

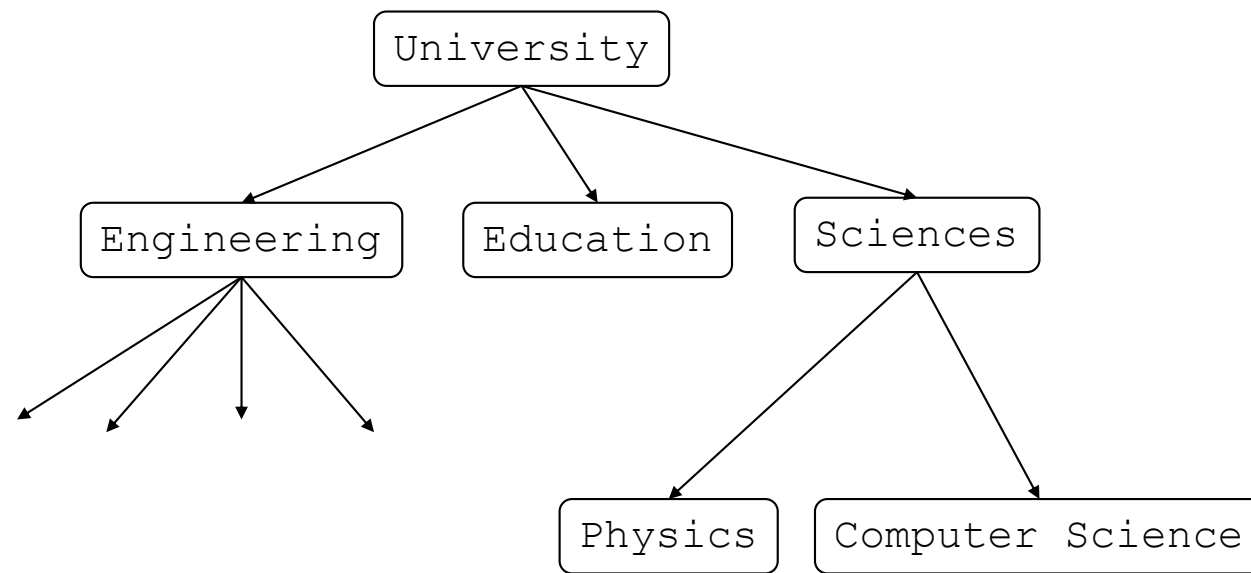
# CMSC 132: Object-Oriented Programming II

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## Binary Trees

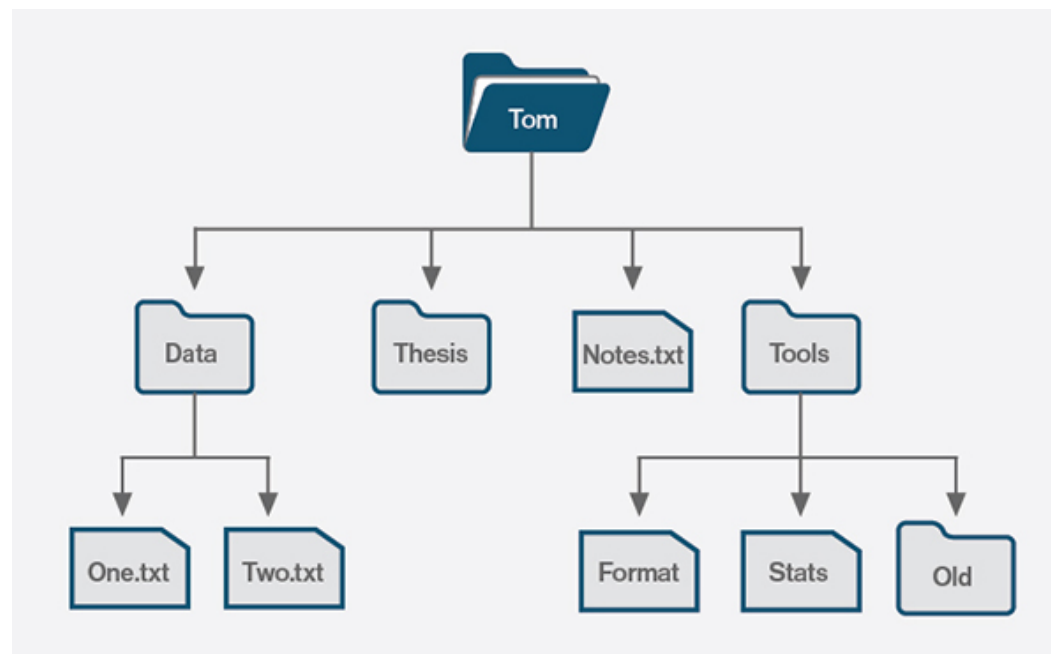
# Trees

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# Trees

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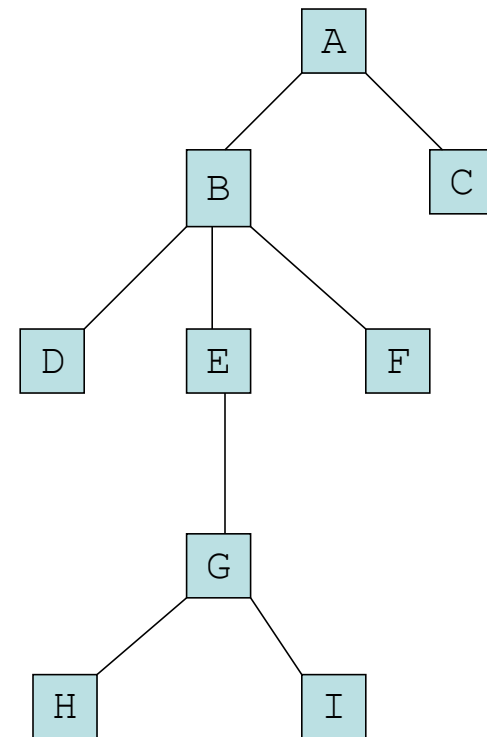
# Trees

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- ▶ A tree is a node with a value and zero or more children.
- ▶ No Cycle

- ▶ **Properties**

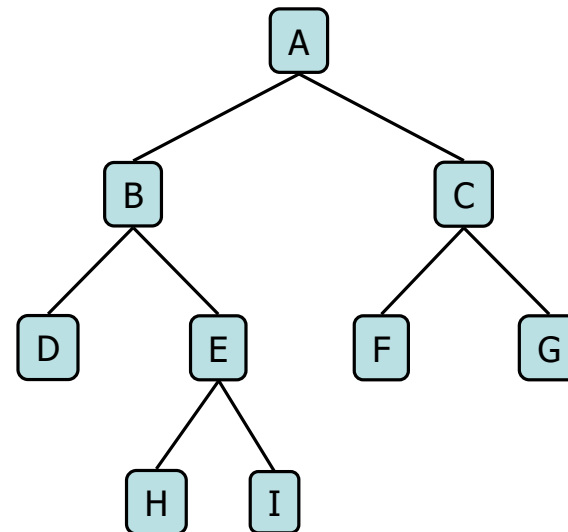
- Number of nodes
- Height
- Root Node
- Leaves
- Interior nodes
- Ancestor
- Descendant
- Siblings
- Subtrees



# Binary Tree

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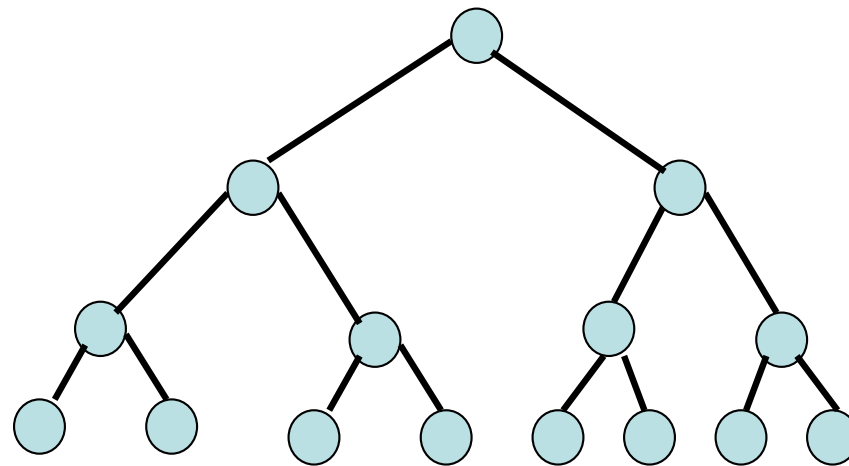
- ▶ Each internal node has at most two children (degree of two)
- ▶ The children of a node are an ordered pair
- ▶ We call the children of an internal node left child and right child
- ▶ Applications:
  - arithmetic expressions
  - decision processes
  - searching



# Full Binary Tree

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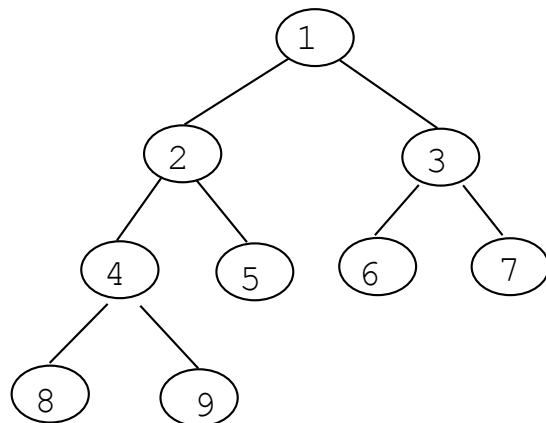
- ▶ A full binary tree is a tree in which every node other than the leaves has two children.
- ▶ A full (perfect) binary tree of a given height  $k$  has  $2^{k+1}-1$  nodes.



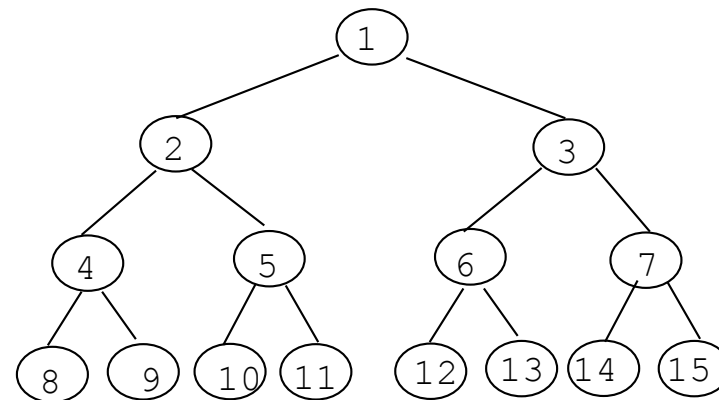
# Complete Binary Trees

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A **complete binary tree** is a **binary tree** in which every level, except possibly the last, is completely filled, and all nodes are as far left as possible.



Complete binary tree

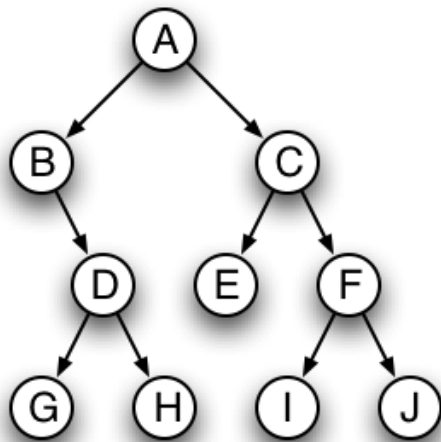


Full binary tree

# Binary Tree Traversal

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Traversal: Process of visiting each node in a tree, exactly once



preOrder:

inOrder:

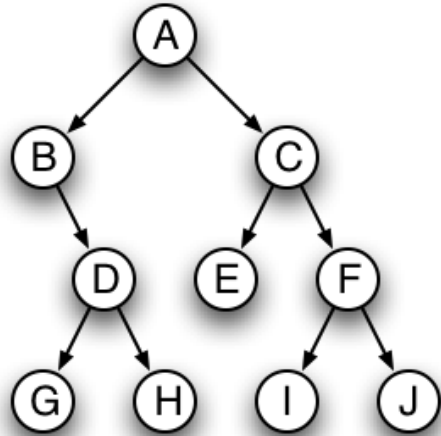
postOrder:

levelOrder:



# Binary Tree Traversal

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preOrder: root, left, right  
A B D G H C E F I J

inOrder: left, root, right  
B G D H A E C I F J

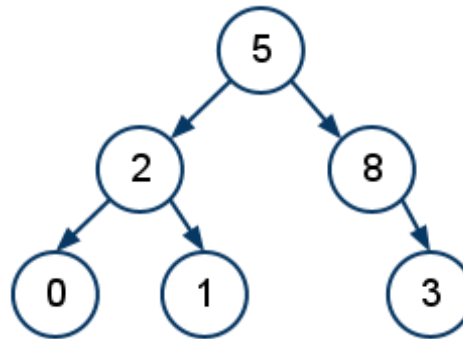
postOrder: left, right, root  
G H D B E I J F C A

Level Order: BFS  
A B C D E F G H I J

## Quiz 1:

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What is the preOrder traversal of this binary tree?

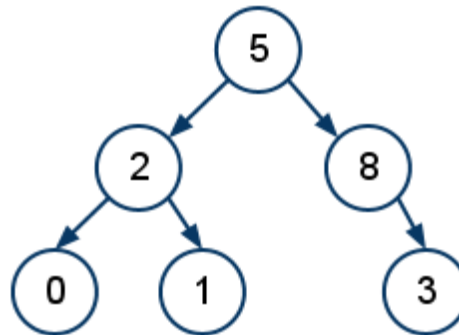


- A. 5 2 8 0 1 3
- B. 5 2 1 0 3 8
- C. 5 2 0 1 8 3
- D. 5 2 0 1 3 8

## Quiz 1:

---

What is the **preOrder** traversal of this binary tree?

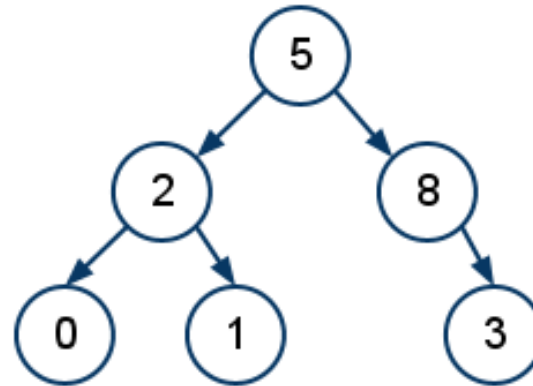


- A. 5 2 8 0 1 3
- B. 5 2 1 0 3 8
- C. 5 2 0 1 8 3**
- D. 5 2 0 1 3 8

## Quiz 2:

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What is the inOrder traversal of this binary tree?

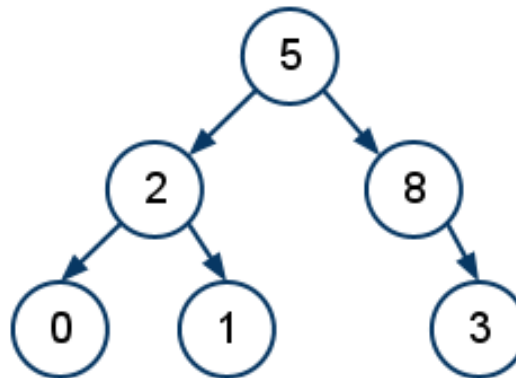


- A. 0 1 2 3 8 5
- B. 0 2 1 5 8 3
- C. 0 2 1 5 3 8
- D. 5 2 0 1 3 8

## Quiz 2:

---

What is the **inOrder** traversal of this binary tree?

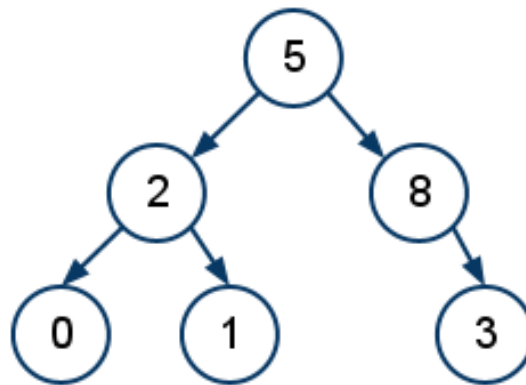


- A. 0 1 2 3 8 5
- B. 0 2 1 5 8 3**
- C. 0 2 1 5 3 8
- D. 5 2 0 1 3 8

## Quiz 3:

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What is the **postOrder** traversal of this binary tree?

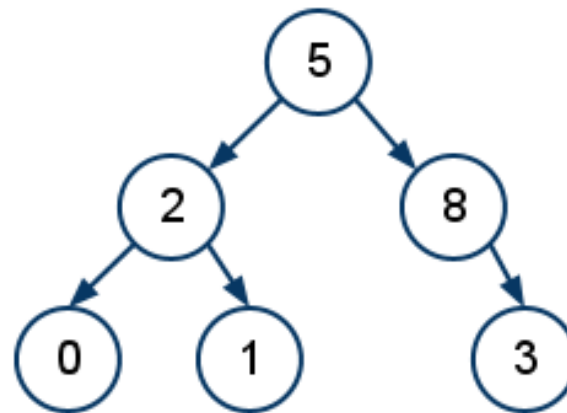


- A. 0 1 2 3 8 5
- B. 0 2 1 5 8 3
- C. 0 1 2 5 3 8
- D. 5 2 0 1 3 8

## Quiz 3:

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What is the **postOrder** traversal of this binary tree?



**A. 0 1 2 3 8 5**

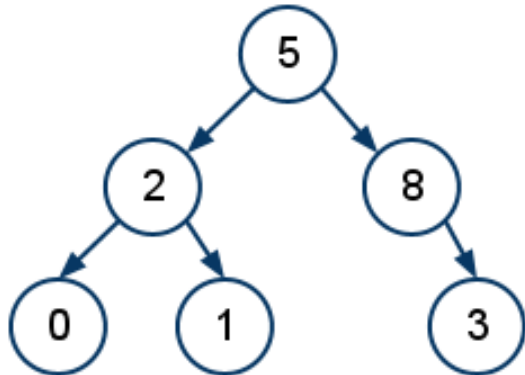
B. 0 2 1 5 8 3

C. 0 1 2 5 3 8

D. 5 2 0 1 3 8

# Binary Tree Traversal

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preOrder: **5 2 0 1 8 3**

inOrder: **0 2 1 5 8 3**

postOrder: **0 1 2 3 8 5**

Level Order: **5 2 8 0 1 3**



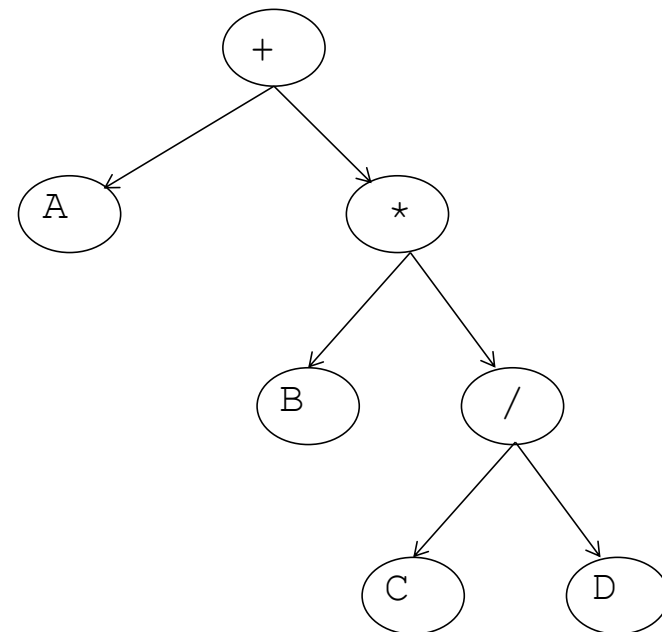
# Arithmetic Expression Trees

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Arithmetic Expression:

$$A + (B * (C / D) )$$

Tree for the above expression:



Used in most compilers

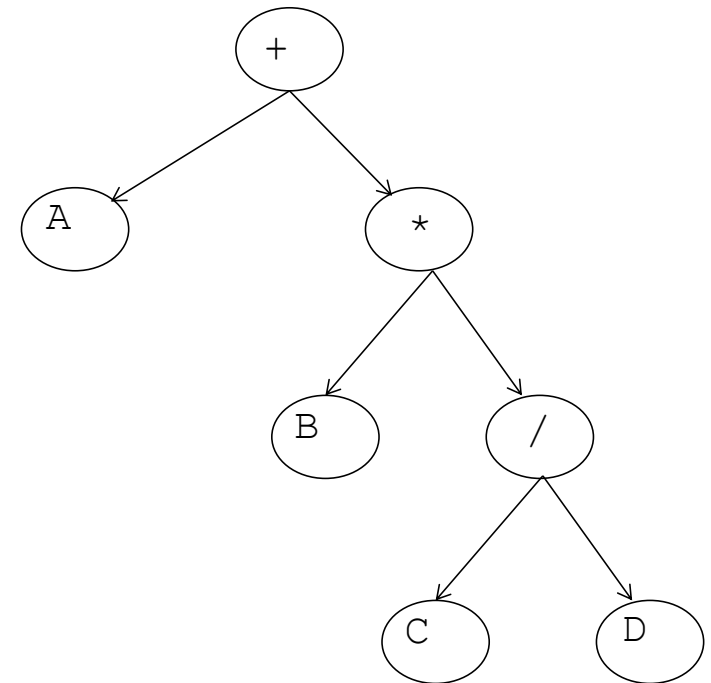
No parenthesis need to evaluate

Calculate by traversing tree

# Traversing Trees

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- ▶ Preorder: Root, then Children
  - $+ A * B / C D$
- ▶ Postorder: Children, then Root
  - $A B C D / * +$
- ▶ Inorder: Left child, Root, Right child
  - $A + B * C / D$



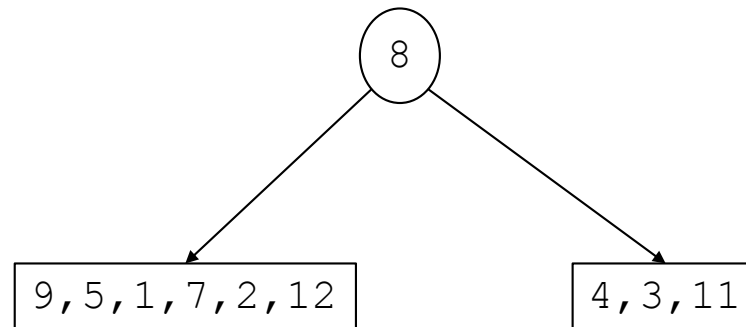
# Build a Binary Tree

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Build a Binary Tree from given inOrder, postOrder

inOrder: 9,5,1,7,2,12,8,4,3,11

postOrder: 9,1,2,12,7,5,3,11,4,8 ← root



# Build a Binary Tree

---

Build a Binary Tree from given `inOrder`, `postOrder`

`inOrder`: 9,5,1,7,2,12,8,4,3,11

`postOrder`: 9,1,2,12,7,5,3,11,4,8

# Build a Binary Tree

---

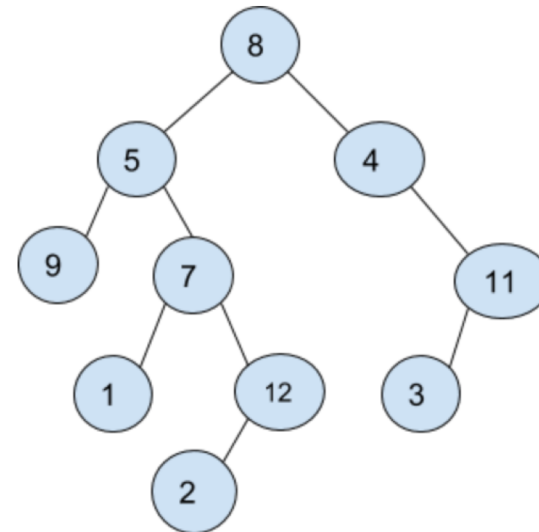
Build a Binary Tree from given **inOrder**, **postOrder**

inOrder: 9,5,1,7,2,12,8,4,3,11

postOrder: 9,1,2,12,7,5,3,11,4,8

preorder: 8,5,9,7,1,12,2,4,11,3

levelOrder: 8,5,4,9,7,11,1,12,3,2



# Build a Binary

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Build Binary Tree from inOrder, preOrder

inOrder: DBHEIAFCG

preOrder: ABDEHICFG

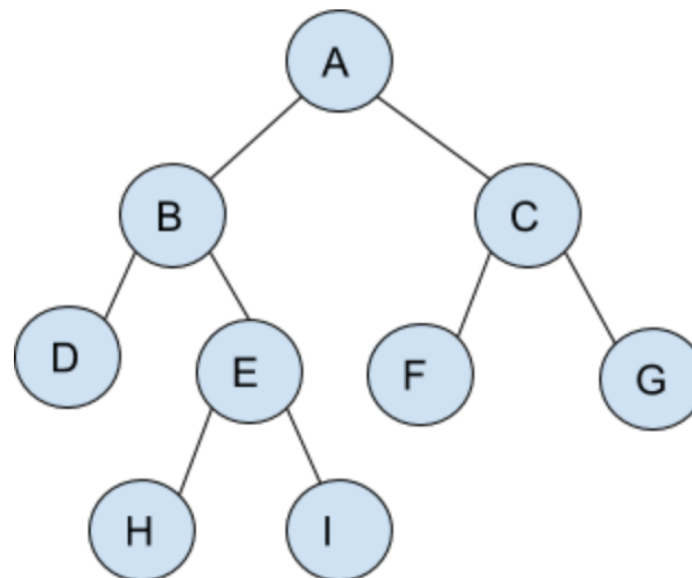
## Build a Binary

---

Build Binary Tree from inOrder, preOrder

inOrder: DBHEIAFCG  
preOrder: ABDEHICFG

postOrder: DHIEBFGCA



# Binary Tree Implementation

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Height:

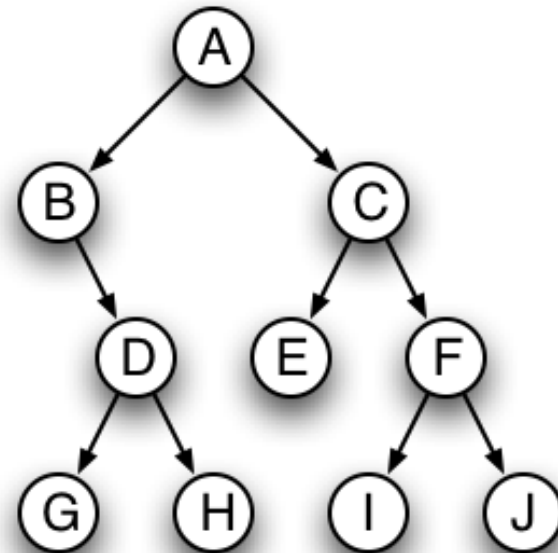
Size:

Diameter:

Mirror:

Path:

Least Common Ancestor (LCA):

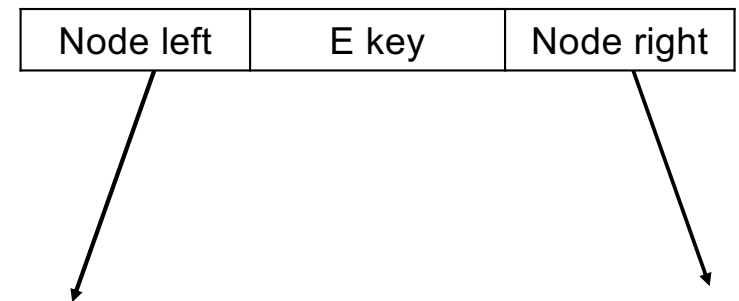




# Binary Tree Node Class

---

```
class Node {  
    private E key;  
    private Node left, right;  
    Node(E key) {  
        this.key = key;  
    }  
}
```



# Binary Tree Class

---

```
public class BinaryTree<E> {  
    private Node root;  
    class Node {  
        private E key;  
        private Node left, right;  
        Node(E key) {  
            this.key = key;  
        }  
    }  
}
```

# Binary Tree Implementation

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Check out the Binary Tree code examples from [github](#)