CMSC 330: Organization of Programming Languages

Tail Recursion and Continuation Passing Style (CPS)
Reverse

```ocaml
let rec rev l = match l with
    | [] -> []
    | (x::xs) -> (rev xs) @ [x]
```

- Pushes a stack frame on each recursive call

```
rev [1;2;3]
→ (rev [2;3]) @ [1]
→ (((rev [3]) @ [2]) @ [1]
→ ((((rev [3]) @ [2]) @ [1]
→ (((rev [3]) @ [2]) @ [1]
→ ([3] @ [2]) @ [1]
→ [3;2] @ [1]
→ [3;2;1]
```
A Clever Version of Reverse

```ocaml
let rec rev_helper l a = match l with
  | [] -> a
  | (x::xs) -> rev_helper xs (x::a)
let rev l = rev_helper l []
```

- No need to push a frame for each call!

  ```
  rev [1;2;3] →
  rev_helper [1;2;3] [] →
  rev_helper [2;3] [1] →
  rev_helper [3] [2;1] →
  rev_helper [] [3;2;1] →
  [3;2;1]
  ```
Tail Recursion

- Whenever a function ends with a recursive call, it is called **tail recursive**
  - Its “tail” is recursive

- Tail recursive functions can be implemented **without requiring a stack frame for each call**
  - No intermediate variables need to be saved, so the compiler overwrites them

- Typical pattern is to use an **accumulator** to build up the result, and return it in the base case
Compare rev and rev_helper

let rec rev l =
  match l with
  | [] -> []
  | (x::xs) -> (rev xs) @ [x]

let rec rev_helper l a =
  match l with
  | [] -> a
  | (x::xs) -> rev_helper xs (x::a)

final result is the result of the recursive call
Quiz #1

True/false: map is tail-recursive.

```ocaml
let rec map f = function
  | [] -> []
  | (h::t) -> (f h)::(map f t)
```

A. True
B. False
True/false: map is tail-recursive.

```
let rec map f = function
  | [] -> []
  | (h::t) -> (f h)::(map f t)
```

A. True
B. False
Quiz #2

True/false: `fold` is tail-recursive

```ocaml
let rec fold f a = function
  | [] -> a
  | (h::t) -> fold f (f a h) t
```

A. True
B. False
Quiz #2

True/false: fold is tail-recursive

let rec fold f a = function
  | []  -> a
  | (h::t) -> fold f (f a h) t

A. True
B. False
Quiz #3

True/false: fold_right is tail-recursive

let rec fold_right f l a =
    match l with
    | [] -> a
    | (h::t) -> f h (fold_right f t a)

A. True
B. False
Quiz #3

True/false: `fold_right` is tail-recursive

```ml
let rec fold_right f l a =
  match l with
  | [] -> a
  | (h::t) -> f h (fold_right f t a)
```

A. True
B. False
Tail Recursion is Important

• Pushing a call frame for each recursive call when operating on a list is dangerous
  – One stack frame for each list element
  – Big list = stack overflow!

• So: favor tail recursion when inputs could be large (i.e., recursion could be deep). E.g.,
  – Prefer `List.fold_left` to `List.fold_right`
    • Library documentation should indicate tail recursion, or not
  – Convert recursive functions to be tail recursive
Tail Recursion Pattern (1 argument)

let func x =
  let rec helper arg acc =
    if (base case) then acc
    else
      let arg’ = (argument to recursive call)
      let acc’ = (updated accumulator)
      helper arg’ acc’ in (* end of helper fun *)
  helper x (initial val of accumulator)
;;
Tail Recursion Pattern with fact

let fact x =
    let rec helper arg acc =
        if arg = 0 then acc
        else
            let arg’ = arg – 1 in
            let acc’ = acc * arg in
            helper arg’ acc’ in (* end of helper fun *)
    helper x 1
;;
Tail Recursion Pattern with \texttt{rev}

\begin{verbatim}
let \texttt{rev} \texttt{x} =
    let rec \texttt{rev\_helper} \texttt{arg} \texttt{acc} =
        match \texttt{arg} with
        | [] \rightarrow \texttt{acc}
        | \texttt{h}::\texttt{t} \rightarrow
            let \texttt{arg'} = \texttt{t} in  \hspace{1cm} \textit{Can generalize to more than one argument, and multiple cases for each recursive call}
            let \texttt{acc'} = \texttt{h}::\texttt{acc} in
            \texttt{rev\_helper} \texttt{arg'} \texttt{acc'} in
        (* end of helper fun *)

\texttt{rev\_helper} \texttt{x} [];;
\end{verbatim}
True/false: this is a tail-recursive map

```ocaml
let map f l =
  let rec helper l a =
    match l with
      | [] -> a
      | h::t -> helper t ((f h)::a)
  in helper l []
```

A. True
B. False
Quiz #4

True/false: this is a tail-recursive map

```
let map f l =
  let rec helper l a =
    match l with
    | []       -> a
    | h::t     -> helper t ((f h)::a)
  in helper l []
```

A. True
B. False (elements are reversed)
A Tail Recursive map

```ocaml
let map f l =
  let rec helper l a =
    match l with
    [] -> a
    | h::t -> helper t ((f h)::a)
  in rev (helper l [])
```

Could instead change \((f \ h) :: a\) to be \(a @ (f \ h)\)

Q: Why is the above implementation a better choice?

A: \(O(n)\) running time, not \(O(n^2)\) (where \(n\) is length of list)
How far does this generalize?

• A function that is tail-recursive returns at most once (to its caller) when completely finished

• Is it possible to convert an arbitrary program into an equivalent one, except where no call ever returns?

• Yes. This is called continuation-passing style