

**Logarithms.**

$$\log_b a = \frac{\log_c a}{\log_c b} \quad a = b^{\log_b a} \quad \log_c(ab) = \log_c a + \log_c b \quad \log_b a^n = n \log_b a$$

$$\log_b(1/a) = -\log_b a \quad \log_b a = \frac{1}{\log_a b} \quad a^{\log_b n} = n^{\log_b a}$$

**Stirling's Formula.**

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

**Probability.**

$$\mathbb{E}[X] = \sum_x x \Pr\{X = x\}, \quad \text{Var}[X] = \mathbb{E}[(X - \mathbb{E}(X))^2] = \mathbb{E}[X^2] - \mathbb{E}^2[X], \quad \sigma[X] = \sqrt{\text{Var}[X]}.$$

**Quadratic Formula.**

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Approximation by integrals:**

$$\int_{m-1}^n f(x)dx \leq \sum_{k=m}^n f(k) \leq \int_m^{n+1} f(x)dx \quad \text{for } f(x) \text{ monotonically increasing}$$

$$\int_m^{n+1} f(x)dx \leq \sum_{k=m}^n f(k) \leq \int_{m-1}^n f(x)dx \quad \text{for } f(x) \text{ monotonically decreasing}$$

**Summations Laws.**

Distribution law:

$$\left(\sum_{i=1}^m a_i\right) \left(\sum_{j=1}^n b_j\right) = \sum_{i=1}^m \left(\sum_{j=1}^n a_i b_j\right)$$

Interchanging order of summation:

$$\sum_{i=1}^m \sum_{j=1}^n a_{ij} = \sum_{j=1}^n \sum_{i=1}^m a_{ij}$$

Splitting range:

$$\sum_{k=1}^n a_k = \sum_{k=1}^r a_k + \sum_{k=r+1}^n a_k$$

Telescoping series:

$$\sum_{k=1}^n (a_k - a_{k-1}) = a_n - a_0$$

**Summations.**

Arithmetic series:

$$\sum_{k=1}^n k = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Quadratic series:

$$\sum_{k=1}^n k^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k(k+1) = 1 \cdot 2 + 2 \cdot 2 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

Geometric series:

$$\sum_{k=0}^n x^k = 1 + x + x^2 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1} \quad x \neq 1$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad |x| < 1$$

Harmonic series:

$$H_n = \sum_{k=1}^n \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} = \ln n + O(1)$$

**Products.**

$$\prod_{k=1}^n a_k = a_1 a_2 \cdots a_n \qquad \log \prod_{k=1}^n a_k = \sum_{k=1}^n \log a_k$$