Problem 1. Consider the sum

$$\sum_{k=1}^{n} k^3 \ .$$

(a) Use a non-integral method to show that the sum is between $n^4/20$ and n^4.

(b) Use the integral method to find better upper and lower bounds.

Problem 2. Consider an array of size nine with the numbers $50, 30, 90, 10, 20, 70, 40, 80, 60$. Assume you execute quicksort using the version of partition from CLRS (and from class).

(a) What is the array after the first partition? How many comparisons did you use? How many exchanges? Do not count the move in the instruction X $\leftarrow$ A[r]. Note that in this algorithm an element might exchange with itself, which counts as one exchange.

(b) How many comparisons and exchanges do you use when you quicksort the entire left side of the pivot?

(c) How many comparisons and exchanges do you use when you quicksort the entire right side of the (original) pivot.

Problem 3. Today is your unlucky day: You run quicksort and the pivot turns out to always be in the first or last quarter of the of the elements, but is otherwise random.

(a) Write a recurrence to *estimate* the number comparisons the algorithm does on average (as we did in class under the standard probabilistic assumptions).

(b) Solve the recurrence using constructive induction (gettting an upper bound as we did in class under the standard probabilistic assumptions). Show your work.

(c) How does this compare to your estimate of quicksort under the standard probabilistic assumptions?

(d) Write a recurrence for the exact number of comparisons the algorithm does on average. To keep things simple assume that the size of list is always a multiple of four.

(e) [9 BONUS points] Get an upper bound for the number of comparisons using constructive induction (as we did in class under the standard probabilistic assumptions). Show your work.

(f) [1 BONUS point] How does this compare to your estimate in Part(b)?