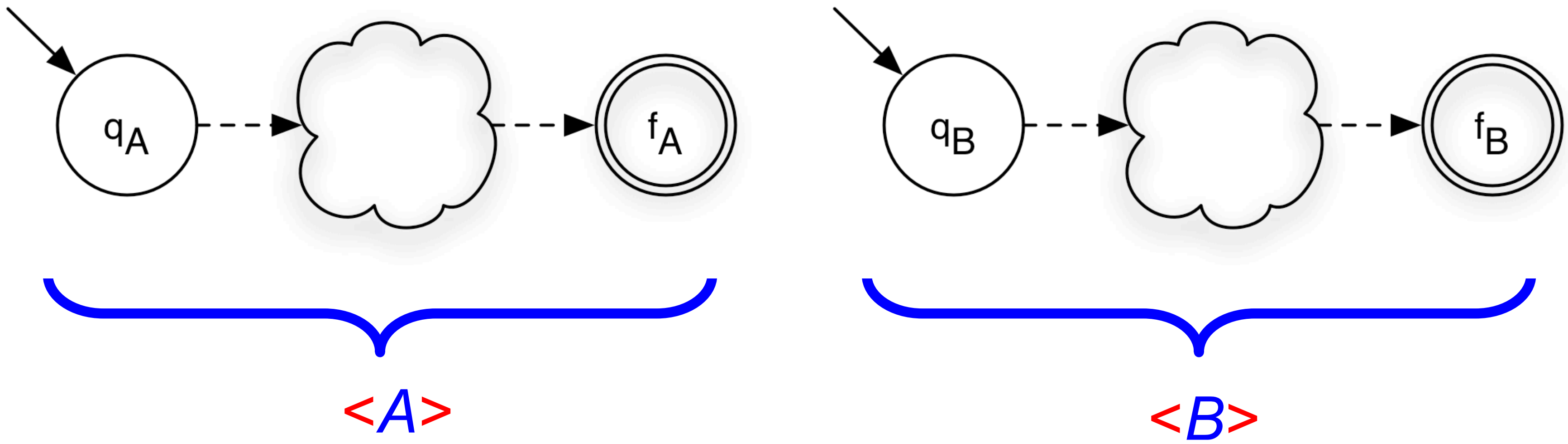
- Base case: ∅



<∅> = (∅, {S0, S1}, S0, {S1}, ∅)

Reduction: Concatenation

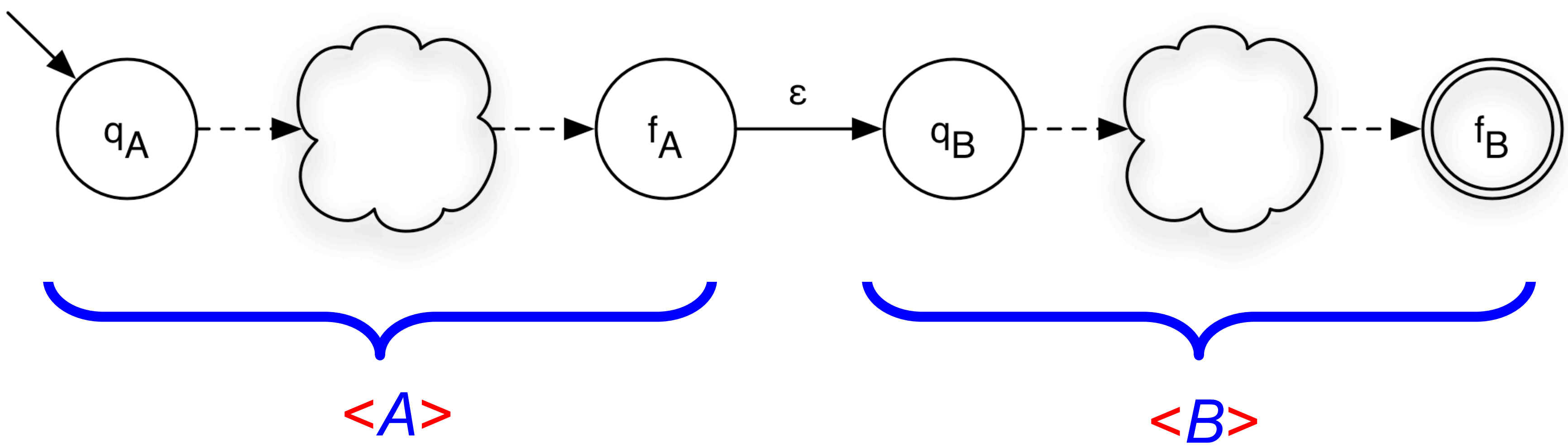
- Induction: *AB*



- $\langle A \rangle = (\Sigma_A, Q_A, q_A, \{f_A\}, \delta_A)$
- $\langle B \rangle = (\Sigma_B, Q_B, q_B, \{f_B\}, \delta_B)$

Reduction: Concatenation

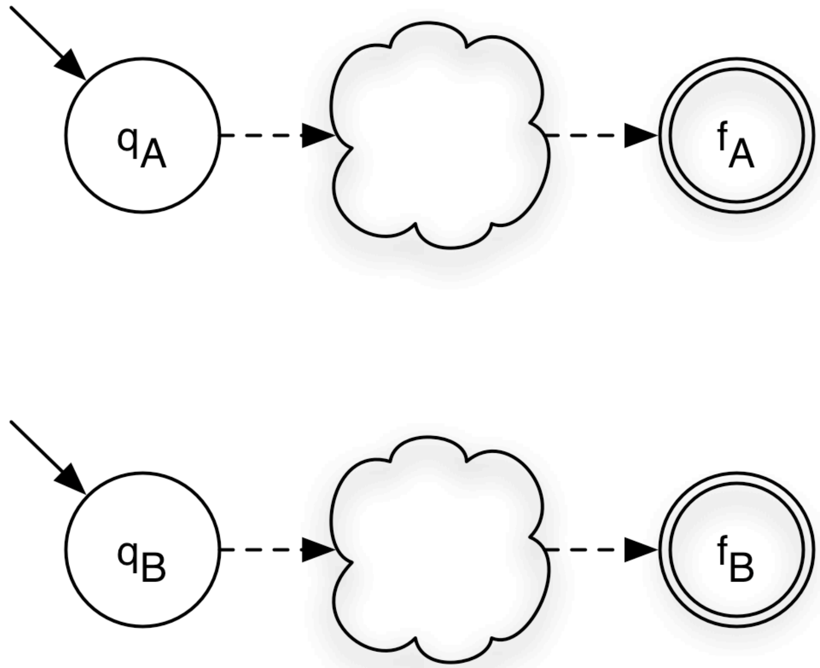
- Induction: *AB*



- $\langle A \rangle = (\Sigma_A, Q_A, q_A, \{f_A\}, \delta_A)$
- $\langle B \rangle = (\Sigma_B, Q_B, q_B, \{f_B\}, \delta_B)$
- $\langle AB \rangle = (\Sigma_A \cup \Sigma_B, Q_A \cup Q_B, q_A, \{f_B\}, \delta_A \cup \delta_B \cup \{(f_A, \varepsilon, q_B)\})$

Reduction: Union

- Induction: $A|B$



  - $\langle A \rangle = (\Sigma_A, Q_A, q_A, \{f_A\}, \delta_A)$
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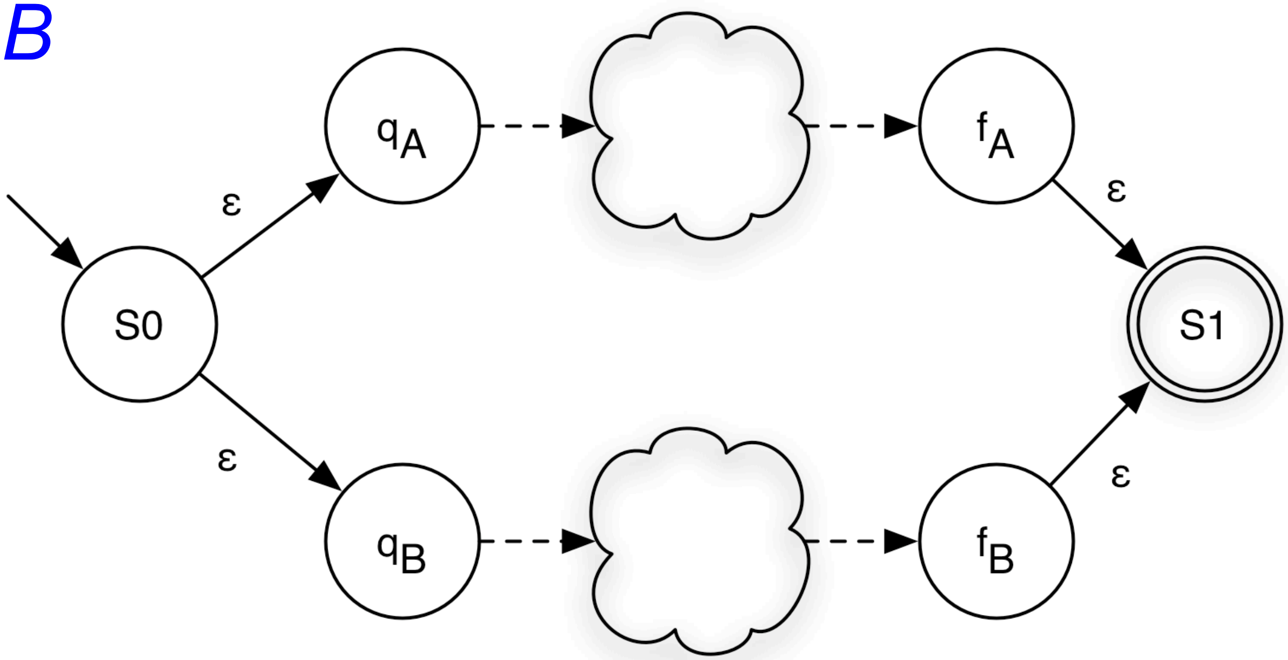
Reduction: Union

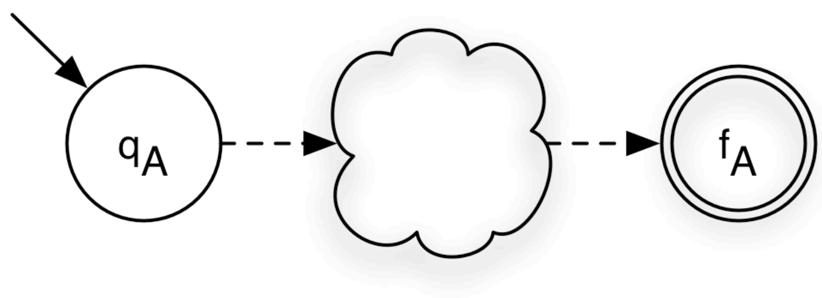
- Induction: *A*|*B*



  - <*A*> = $(\Sigma_A, Q_A, q_A, \{f_A\}, \delta_A)$
  - <*B*> = $(\Sigma_B, Q_B, q_B, \{f_B\}, \delta_B)$
  - <*A*|*B*> = $(\Sigma_A \cup \Sigma_B, Q_A \cup Q_B \cup \{S0, S1\}, S0, \{S1\},$
    $\delta_A \cup \delta_B \cup \{(S0,\varepsilon,q_A), (S0,\varepsilon,q_B), (f_A,\varepsilon,S1), (f_B,\varepsilon,S1)\})$

Reduction: Closure

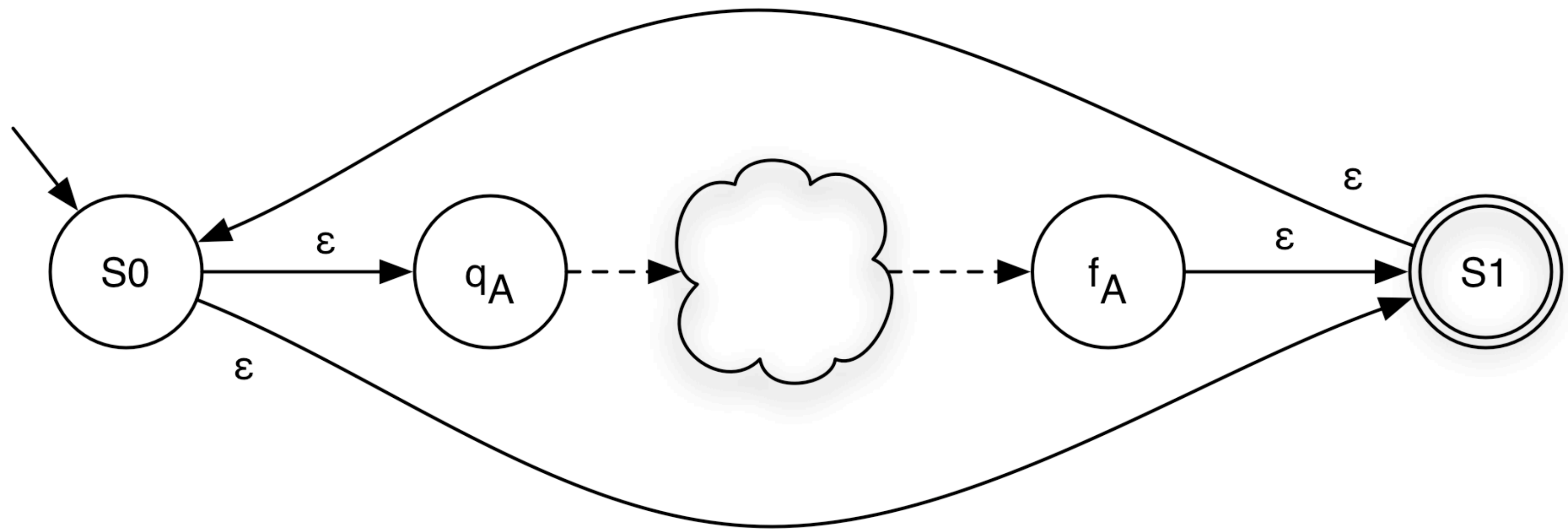
- Induction: *A**



- <*A*> = $(\Sigma_A, Q_A, q_A, \{f_A\}, \delta_A)$

Reduction: Closure

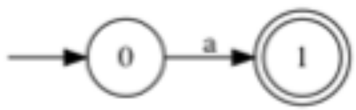
- Induction: A^*



- $\langle A \rangle = (\Sigma_A, Q_A, q_A, \{f_A\}, \delta_A)$
- $\langle A^* \rangle = (\Sigma_A, Q_A \cup \{S0, S1\}, S0, \{S1\},$
  $\delta_A \cup \{(f_A, \varepsilon, S1), (S0, \varepsilon, q_A), (S0, \varepsilon, S1), (S1, \varepsilon, S0)\})$

Quiz 2: Which NFA matches **a*** ?

A.



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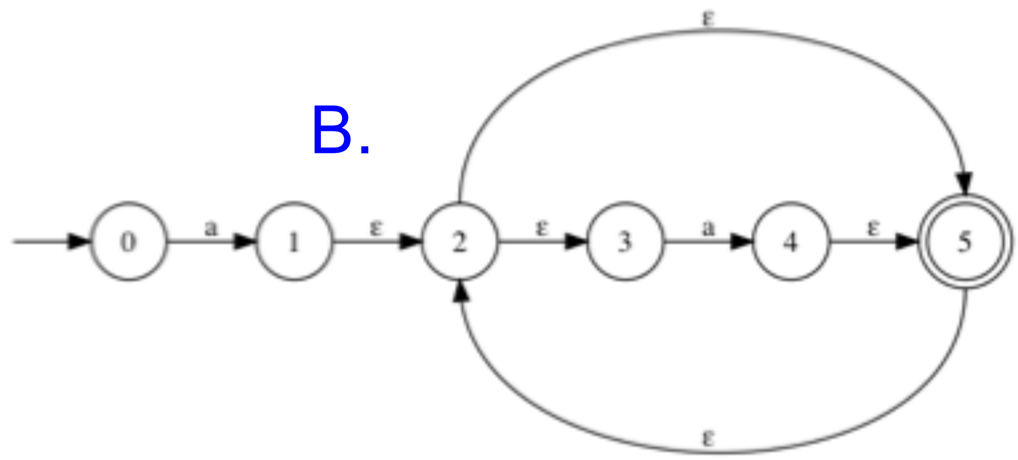

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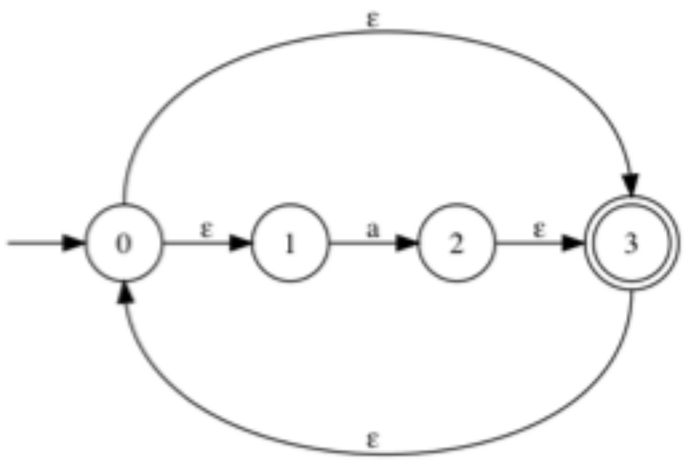
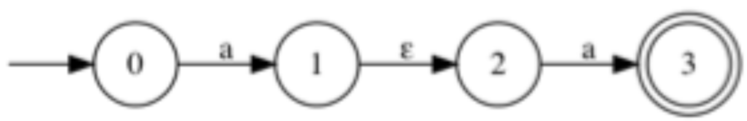


D.



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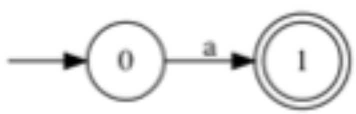

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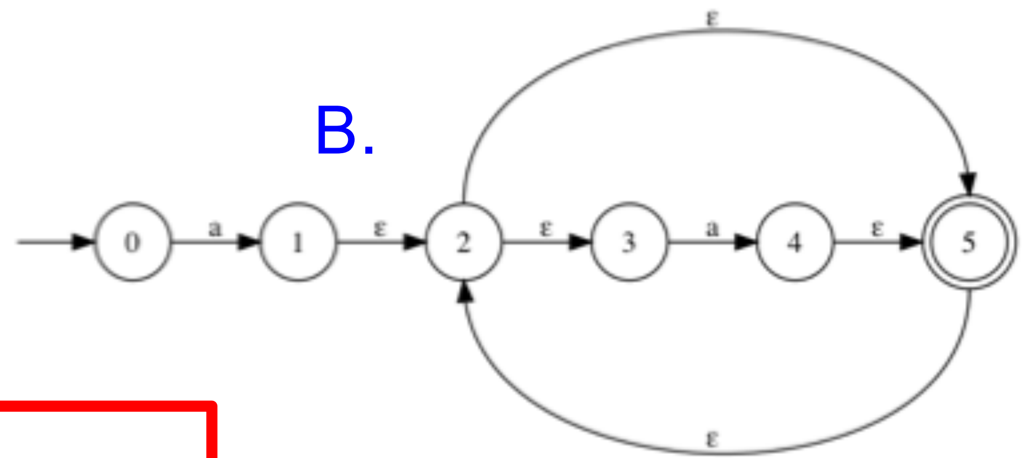

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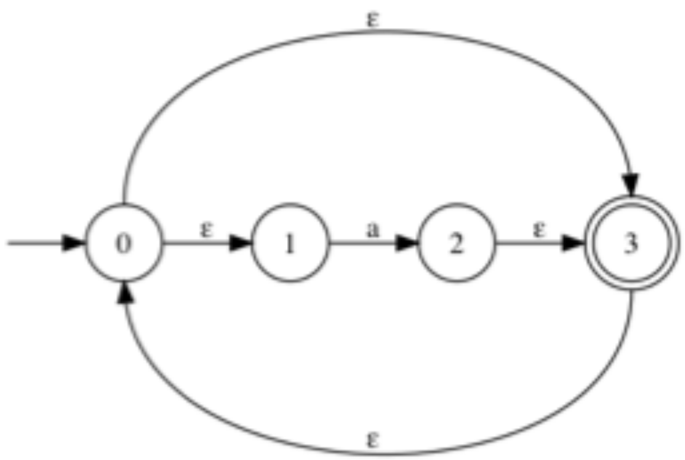

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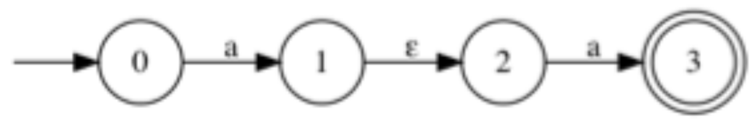

Quiz 3: Which NFA matches **a|b*** ?

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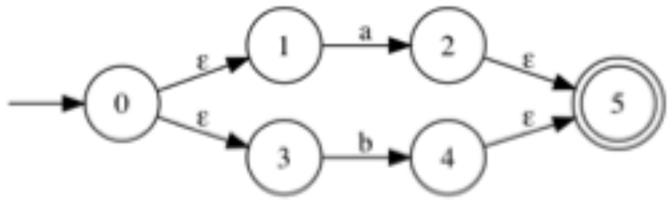

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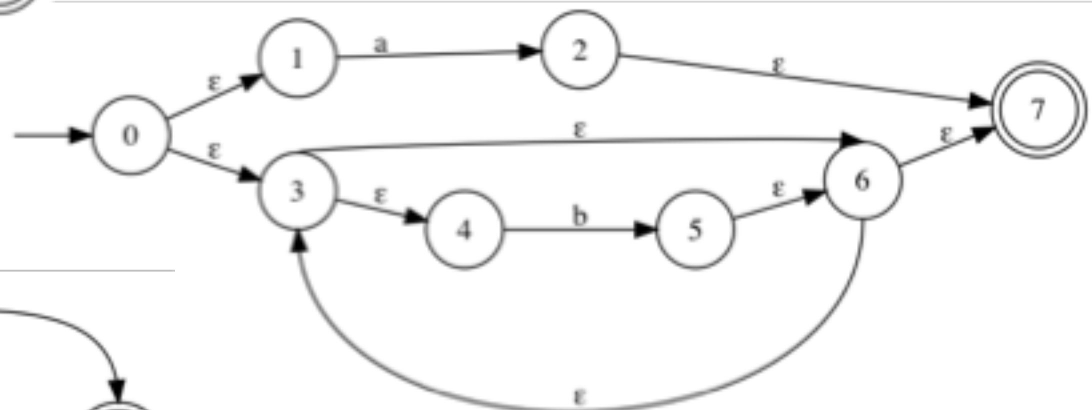

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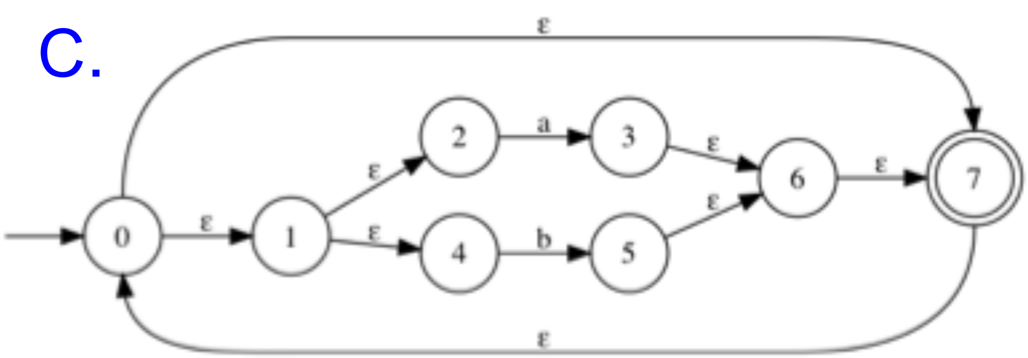
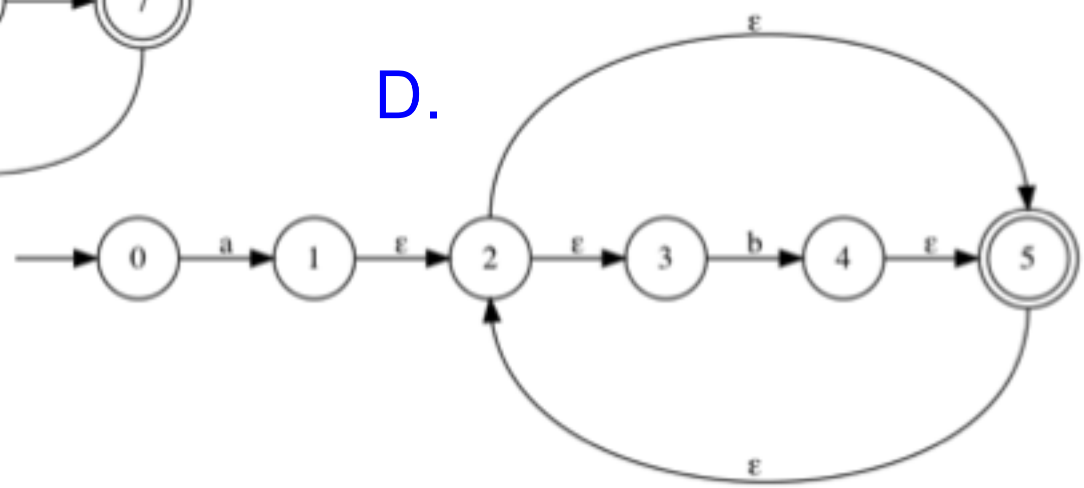


D.



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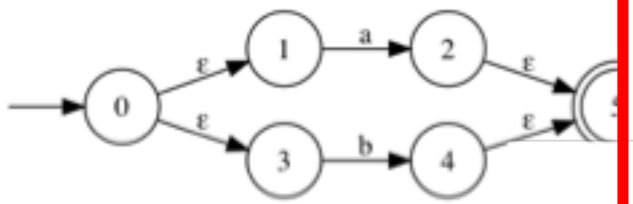

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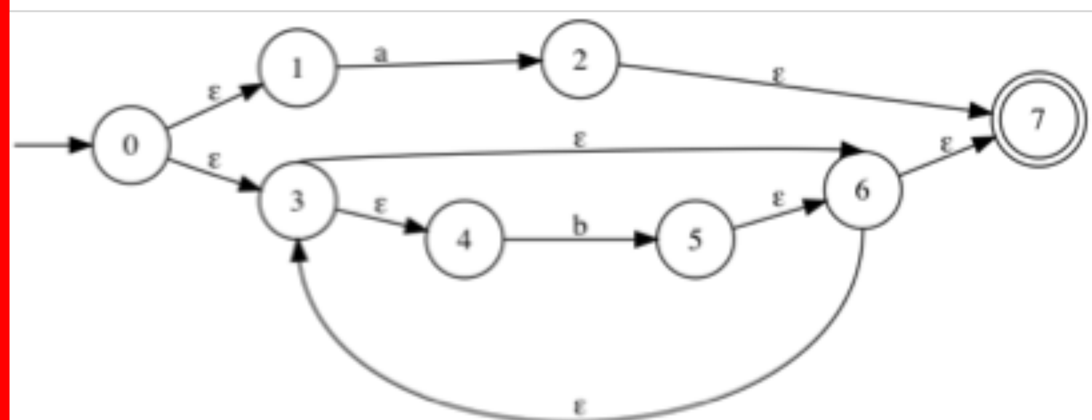

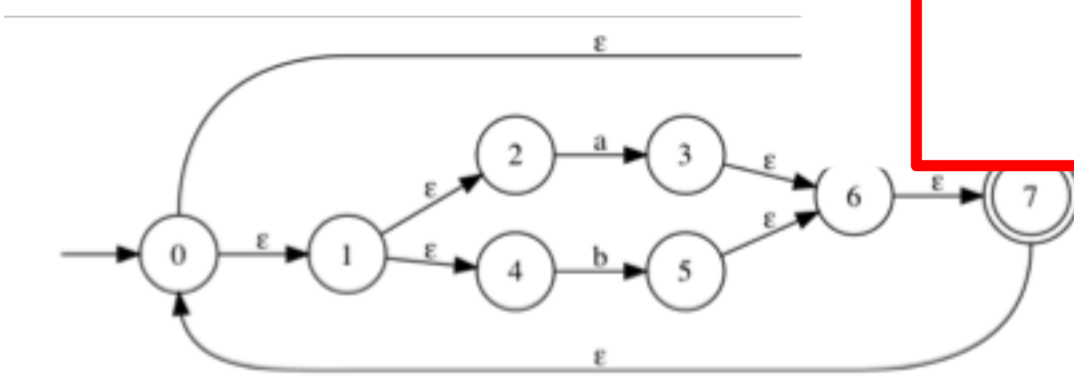

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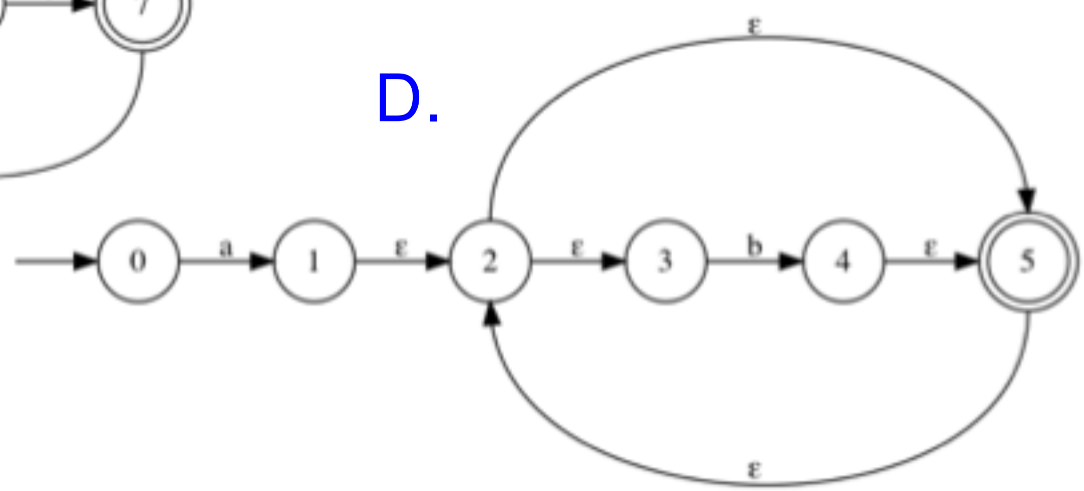


RE → NFA

Draw NFAs for the regular expression (0|1)*110*

RE → NFA

Draw NFAs for the regular expression (ab*c|d*a|ab)d

Reduction Complexity

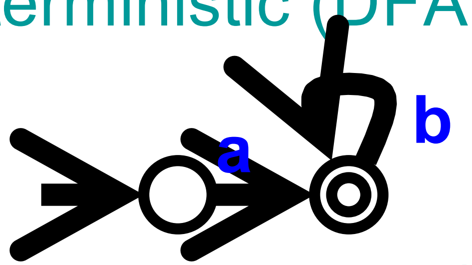
- Given a regular expression *A* of size n...

  Size = # of symbols + # of operations

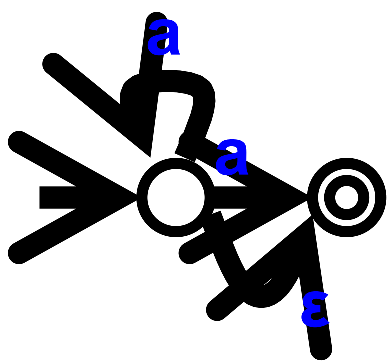
- How many states does <*A*> have?
  - Two added for each **|**, two added for each *****
  - O(n)
  - That’s pretty good!

Recap

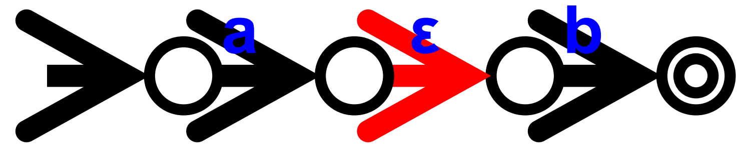
- Finite automata
  - Alphabet, states…
  - $(\Sigma, Q, q_0, F, \delta)$
- Types
  - Deterministic (DFA)



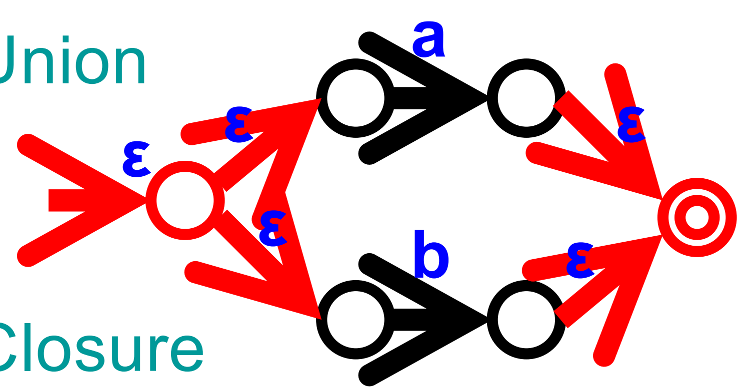
  - Non-deterministic (NFA)



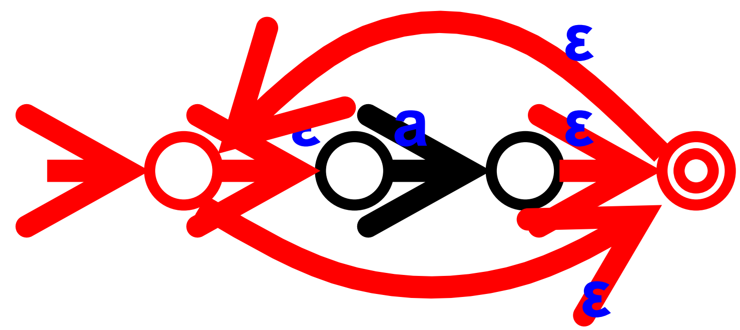
- Reducing RE to NFA
  - Concatenation



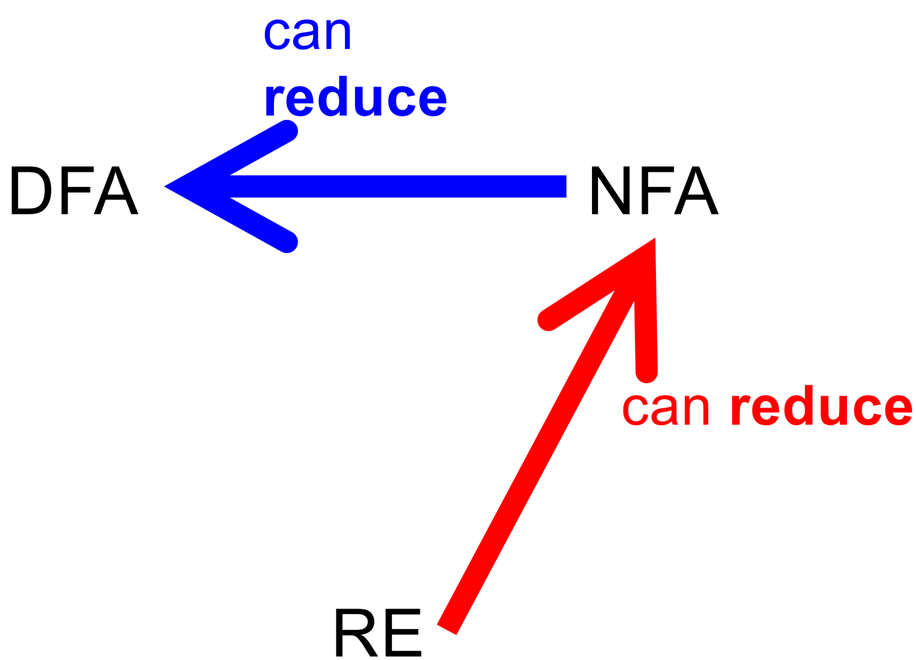
  - Union



  - Closure

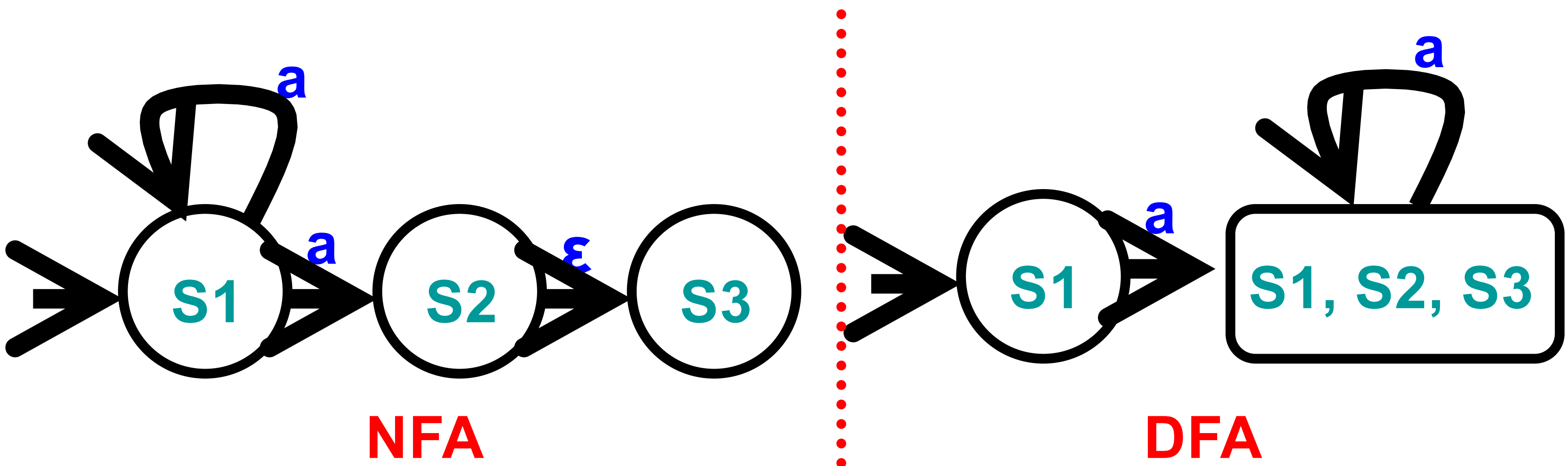


Reducing NFA to DFA



Reducing NFA to DFA

- NFA may be reduced to DFA
  - By explicitly tracking the set of NFA states
- Intuition
  - Build DFA where
    - Each DFA state represents a set of NFA “current states”
- Example



Algorithm for Reducing NFA to DFA

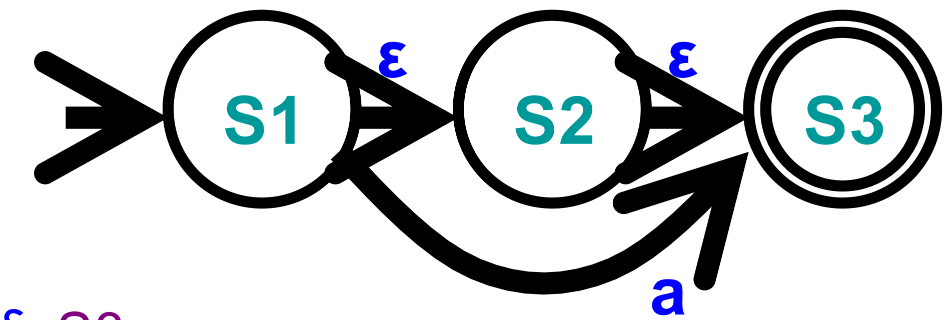
- Reduction applied using the subset algorithm
  - DFA state is a subset of set of all NFA states
- Algorithm
  - Input
    - NFA (Σ, Q, q_0, F_n, δ)
  - Output
    - DFA (Σ, R, r_0, F_d, δ)
  - Using two subroutines
    - ε-closure(δ, p) (and ε-closure(δ, S))
    - move(δ, p, a) (and move(δ, S, a))

ε-transitions and ε-closure

- We say $p \xrightarrow{\varepsilon} q$
  - If it is possible to go from state p to state q by taking only ε-transitions in δ
  - If $\exists$ $p, p_1, p_2, \ldots p_n, q \in Q$ such that
    - $\{p,\varepsilon,p_1\} \in \delta$, $\{p_1,\varepsilon,p_2\} \in \delta$, … , $\{p_n,\varepsilon,q\} \in \delta$
- ε-closure(δ, p)
  - Set of states reachable from p using ε-transitions alone
    - Set of states q such that $p \xrightarrow{\varepsilon} q$ according to δ
    - ε-closure(δ, p) = $\{q \mid p \xrightarrow{\varepsilon} q \text{ in } \delta\}$
    - ε-closure(δ, Q) = $\{ q \mid p \in Q, p \xrightarrow{\varepsilon} q \text{ in } \delta \}$
  - Notes
    - ε-closure(δ, p) always includes p
    - We write ε-closure(p) or ε-closure(Q) when δ is clear from context

ε-closure: Example 1

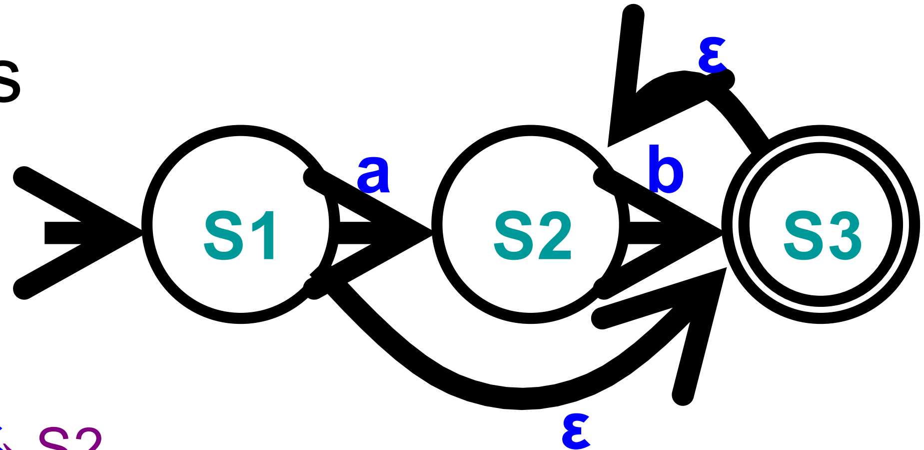
- Following NFA contains
  - $S1 \xrightarrow{\varepsilon} S2$
  - $S2 \xrightarrow{\varepsilon} S3$
  - $S1 \xrightarrow{\varepsilon} S3$
    - Since $S1 \xrightarrow{\varepsilon} S2$ and $S2 \xrightarrow{\varepsilon} S3$



- ε-closures
  - ε-closure(S1) = { S1, S2, S3 }
  - ε-closure(S2) = { S2, S3 }
  - ε-closure(S3) = { S3 }
  - ε-closure({ S1, S2 }) = { S1, S2, S3 } ∪ { S2, S3 }

ε-closure: Example 2

- Following NFA contains
  - S1 $\xrightarrow{\varepsilon}$ S3
  - S3 $\xrightarrow{\varepsilon}$ S2
  - S1 $\xrightarrow{\varepsilon}$ S2
    - Since S1 $\xrightarrow{\varepsilon}$ S3 and S3 $\xrightarrow{\varepsilon}$ S2



- ε-closures
  - ε-closure(S1) = { S1, S2, S3 }
  - ε-closure(S2) = { S2 }
  - ε-closure(S3) = { S2, S3 }
  - ε-closure({ S2,S3 }) = { S2 } ∪ { S2, S3 }

ε-closure Algorithm: Approach

- Input: NFA $(\Sigma, Q, q_0, F_n, \delta)$, State Set R
- Output: State Set R'
- Algorithm

```
Let R' = R                                   // start states
Repeat
    Let R = R'                               // continue from previous
    Let R' = R ∪ {q | p ∈ R, (p, ε, q) ∈ δ}  // new ε-reachable states
Until R = R'                                 // stop when no new states
```

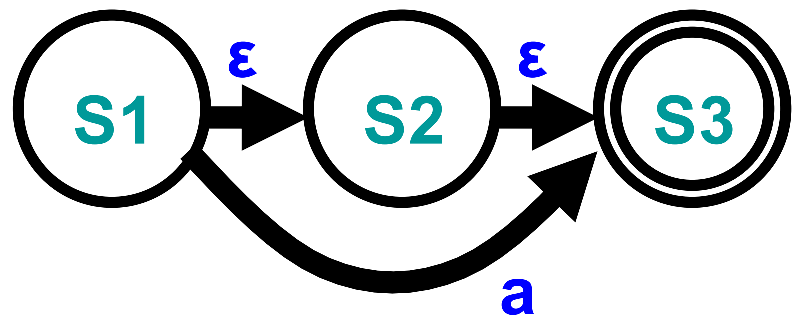
This algorithm computes a fixed point

- see note linked from project description

ε-closure Algorithm Example

- Calculate ε-closure(δ,{S1})

R	R'
{S1}	{S1}
{S1}	{S1, S2}
{S1, S2}	{S1, S2, S3}
{S1, S2, S3}	{S1, S2, S3}



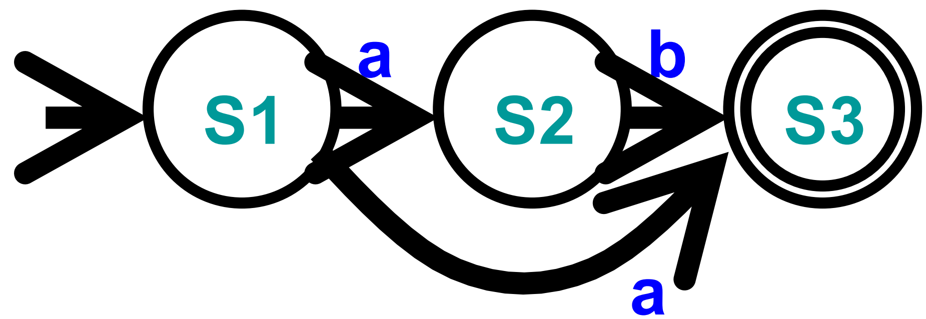
Let R' = R
Repeat
  Let R= R'
  Let R' = R $\cup$ {q | p $\in$ R, (p, ε, q) $\in$ δ}
Until R = R'

Calculating move(p,a)

- move(δ,p,a)
  - Set of states reachable from p using exactly one transition on a
    - Set of states q such that $\{p, a, q\} \in \delta$
    - $move(\delta,p,a) = \{ q \mid \{p, a, q\} \in \delta \}$
    - $move(\delta,Q,a) = \{ q \mid p \in Q, \{p, a, q\} \in \delta \}$
      - i.e., can “lift” move() to start from a *set* of states Q
  - Notes:
    - move(δ,p,a) is Ø if no transition $(p,a,q) \in \delta$, for any q
    - We write move(p,a) or move(R,a) when δ clear from context

move(a,p) : Example 1

- Following NFA
  - Σ = { a, b }

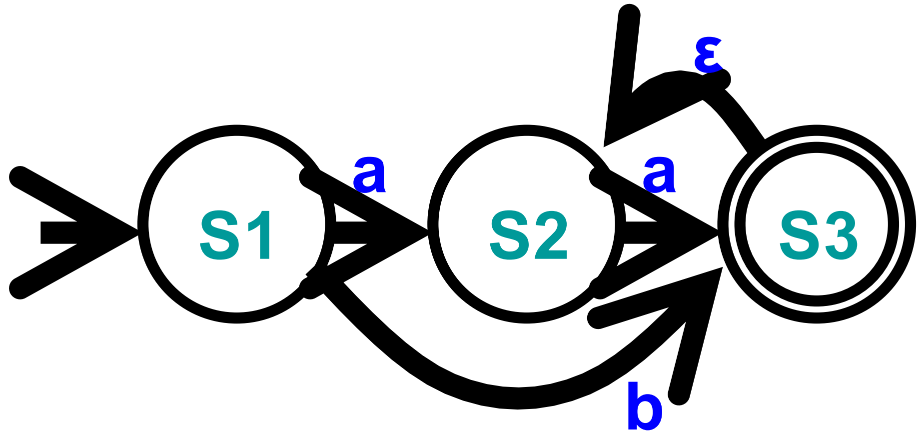


- Move
  - move(S1, a) = { S2, S3 }
  - move(S1, b) = Ø
  - move(S2, a) = Ø
  - move(S2, b) = { S3 }
  - move(S3, a) = Ø
  - move(S3, b) = Ø

move({S1,S2},b) = { S3 }

move(a,p) : Example 2

- Following NFA
  - Σ = { a, b }



- Move
  - move(S1, a) = { S2 }
  - move(S1, b) = { S3 }
  - move(S2, a) = { S3 }
  - move(S2, b) = ∅
  - move(S3, a) = ∅
  - move(S3, b) = ∅

move({S1,S2},a) = {S2,S3}

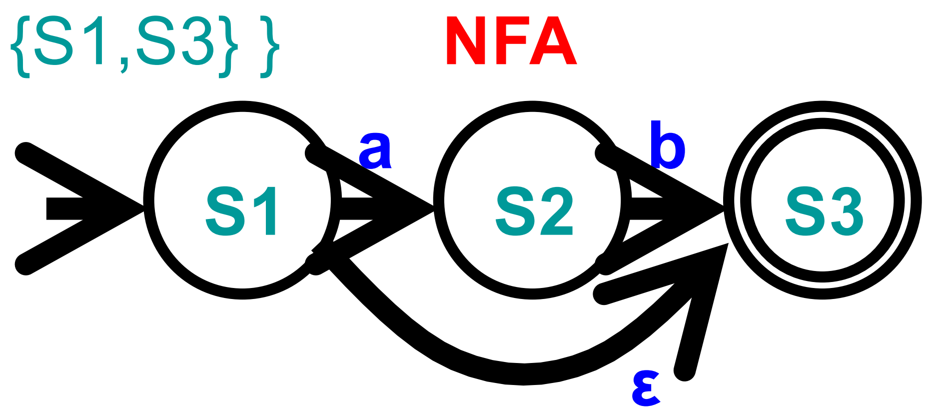
NFA $\rightarrow$ DFA Reduction Algorithm ("subset")

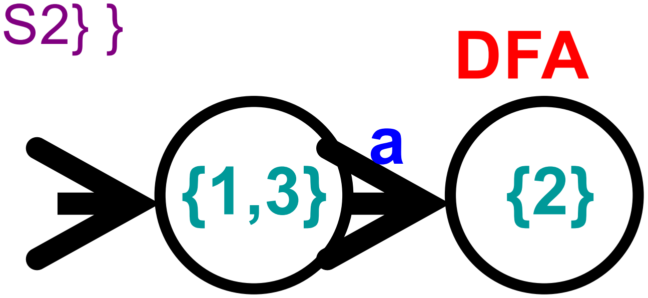
- Input NFA $(\Sigma, Q, q_0, F_n, \delta)$, Output DFA $(\Sigma, R, r_0, F_d, \delta')$
- Algorithm

Let r_0 = ε-closure(δ,q_0), add it to R	// DFA start state
While $\exists$ an unmarked state $r \in R$	// process DFA state r
Mark r	// each state visited once
For each $a \in \Sigma$	// for each letter a
Let E = move(δ,r,a)	// states reached via a
Let e = ε-closure(δ,E)	// states reached via ε
If $e \notin R$	// if state e is new
Let $R = R \cup \{e\}$	// add e to R (unmarked)
Let $\delta' = \delta' \cup \{r, a, e\}$	// add transition $r \rightarrow e$
Let $F_d = \{r \mid \exists s \in r \text{ with } s \in F_n\}$	// final if include state in F_n

NFA → DFA Example 1

- Start = ε-closure(δ,S1) = { {S1,S3} }
- R = { {S1,S3} }
- r ∈ R = {S1,S3}
- move(δ,{S1,S3},a) = {S2}
  - e = ε-closure(δ,{S2}) = {S2}
  - R = R ∪ {{S2}} = { {S1,S3}, {S2} }
  - δ' = δ' ∪ {{S1,S3}, a, {S2}}
- move(δ,{S1,S3},b) = Ø

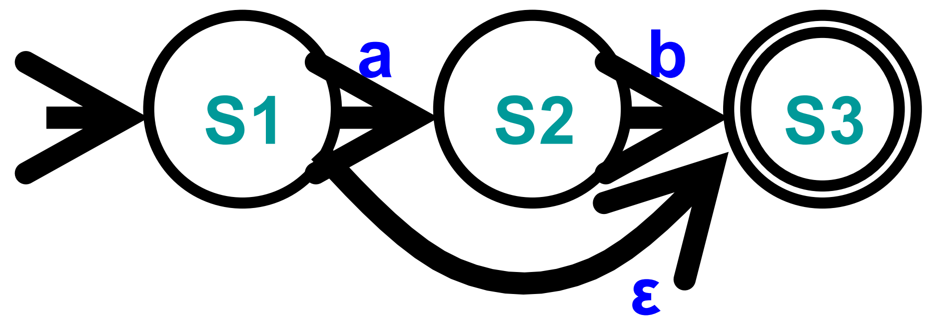




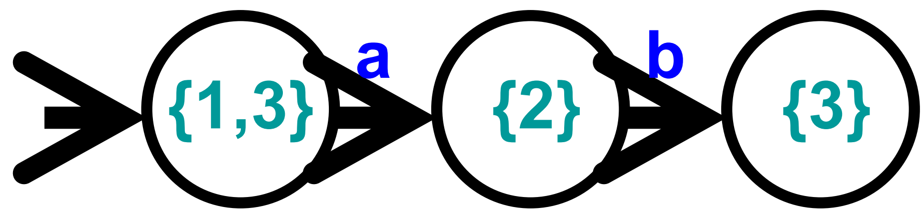
NFA $\rightarrow$ DFA Example 1 (cont.)

- R = { {S1,S3}, {S2} }
- r $\in$ R = {S2}
- move(δ,{S2},a) = Ø
- move(δ,{S2},b) = {S3}
  - e = ε-closure(δ,{S3}) = {S3}
  - R = R $\cup$ {{S3}} = { {S1,S3}, {S2}, {S3} }
  - δ' = δ' $\cup$ {{S2}, b, {S3}}

NFA

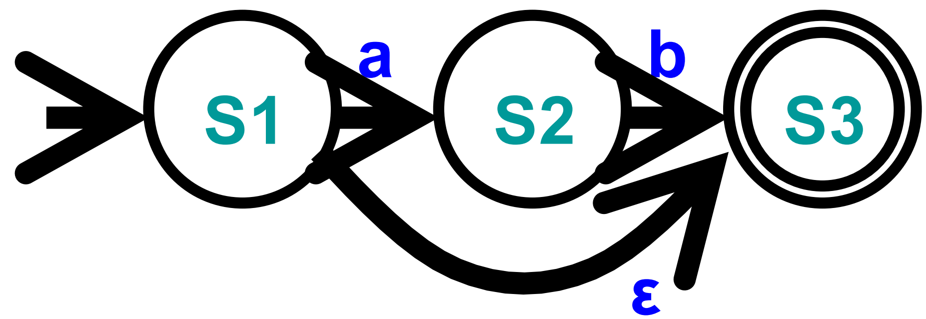


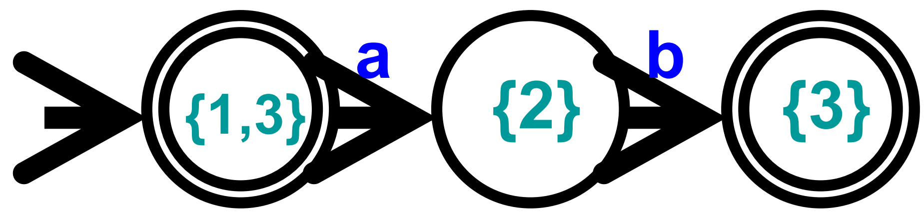
DFA



NFA → DFA Example 1 (cont.)

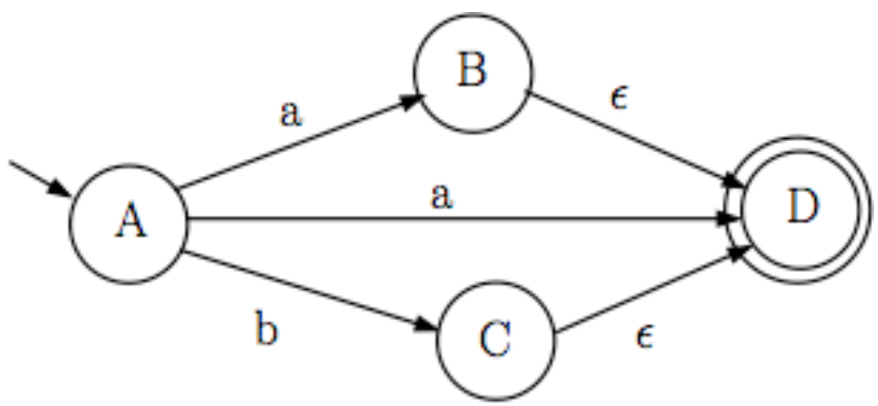
- R = { {S1,S3}, {S2}, {S3} }
- r ∈ R = {S3}
- Move({S3},a) = Ø
- Move({S3},b) = Ø
- Mark {S3}, exit loop
- F_d = {{S1,S3}, {S3}}
  - Since S3 ∈ F_n
- Done!



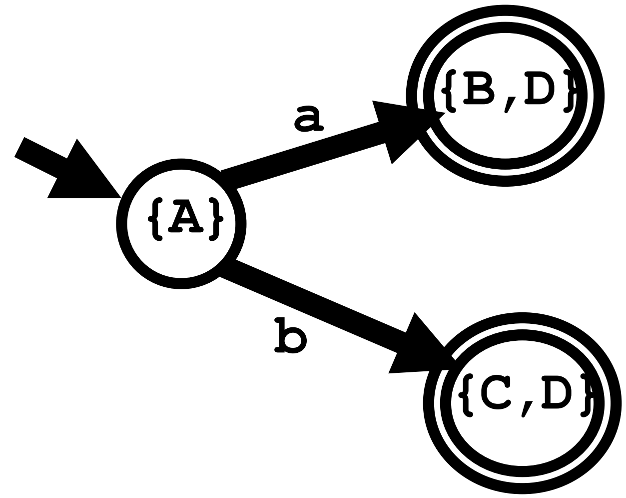


NFA → DFA Example 2

- NFA

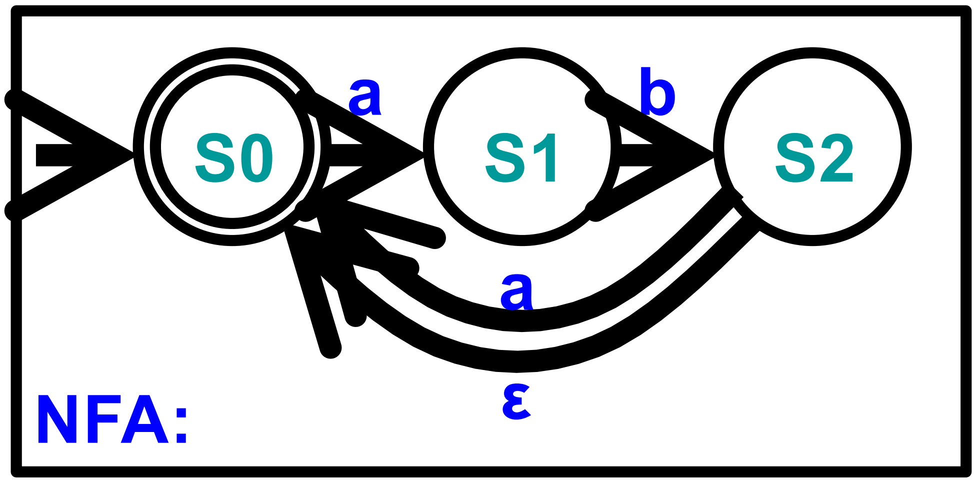


- DFA

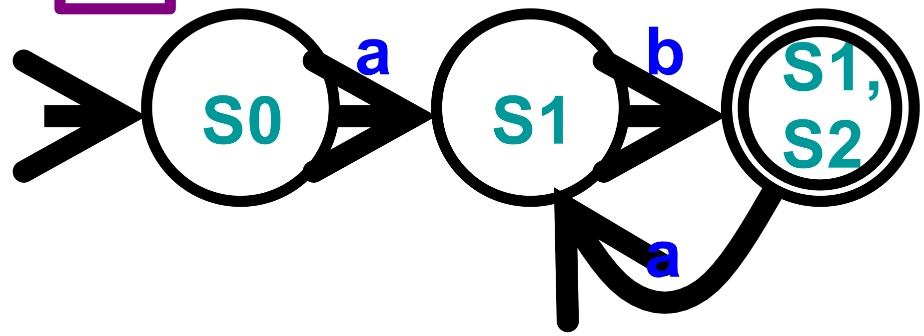


R = { $\boxed{\{A\}}$, $\boxed{\{B,D\}}$, $\boxed{\{C,D\}}$ }

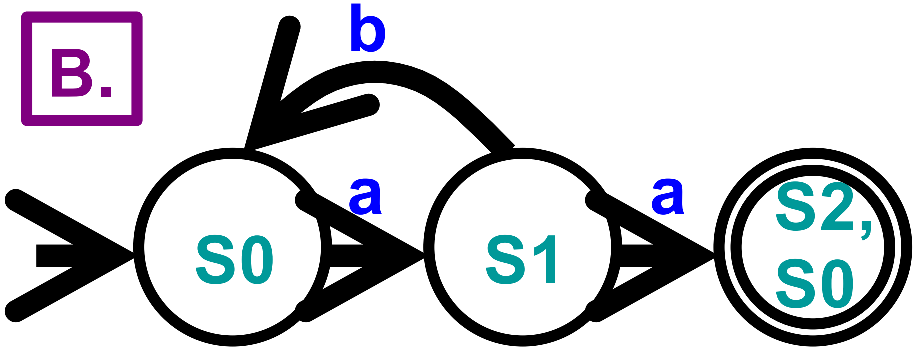
Quiz 4: Which DFA is equiv to this NFA?



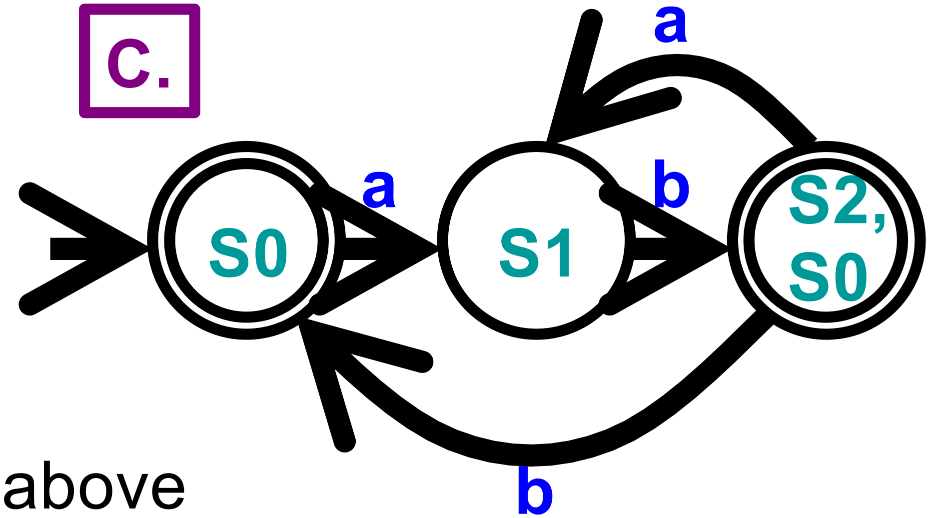
A.



B.

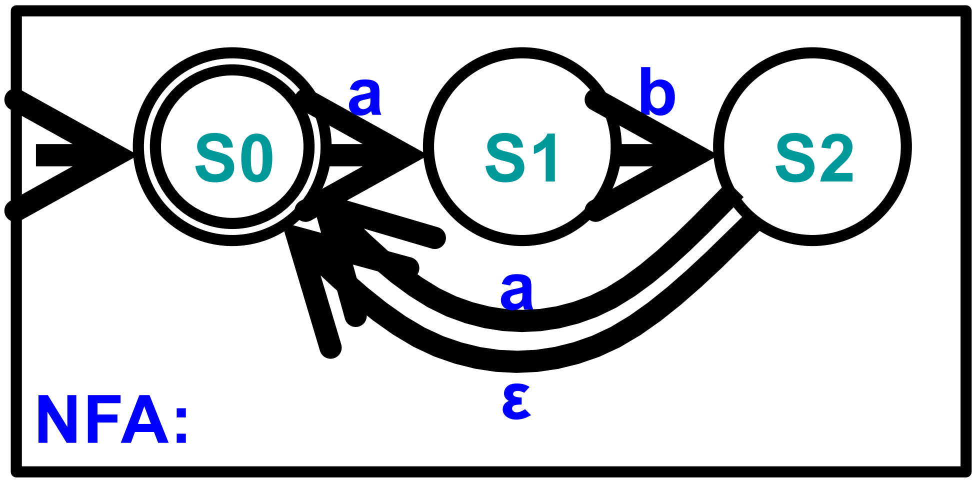


C.

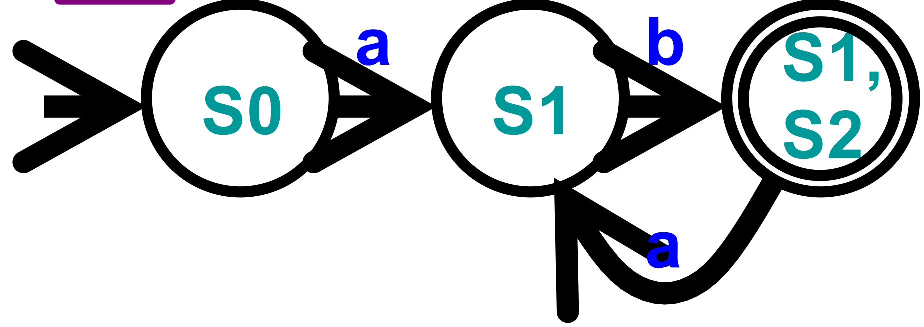


D. None of the above

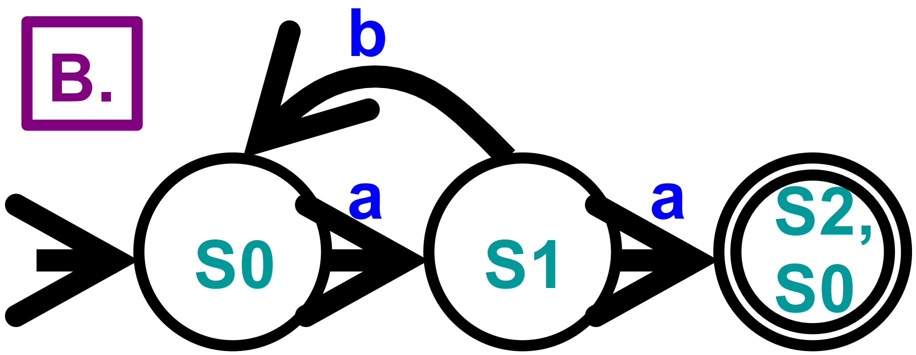
Quiz 4: Which DFA is equiv to this NFA?



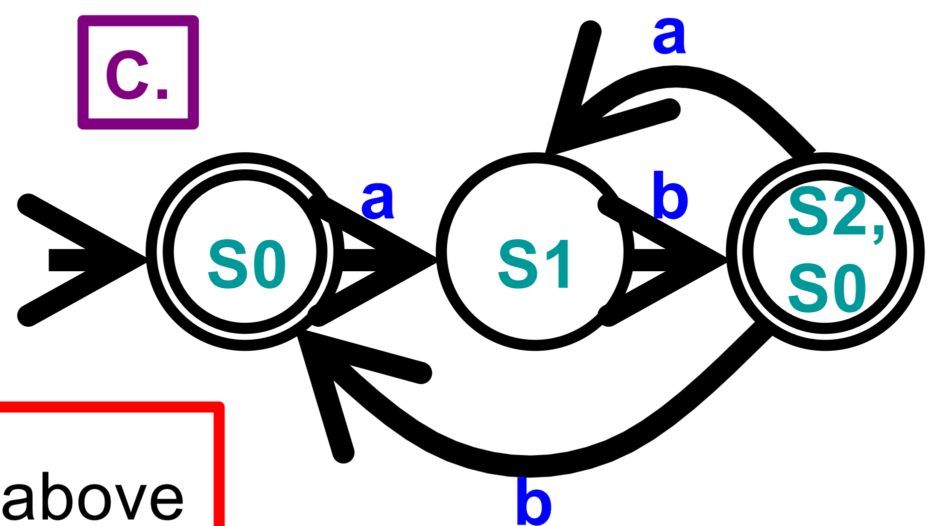
A.



B.

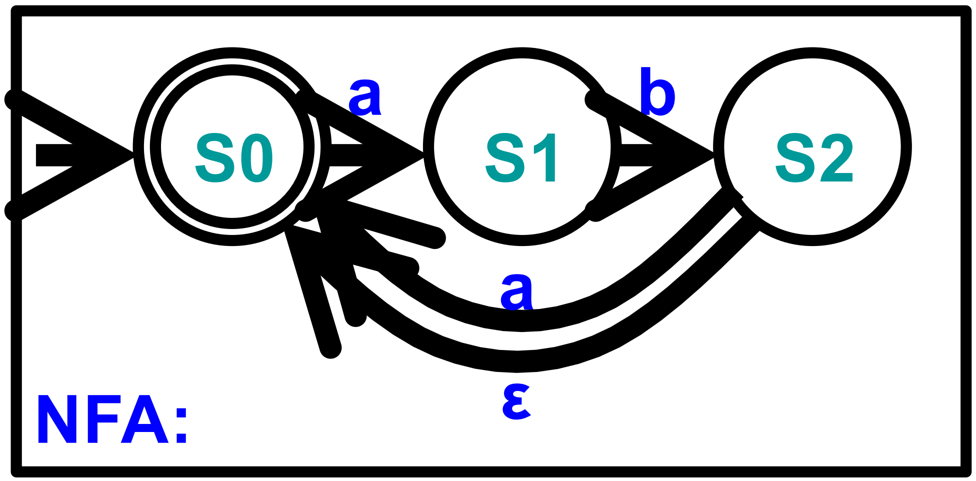


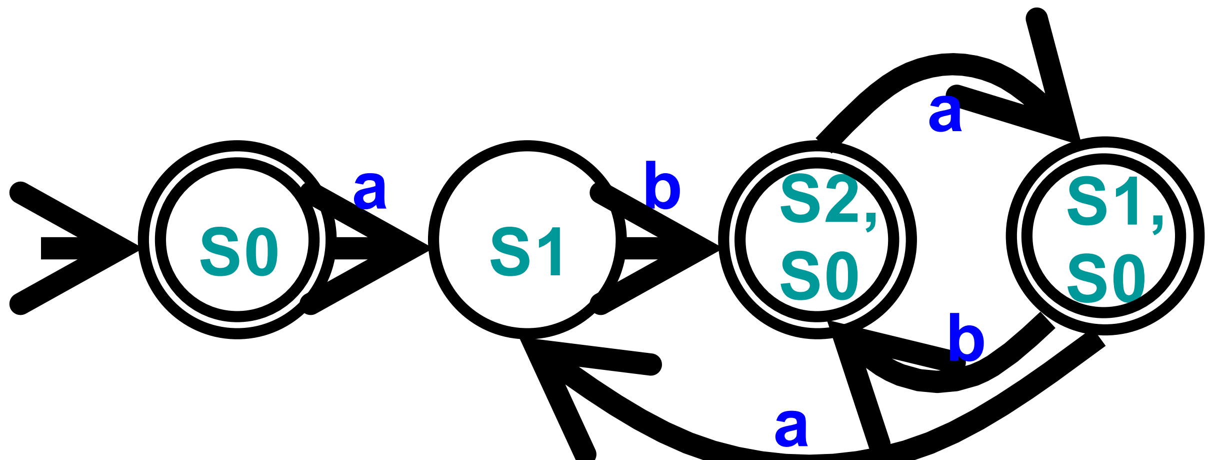
C.



D. None of the above

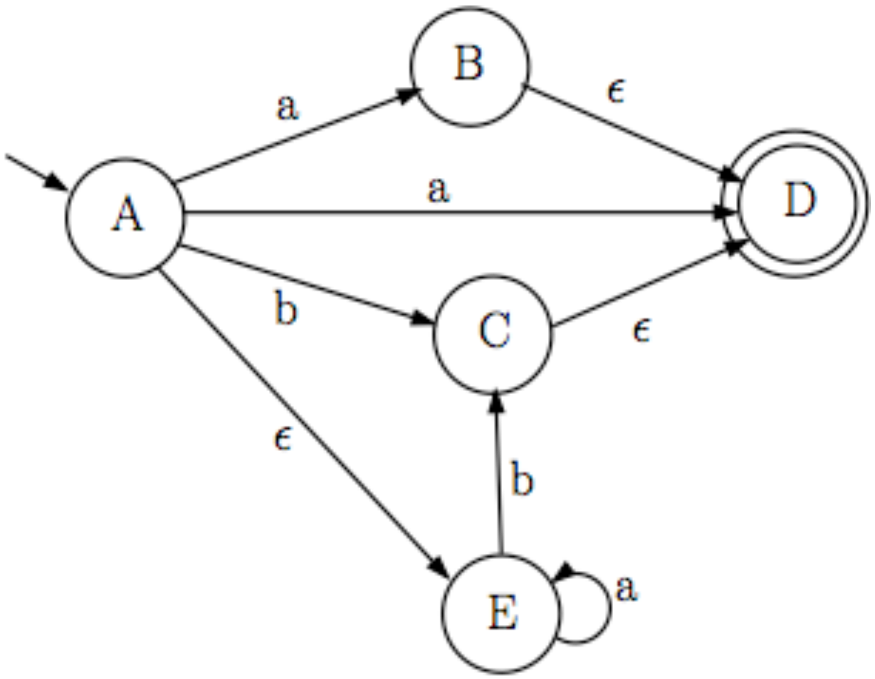
Actual Answer



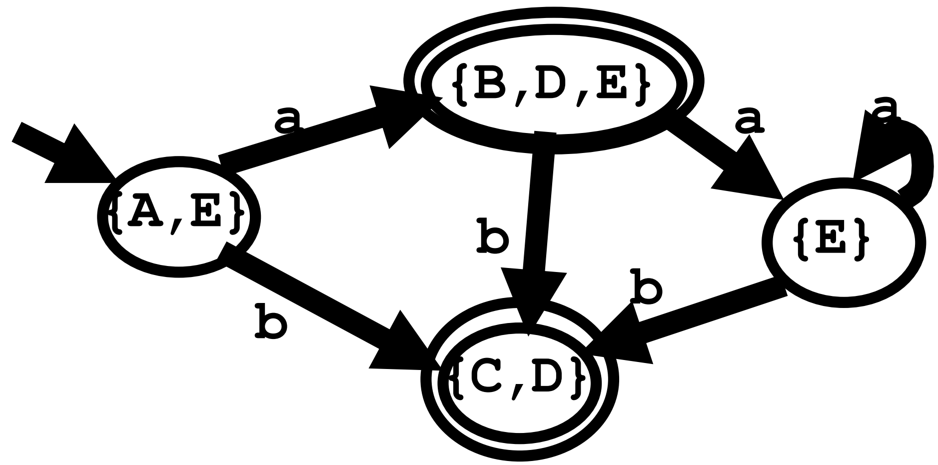


NFA → DFA Example 3

- NFA

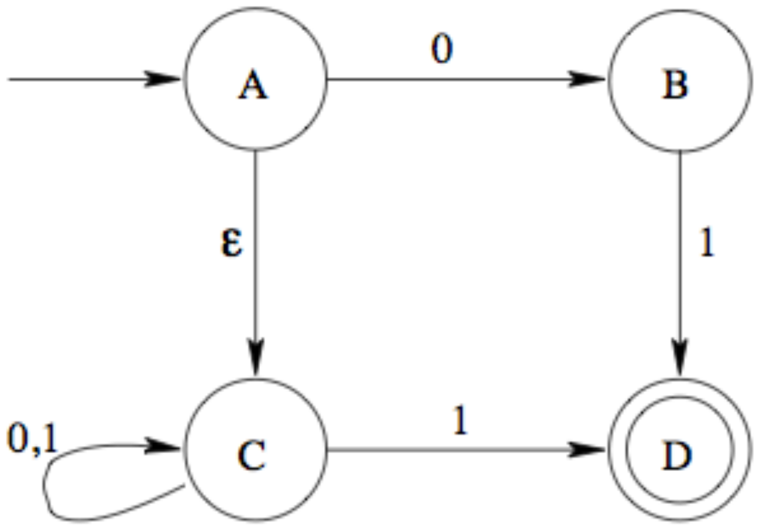


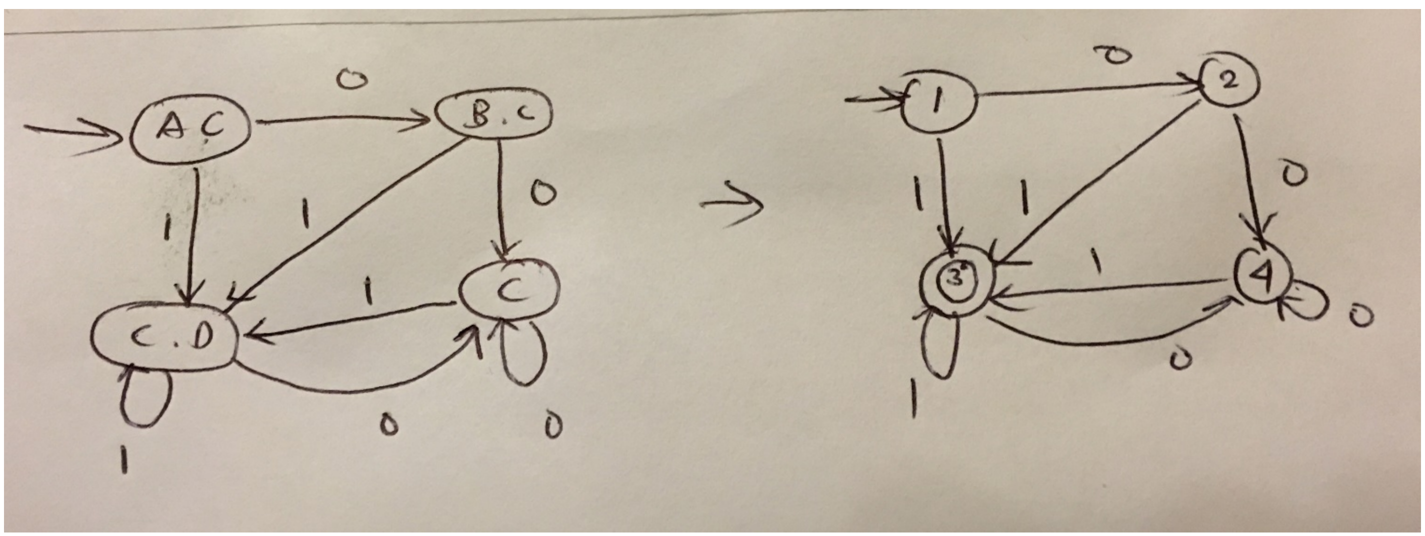
- DFA



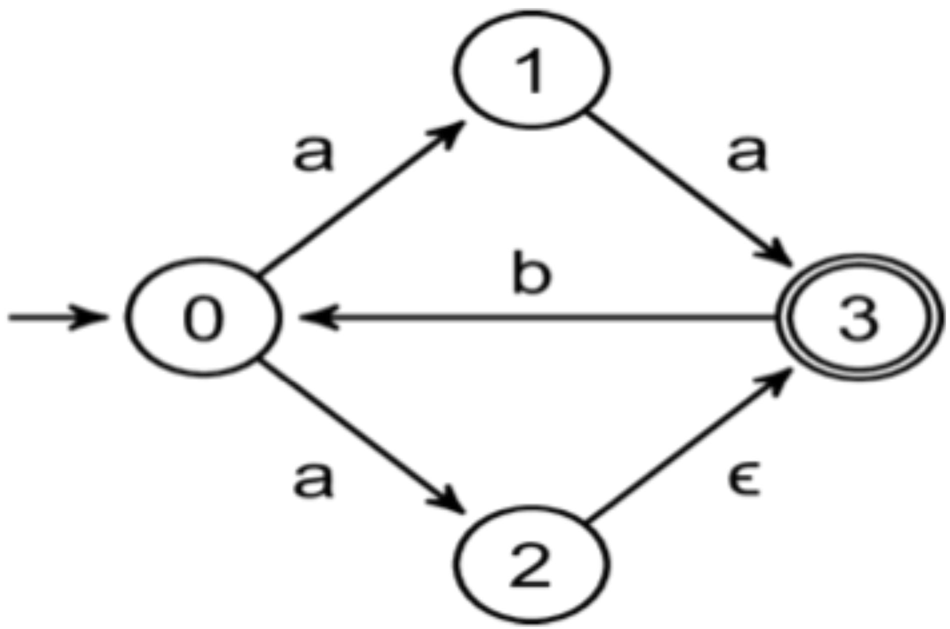
R = { {A,E}, {B,D,E}, {C,D}, {E} }

NFA → DFA Example

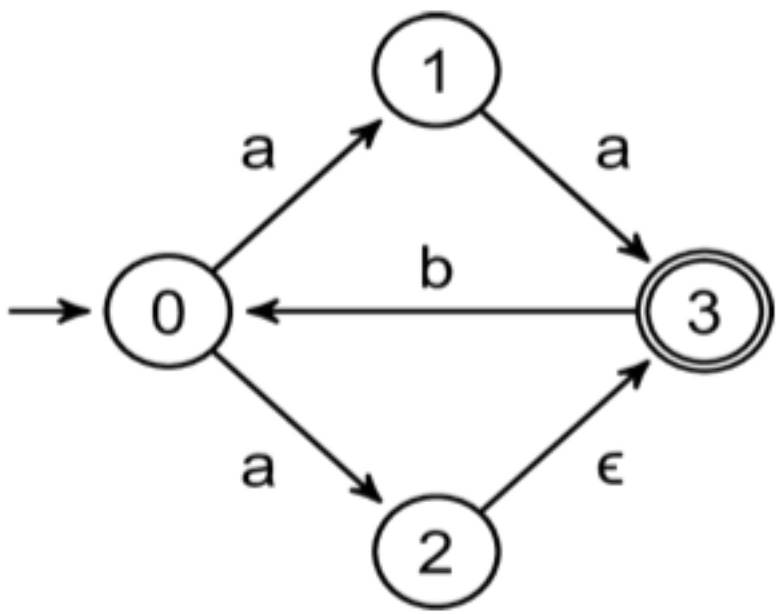


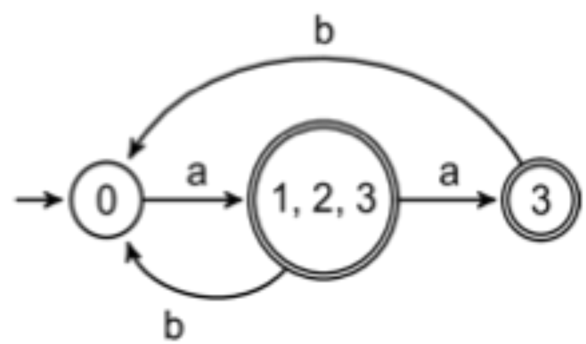


NFA $\rightarrow$ DFA Practice



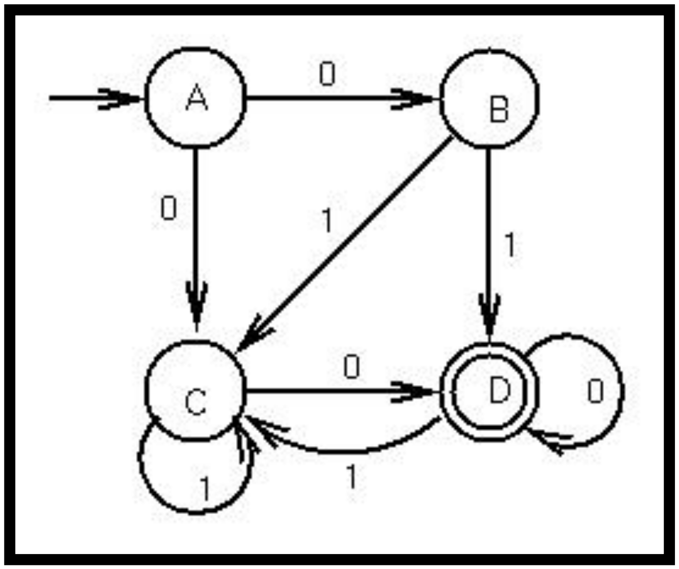
NFA → DFA Practice



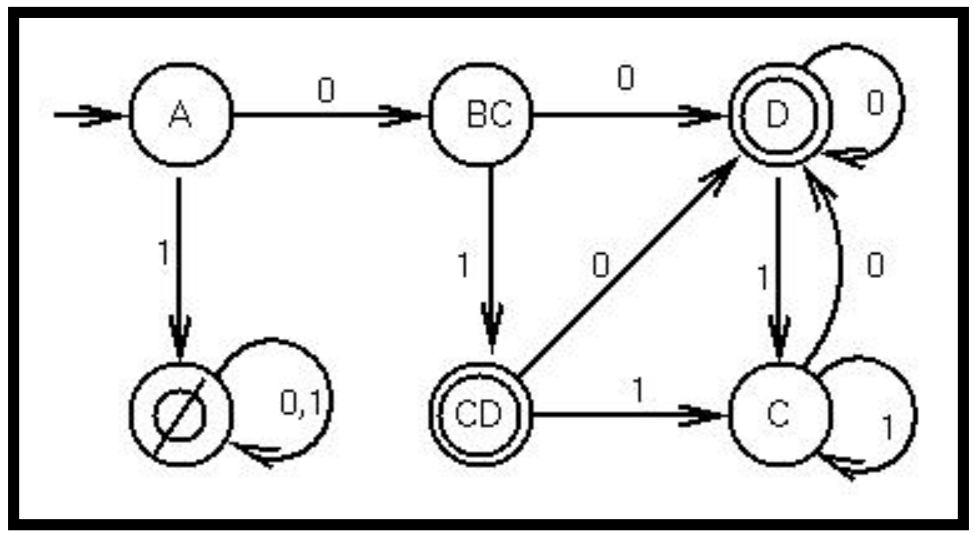


Analyzing the reduction

- Any string from {A} to either {D} or {CD}
  - Represents a path from A to D in the original NFA



NFA



DFA

Subset Algorithm as a Fixed Point

- Input: NFA $(\Sigma, Q, q_0, F, \delta)$
- Output: DFA M'
- Algorithm

Let $q_0' = \varepsilon\text{-closure}(\delta, q_0)$

Let $F' = \{q_0'\}$ if $q_0' \cap F \neq \emptyset$, or $\emptyset$ otherwise

Let $M' = (\Sigma, \{q_0'\}, q_0', F', \emptyset)$ // starting approximation of DFA

Repeat

Let $M = M'$ // current DFA approx

For each $q \in \text{states}(M)$, $a \in \Sigma$ // for each DFA state q and letter a

Let $s = \varepsilon\text{-closure}(\delta, \text{move}(\delta, q, a))$ // new subset from q

Let $F' = \{s\}$ if $s \cap F \neq \emptyset$, or $\emptyset$ otherwise, // subset contains final?

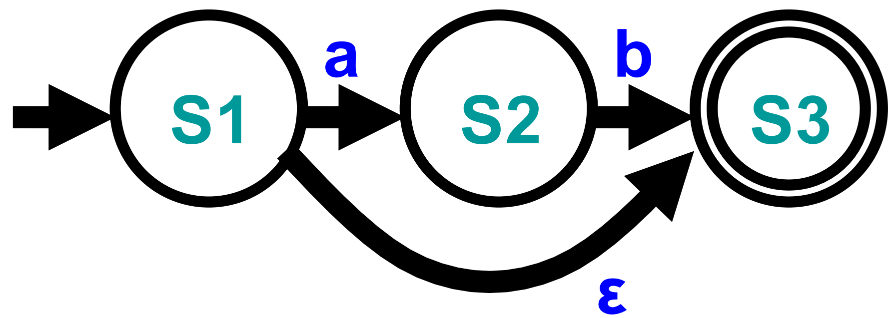
$M' = M' \cup (\emptyset, \{s\}, \emptyset, F', \{(q, a, s)\})$ // update DFA

Until $M' = M$ // reached fixed point

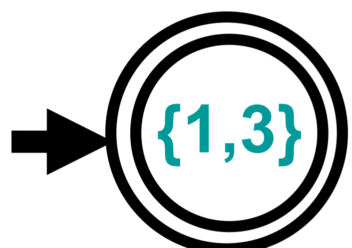
Redux: DFA to NFA Example 1

- $q_0' = \varepsilon\text{-closure}(\delta, S1) = \{S1,S3\}$
- $F' = \{\{S1,S3\}\}$ since $\{S1,S3\} \cap \{S3\} \neq \emptyset$

NFA



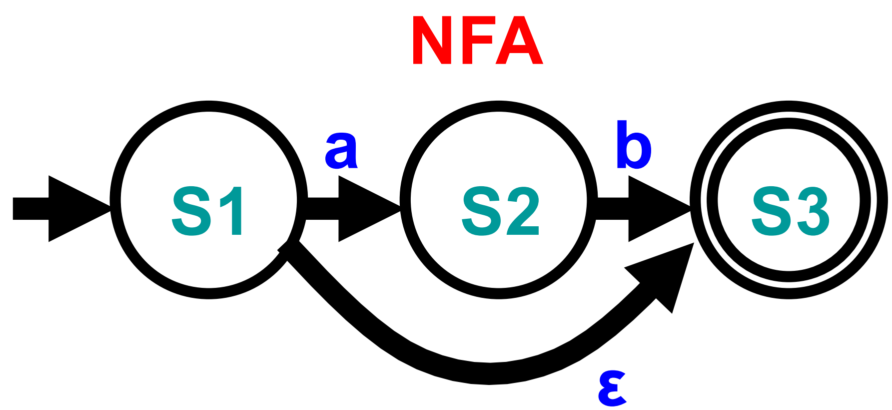
DFA

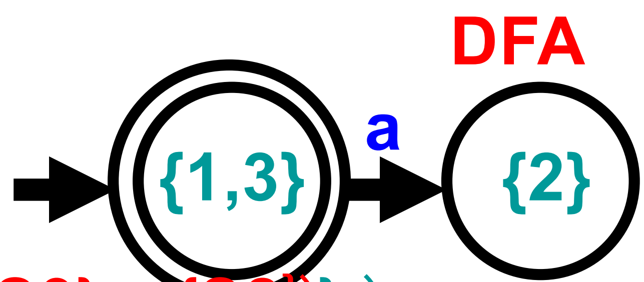


- $M' = \{\ \Sigma,\ \underbrace{\{\{S1,S3\}\}}_{Q'},\ \underbrace{\{S1,S3\}}_{q_0'},\ \underbrace{\{\{S1,S3\}\}}_{F'},\ \underbrace{\emptyset}_{\delta'}\ \}$

Redux: DFA to NFA Example 1 (cont)

- M' = { Σ, {{S1,S3}}, {S1,S3}, {{S1,S3}}, ∅ }
  - q = {S1, S3}
  - a = a
  - s = {S2}
    - since move(δ,{S1, S3},a) = {S2}
    - and ε-closure(δ,{S2}) = {S2}
  - F' = ∅
    - Since {S2} ∩ {S3} = ∅
    - where s = {S2} and F = {S3}

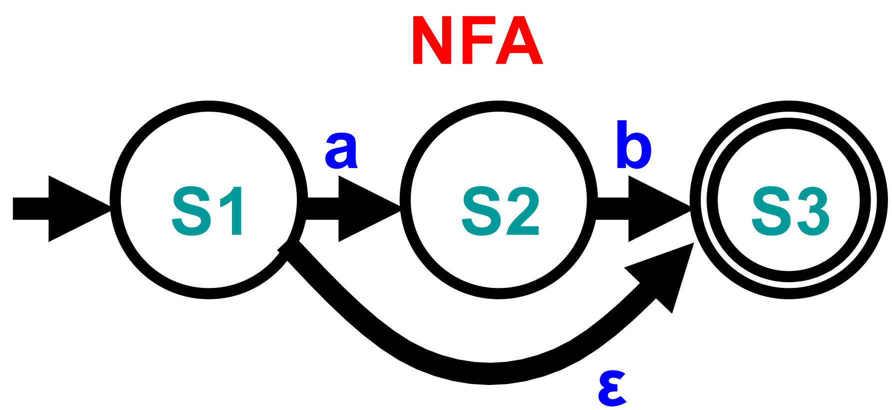


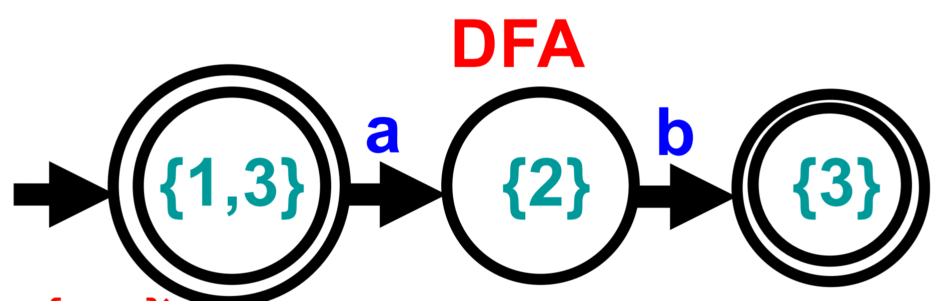


- M' = M' ∪ (∅, {{S2}}, ∅, ∅, {({S1,S3},a,{S2})})
- $= \{\ \Sigma,\ \underbrace{\{\{S1,S3\},\{S2\}\}}_{Q'},\ \underbrace{\{S1,S3\}}_{q_0'},\ \underbrace{\{\{S1,S3\}\}}_{F'},\ \underbrace{\{(\{S1,S3\},a,\{S2\})\}}_{\delta'}\ \}$

Redux: DFA to NFA Example 1 (cont)

- M’ = { Σ, {{S1,S3},{S2}}, {S1,S3}, {{S1,S3}}, {({S1,S3},a,{S2})} }
  - q = {S2}
  - a = b
  - s = {S3}
    - since move(δ,{S2},b) = {S3}
    - and ε-closure(δ,{S3}) = {S3}
  - F’ = {{S3}}
    - Since {S3} ∩ {S3} = {S3}
    - where s = {S3} and F = {S3}





- M’ = M’ ∪

(∅, {{S3}}, ∅, {{S3}}, {({S2},b,{S3})})

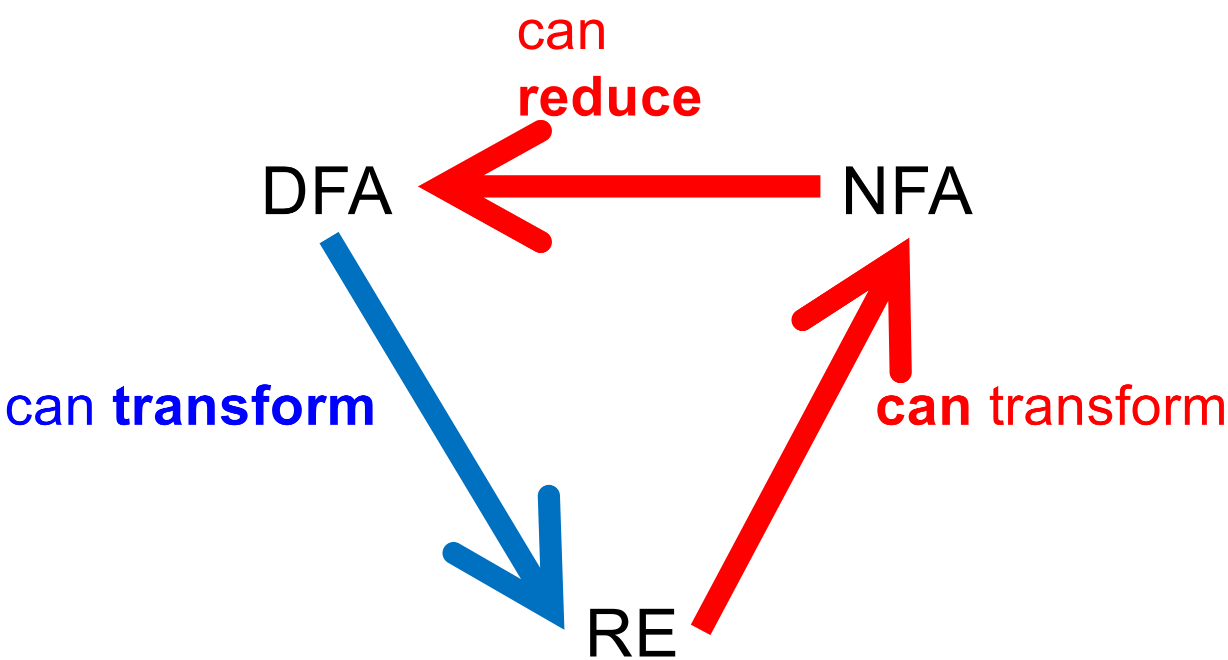
= { Σ, {{S1,S3},{S2},{S3}}, {S1,S3}, {{S1,S3},{S3}}, {({S1,S3},a,{S2}), ({S2},b,{S3})} }

Q’ q₀' F’ δ'

Analyzing the Reduction

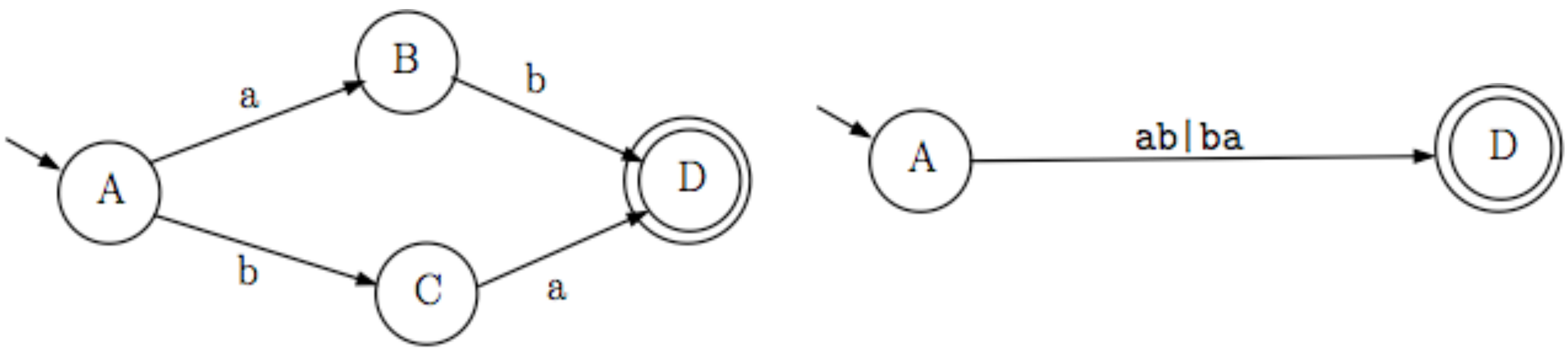
- Can reduce any NFA to a DFA using subset alg.
- How many states in the DFA?
  - Each DFA state is a subset of the set of NFA states
  - Given NFA with n states, DFA may have 2^n states
    - Since a set with n items may have 2^n subsets
  - Corollary
    - Reducing a NFA with n states may be $O(2^n)$

Reducing DFA to RE



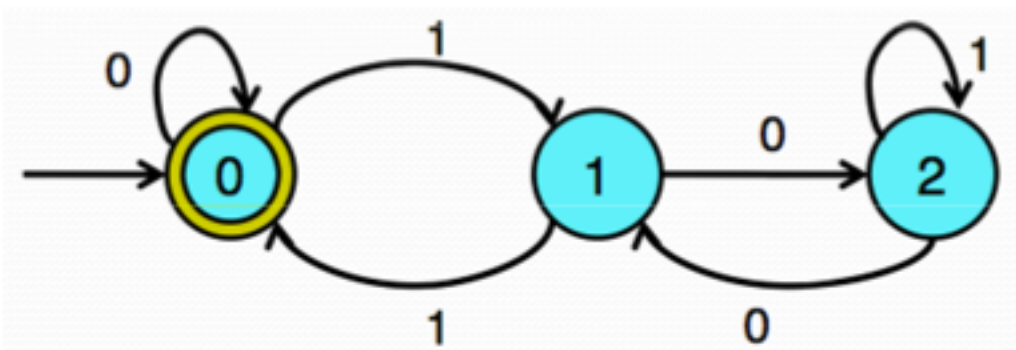
Reducing DFAs to REs

- General idea
  - Remove states one by one, labeling transitions with regular expressions
  - When two states are left (start and final), the transition label is the regular expression for the DFA



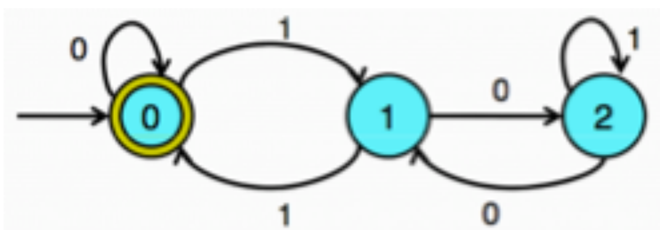
DFA to RE example

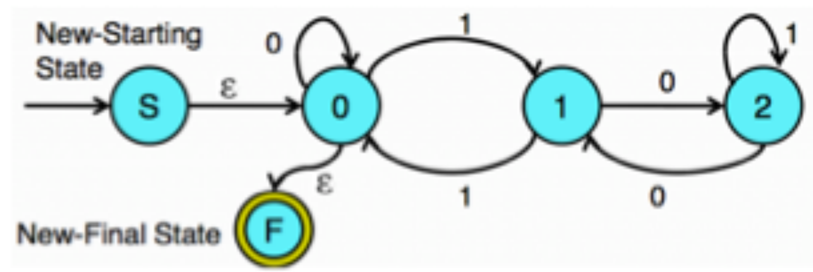
Language over Σ = {0,1} such that every string is a multiple of 3 in binary

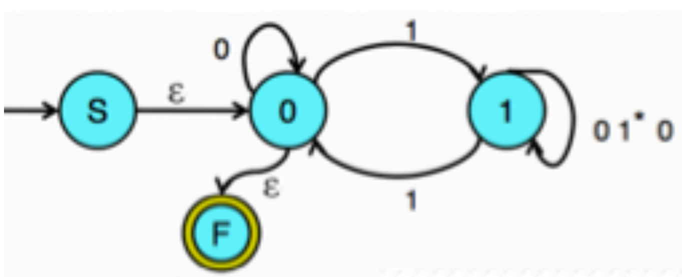


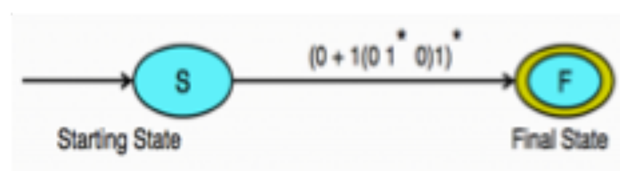
DFA to RE example

Language over $\Sigma = \{0,1\}$ such that every string is a multiple of 3 in binary









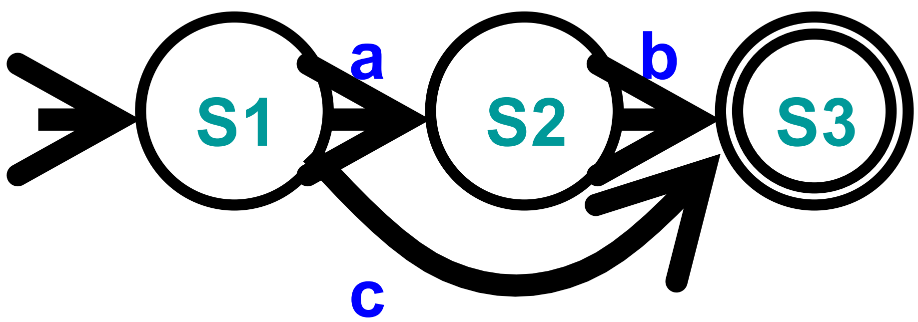
$$(0 + 1(0\,1^{*}\,0)1)^{*}$$

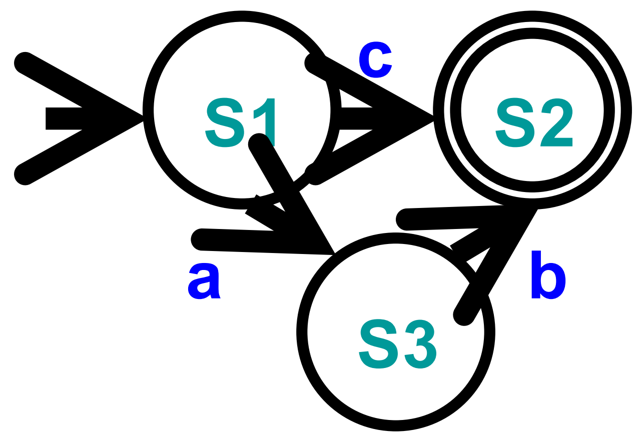
Other Topics

- Minimizing DFA
  - Hopcroft reduction
- Complementing DFA
- Implementing DFA

Minimizing DFAs

- Every regular language is recognizable by a unique minimum-state DFA
  - Ignoring the particular names of states
- In other words
  - For every DFA, there is a unique DFA with minimum number of states that accepts the same language





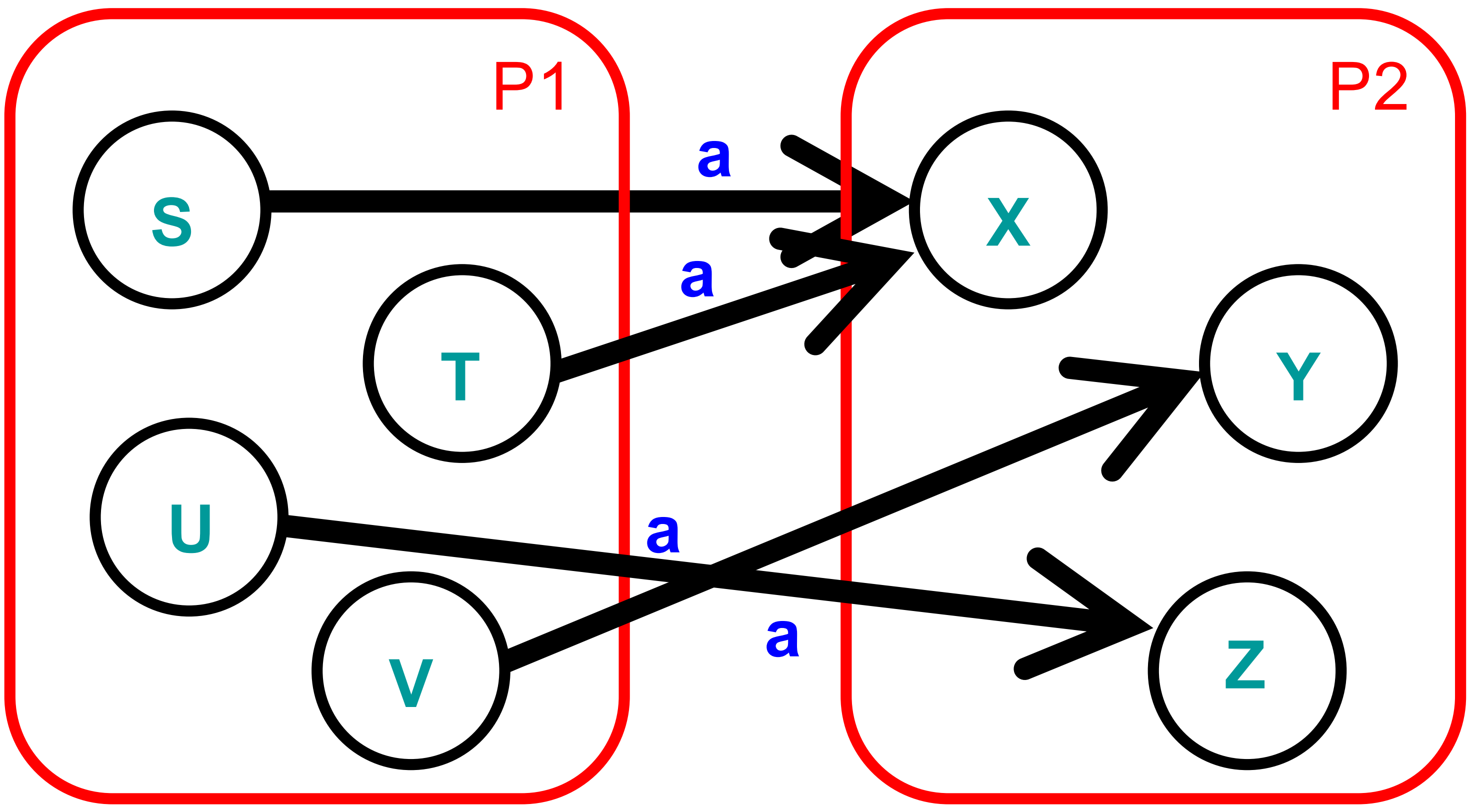
J. Hopcroft, “An n log n algorithm for minimizing states in a finite automaton,” 1971

Minimizing DFA: Hopcroft Reduction

- Intuition
  - Look to distinguish states from each other
    - End up in different accept / non-accept state with identical input
- Algorithm
  - Construct initial partition
    - Accepting & non-accepting states
  - Iteratively split partitions (until partitions remain fixed)
    - Split a partition if **members in partition have transitions to different partitions for same input**
      - Two states x, y belong in same partition if and only if for all symbols in Σ they transition to the same partition
  - Update transitions & remove dead states

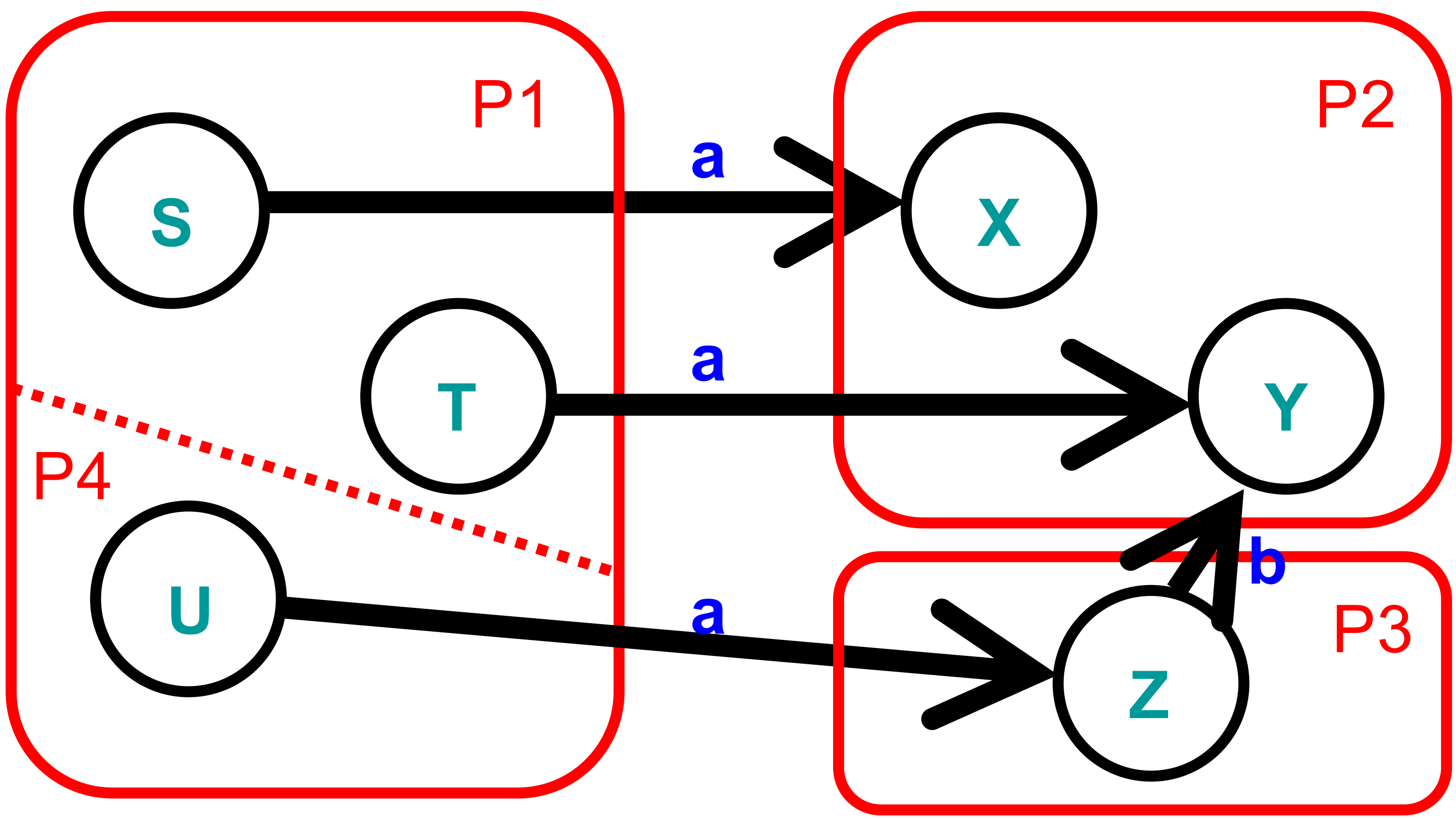
Splitting Partitions

- No need to split partition {S,T,U,V}
  - All transitions on a lead to identical partition P2
  - Even though transitions on a lead to different states



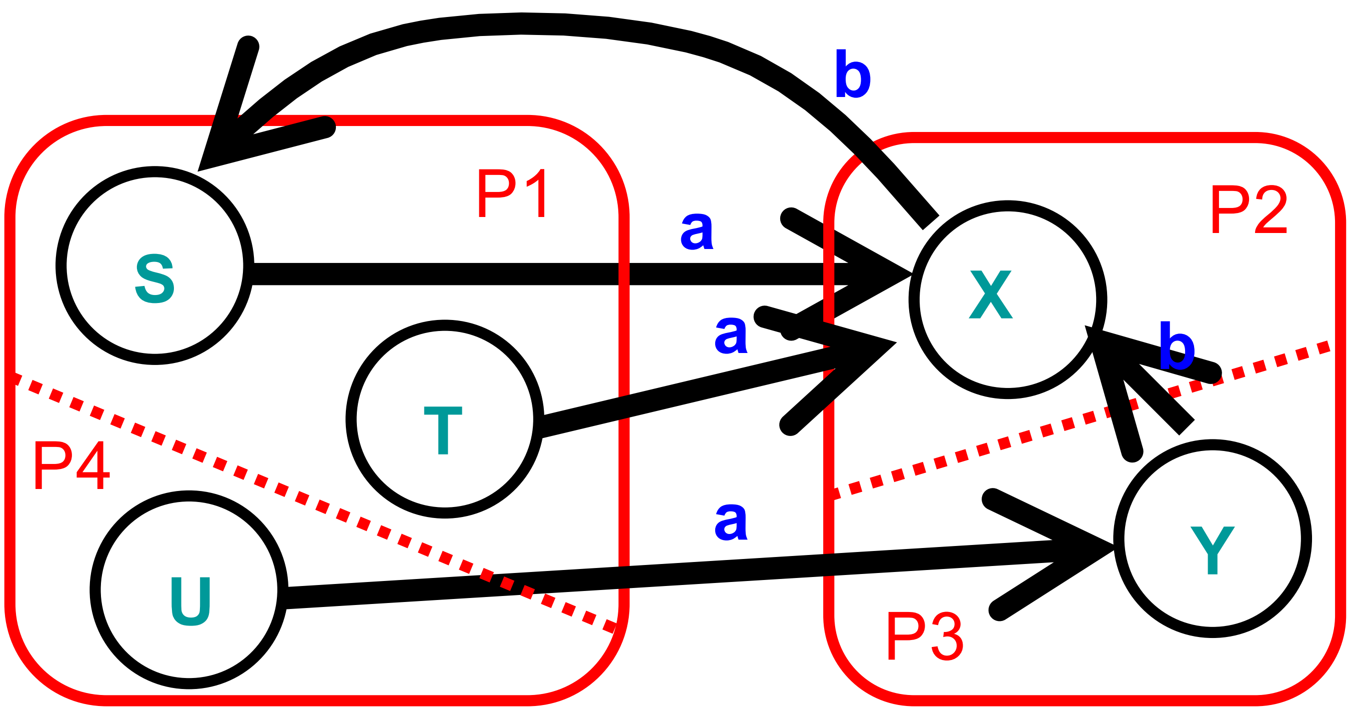
Splitting Partitions (cont.)

- Need to split partition {S,T,U} into {S,T}, {U}
  - Transitions on a from S,T lead to partition P2
  - Transition on a from U lead to partition P3



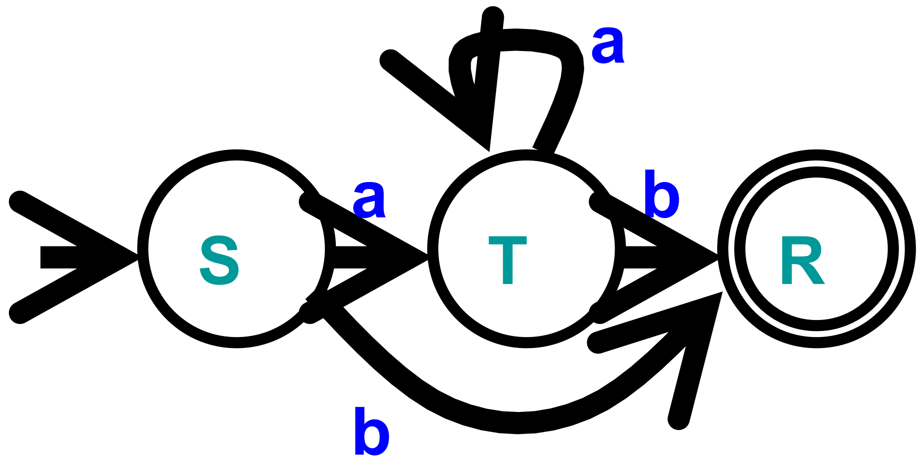
Resplitting Partitions

- Need to reexamine partitions after splits
  - Initially no need to split partition {S,T,U}
  - After splitting partition {X,Y} into {X}, {Y} we need to split partition {S,T,U} into {S,T}, {U}



Minimizing DFA: Example 1

- DFA

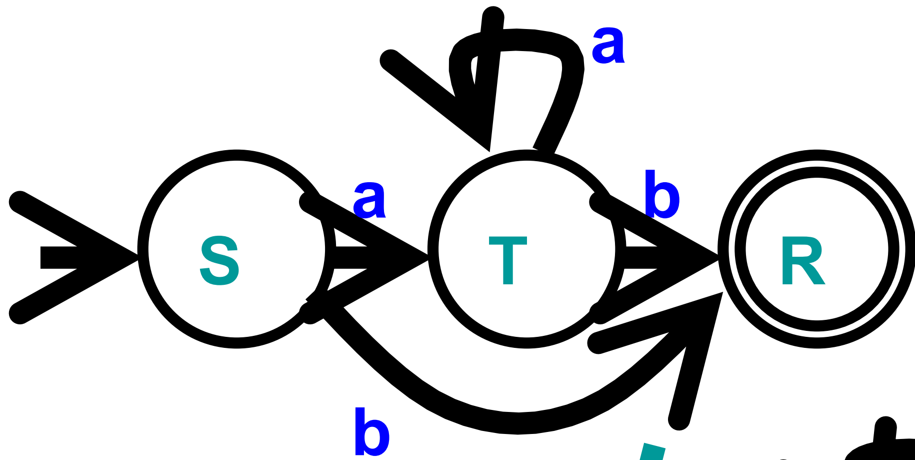


- Initial partitions

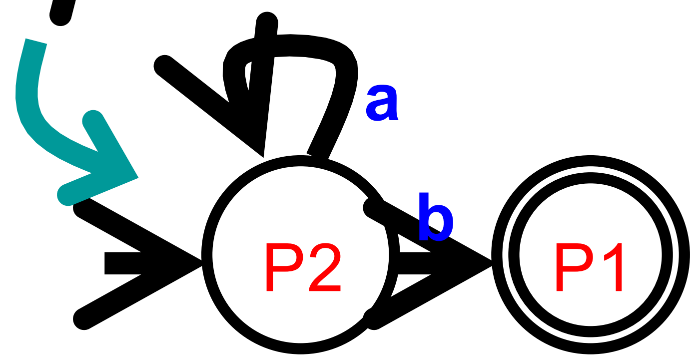
- Split partition

Minimizing DFA: Example 1

- DFA

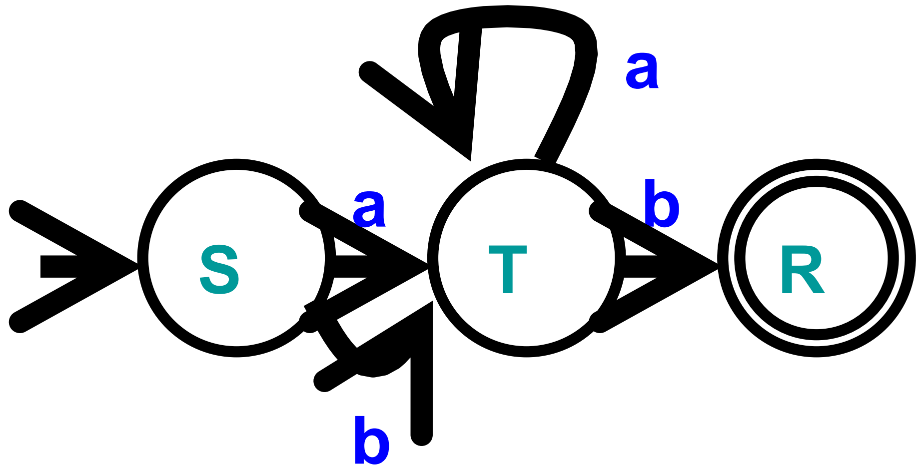


- Initial partitions
  - Accept { R } = P1
  - Reject { S, T } = P2



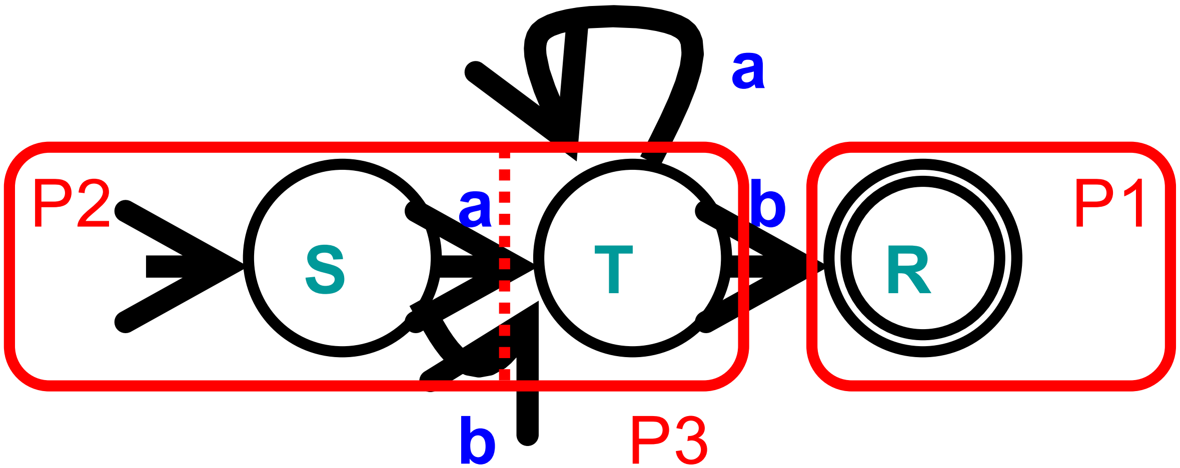
- Split partition? → Not required, minimization done
  - move(S,a) = T ∊ P2
    - move(S,b) = R ∊ P1
  - move(T,a) = T ∊ P2
    - move (T,b) = R ∊ P1

Minimizing DFA: Example 2



Minimizing DFA: Example 2

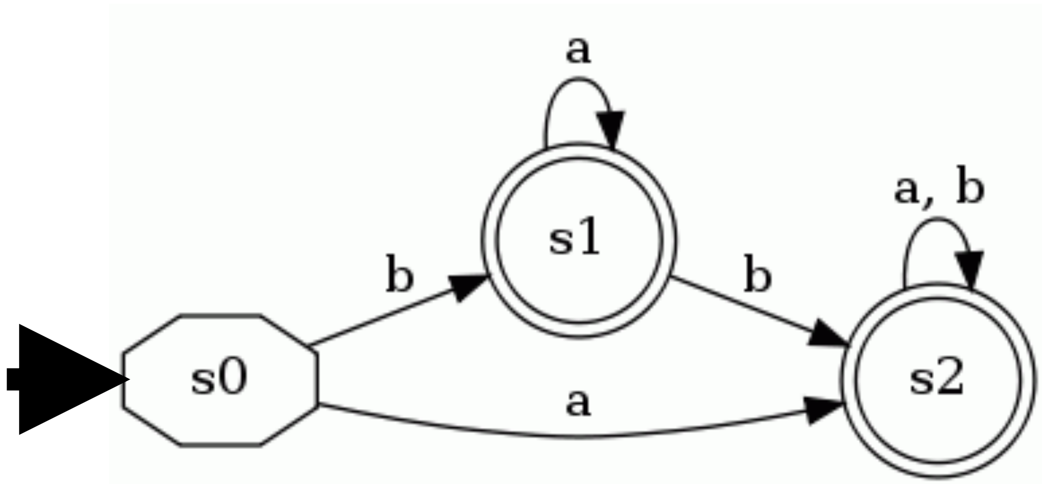
- DFA



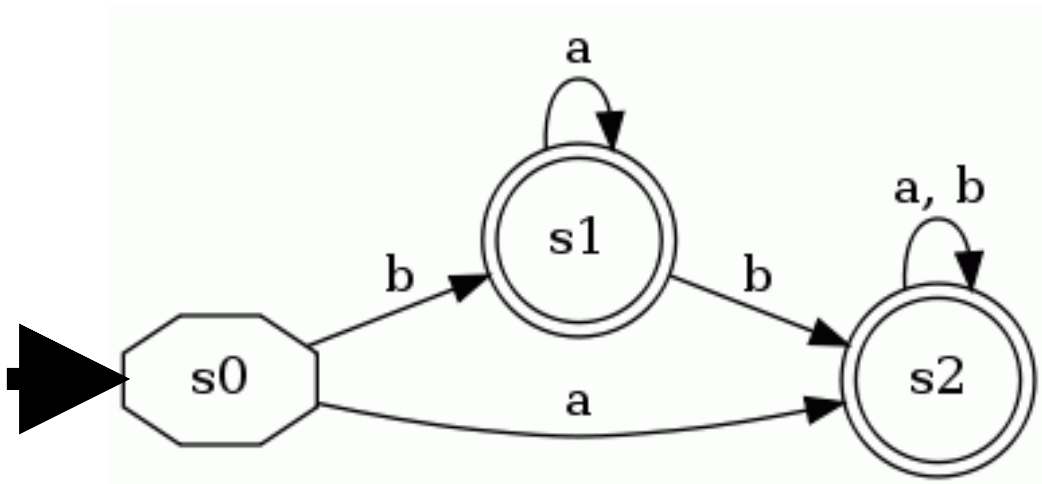
DFA already minimal

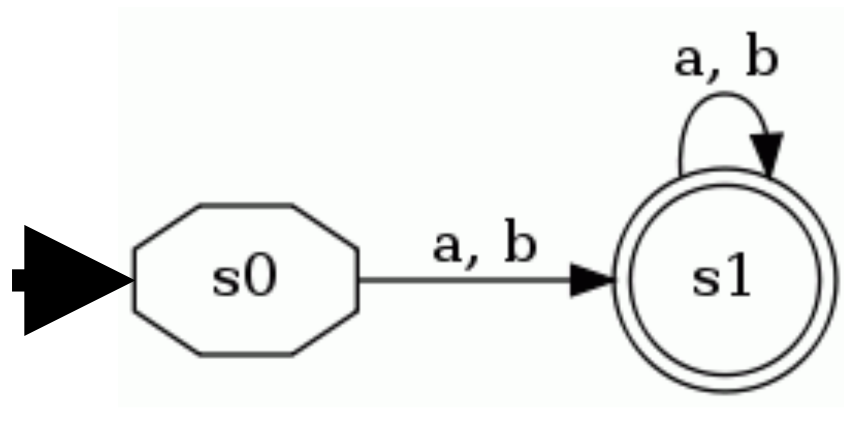
- Initial partitions
  - Accept { R } = P1
  - Reject { S, T } = P2
- Split partition? → Yes, different partitions for B
  - move(S,a) = T ∈ P2
  - move(T,a) = T ∈ P2
    – move(S,b) = T ∈ P2
    – move (T,b) = R ∈ P1

Minimizing DFA: Example 3



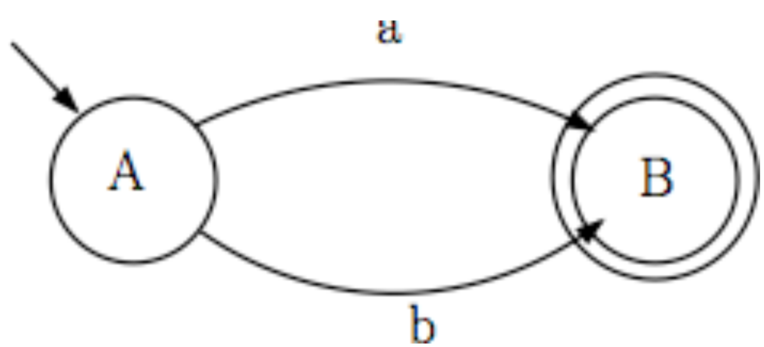
Minimizing DFA: Example 3





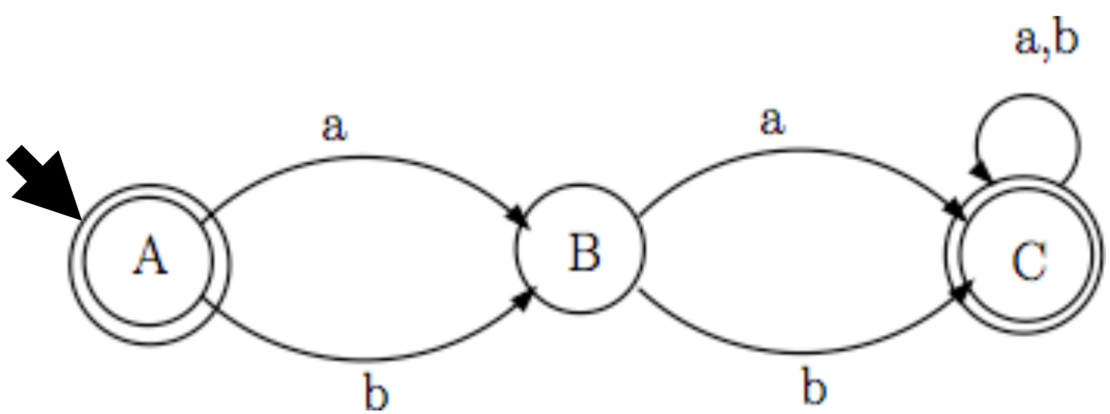
Complement of DFA

- Given a DFA accepting language L
  - How can we create a DFA accepting its complement?
  - Example DFA
    - $\Sigma = \{a,b\}$



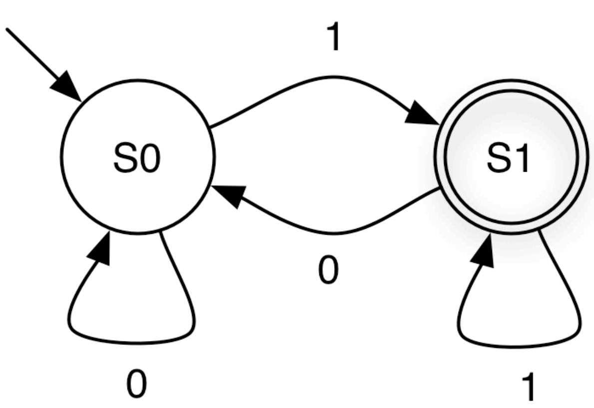
Complement of DFA

- Algorithm
  - Add explicit transitions to a dead state
  - Change every accepting state to a non-accepting state & every non-accepting state to an accepting state
- Note this only works with DFAs
  - Why not with NFAs?



Implementing DFAs (one-off)

It's easy to build a program which mimics a DFA




```
cur_state = 0;
while (1) {

  symbol = getchar();

  switch (cur_state) {

    case 0: switch (symbol) {
              case '0':  cur_state = 0; break;
              case '1':  cur_state = 1; break;
              case '\n': printf("rejected\n"); return 0;
              default:   printf("rejected\n"); return 0;
            }
            break;

    case 1: switch (symbol) {
              case '0':  cur_state = 0; break;
              case '1':  cur_state = 1; break;
              case '\n': printf("accepted\n"); return 1;
              default:   printf("rejected\n"); return 0;
            }
            break;

    default: printf("unknown state; I'm confused\n");
             break;
  }
}
```

Implementing DFAs (generic)

More generally, use generic table-driven DFA

```
given components (Σ, Q, q0, F, δ) of a DFA:
let q = q0
while (there exists another symbol s of the input string)
   q := δ(q, s);
if q ∈  F then
   accept
else reject
```

- q is just an integer
- Represent δ using arrays or hash tables
- Represent F as a set

Running Time of DFA

- How long for DFA to decide to accept/reject string s?
  - Assume we can compute $\delta(q, c)$ in constant time
  - Then time to process s is $O(|s|)$
    - Can't get much faster!
- Constructing DFA for RE A may take $O(2^{|A|})$ time
  - But usually not the case in practice
- So there's the initial overhead
  - But then processing strings is fast

Regular Expressions in Practice

- Regular expressions are typically “compiled” into tables for the generic algorithm
  - Can think of this as a simple byte code interpreter
  - But really just a representation of $(\Sigma, Q_A, q_A, \{f_A\}, \delta_A)$, the components of the DFA produced from the RE
- Regular expression implementations often have extra constructs that are non-regular
  - I.e., can accept more than the regular languages
  - Can be useful in certain cases
  - Disadvantages
    - Nonstandard, plus can have higher complexity

Summary of Regular Expression Theory

- Finite automata
  - DFA, NFA
- Equivalence of RE, NFA, DFA
  - RE → NFA
    - Concatenation, union, closure
  - NFA → DFA
    - ε-closure & subset algorithm
- DFA
  - Minimization, complement
  - Implementation