CMSC 132: Object-Oriented Programming II

Sorting

What Is Sorting?

- To arrange a collection of items in some specified order.
  - Numerical order
  - Lexicographical order
- ***Input:*** sequence $<a_1, a_2, \ldots, a_n>$ of numbers.
- ***Output:*** permutation $<a'_1, a'_2, \ldots, a'_n>$ such that $a'_1 \le a'_2 \le \cdots \le a'_n$.
- Example
  - Start → 1 23 2 56 9 8 10 100
  - End → 1 2 8 9 10 23 56 100

Why Sort?

- A classic problem in computer science.
  - Data requested in sorted order
    - e.g., list students in increasing GPA order
- Searching
  - To find an element in an array of a million elements
    - Linear search: average **500,000** comparisons
    - Binary search: worst case **20** comparisons
  - Database, Phone book
- Eliminating duplicate copies in a collection of records
- Finding a missing element, Max, Min

Sorting Algorithms

- **Selection Sort** **Bubble Sort**
- **Insertion Sort** **Shell Sort**
- $T(n)=O(n^2)$ **Quadratic growth**
- In clock time

10,000	**3 sec**	**20,000**	**17 sec**
50,000	77 sec	100,000	5 min

- Double input -> 4X time
  - Feasible for small inputs, quickly unmanagable
- Halve input -> 1/4 time
  - Hmm... can recursion save the day?
  - If have two sorted halves, how to produce sorted full result?

Divide and Conquer

1. Base case: the problem is small enough, solve directly
2. Divide the problem into two or more similar and smaller subproblems
3. Recursively solve the subproblems
4. Combine solutions to the subproblems

Merge Sort

- Divide and conquer algorithm
- Worst case: O(nlogn)
- Stable
  - maintain the relative order of records with equal values
  - Input: 12, 5, 8, 13, 8, 27
  - Stable: 5, 8, 8, 12, 13, 27
  - Not Stable:5, 8, 8, 12, 13, 27

Stable Sort Example

$(Dave, A)$
$(Alice, B)$
(Ken, A)
$(Eric, B)$
$(Carol, A)$

`Sort by name` $\longrightarrow$

$(Alice, B)$
$(Carol, A)$
$(Dave, A)$
$(Eric, B)$
(Ken, A)

`Now, sort by section`

$(Carol, A)$
$(Dave, A)$
(Ken, A)
$\boxed{(Eric, B)}$
$\boxed{(Alice, B)}$

Not Stable

$(Carol, A)$
$(Dave, A)$
(Ken, A)
$\boxed{(Alice, B)}$
$\boxed{(Eric, B)}$

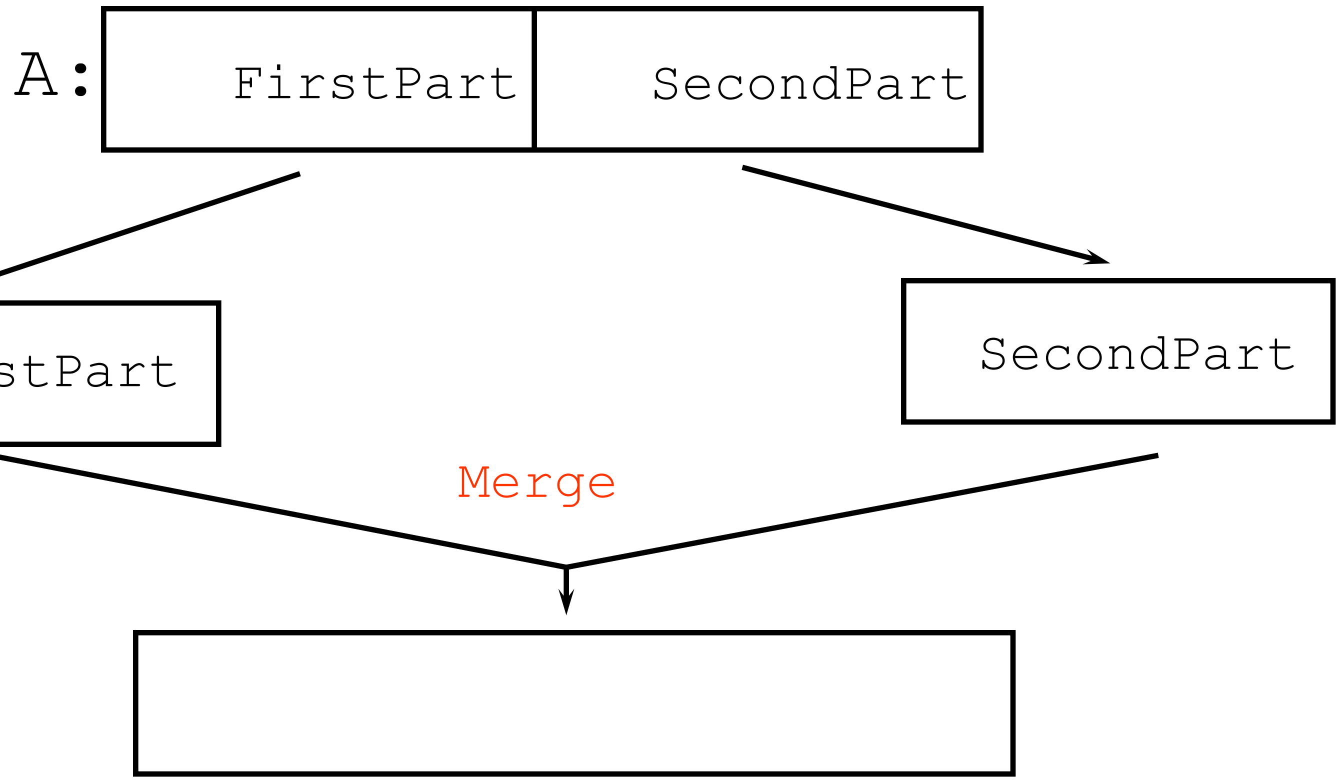
Stable

Merge Sort: Idea

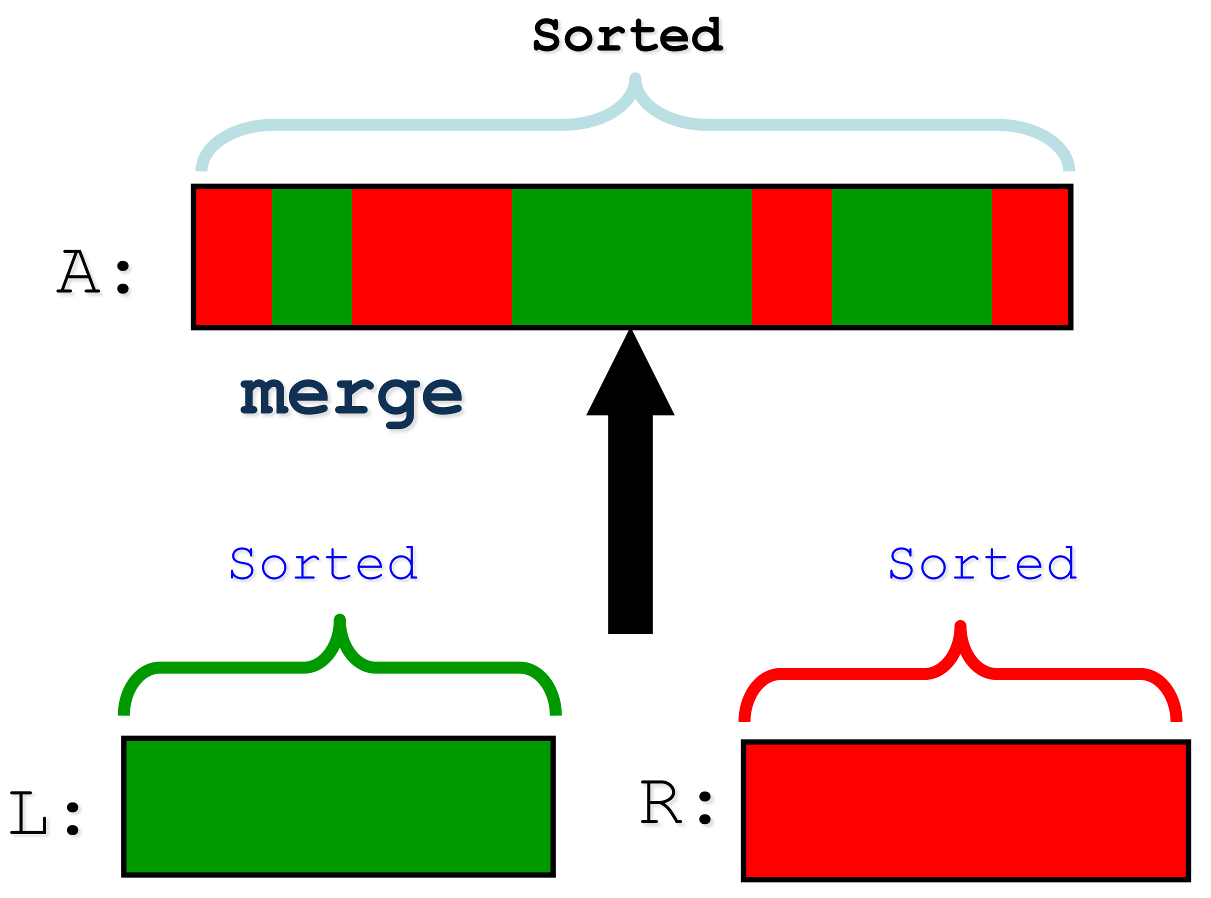
Divide into two halves

Recursively sort

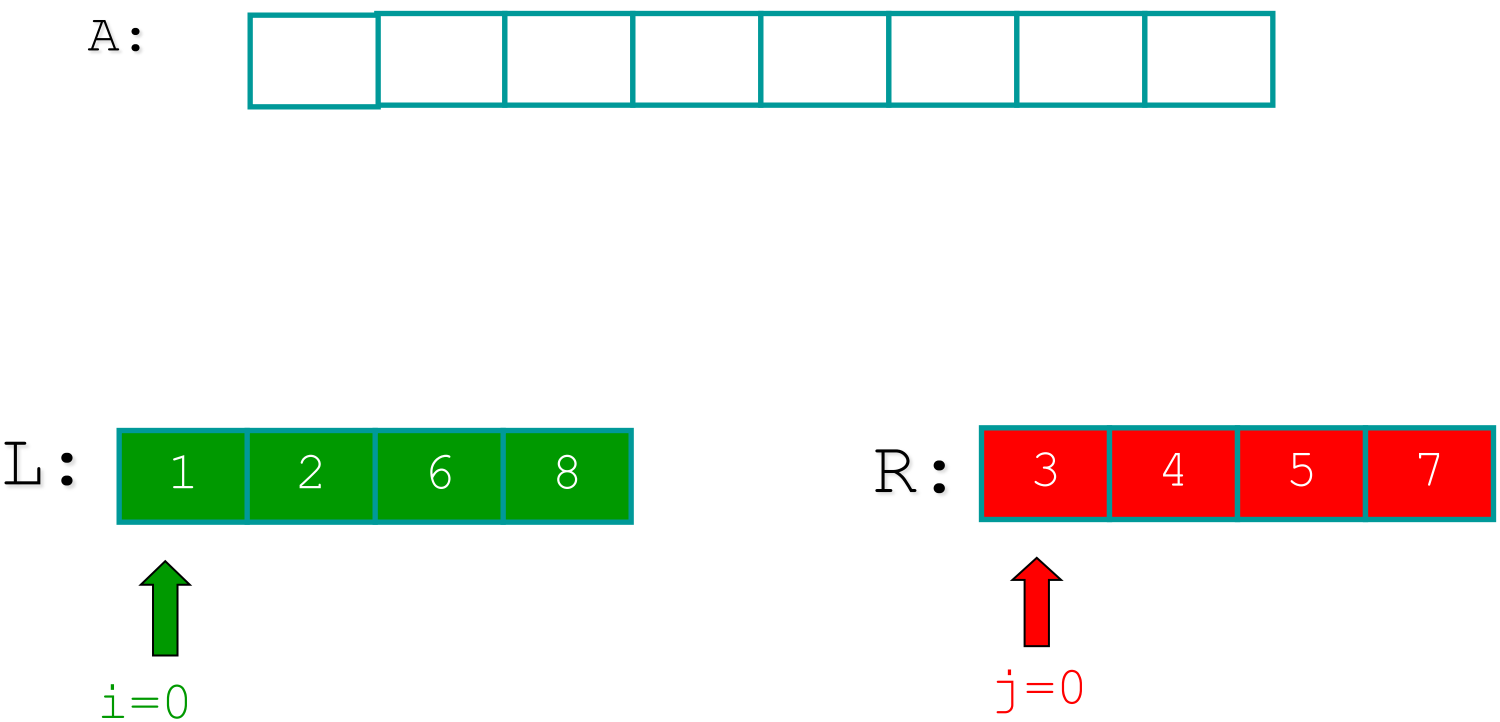
A is sorted!



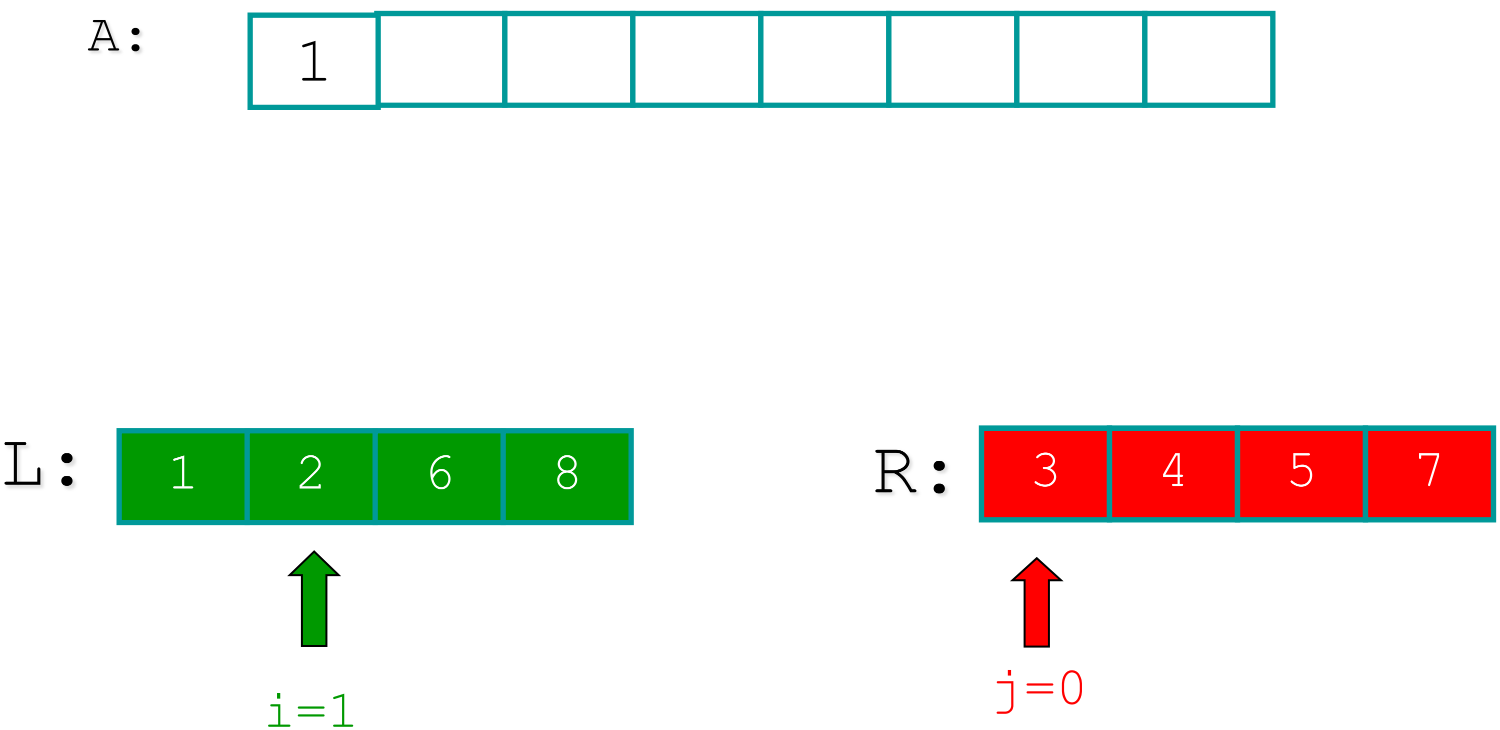
Merge-Sort: Merge



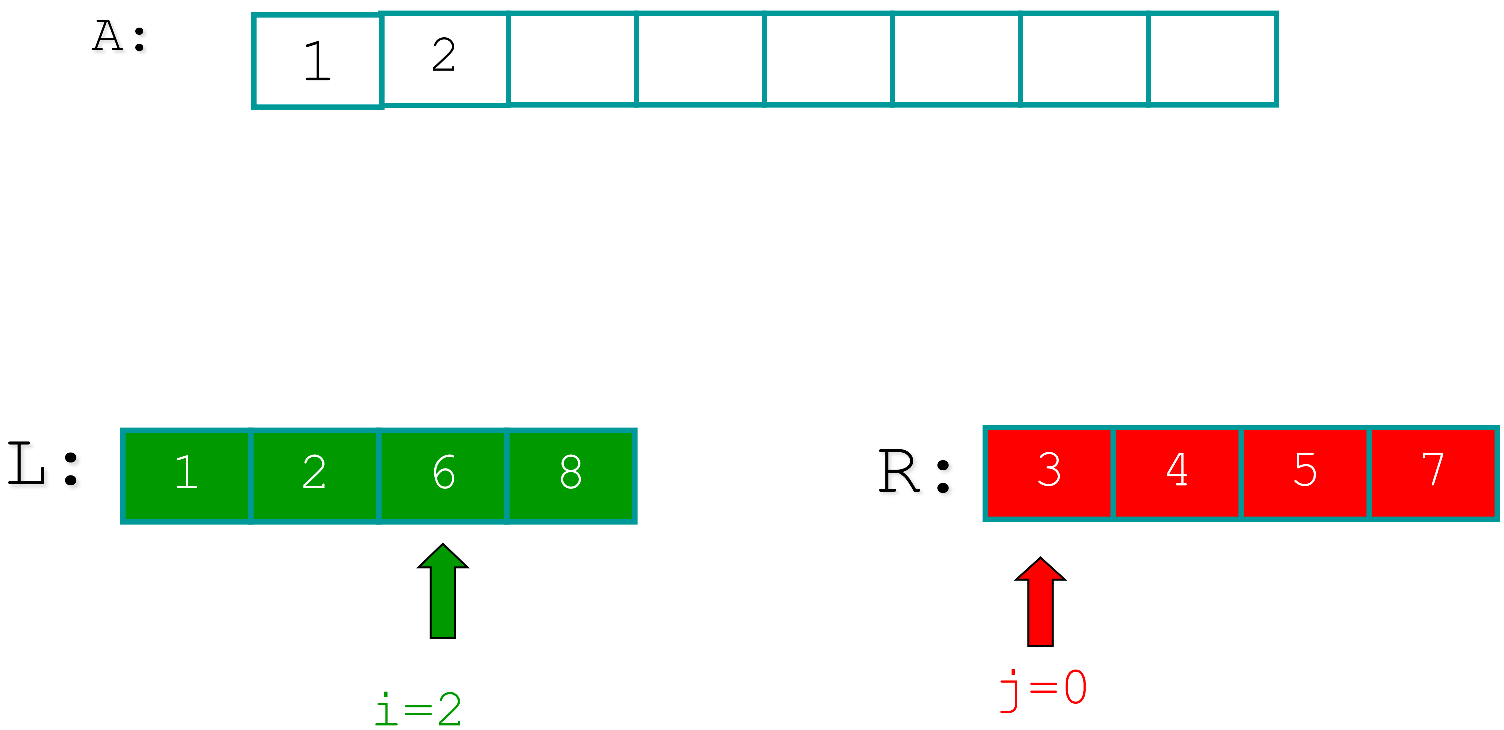
Merge Example



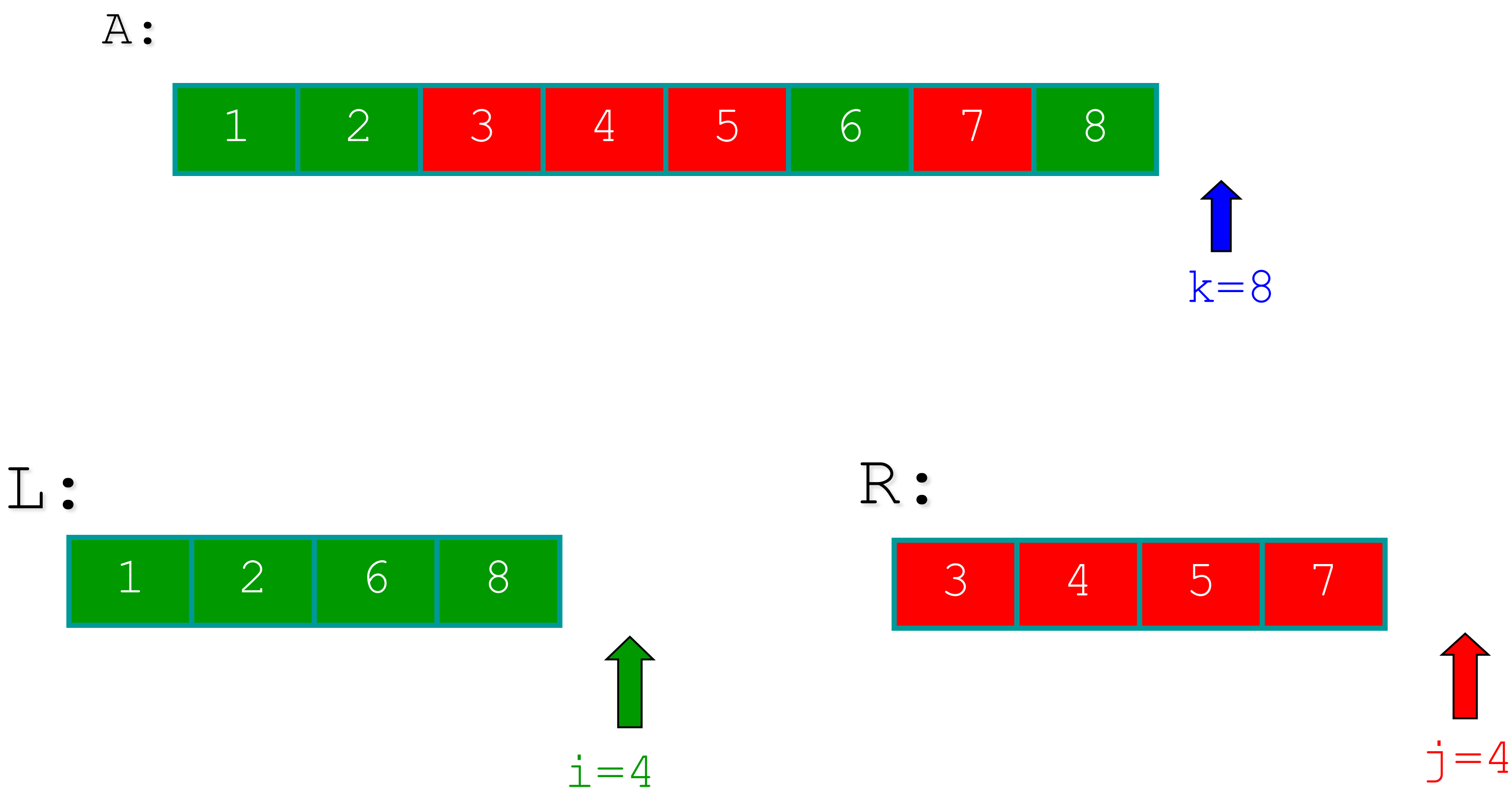
Merge Example



Merge Example



Merge Example cont.



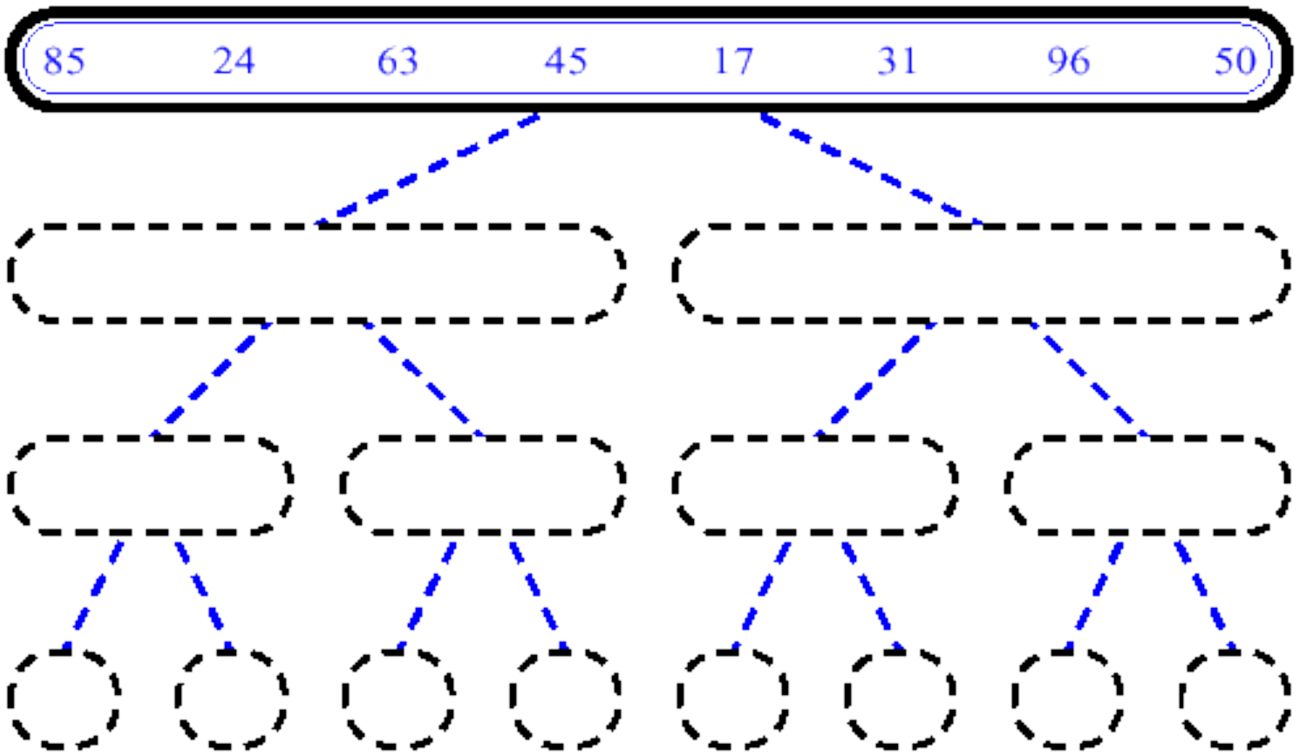
Merge sort algorithm

MERGE-SORT ***A*[1 . . *n*]**

1. If $n = 1$, done.
2. Recursively sort $A[\,1\,..\,\lceil n/2 \rceil\,]$ and $A[\,\lceil n/2 \rceil + 1\,..\,n\,]$.
3. "***Merge***" the 2 sorted lists.

Key subroutine: **MERGE**

Merge sort (Example)



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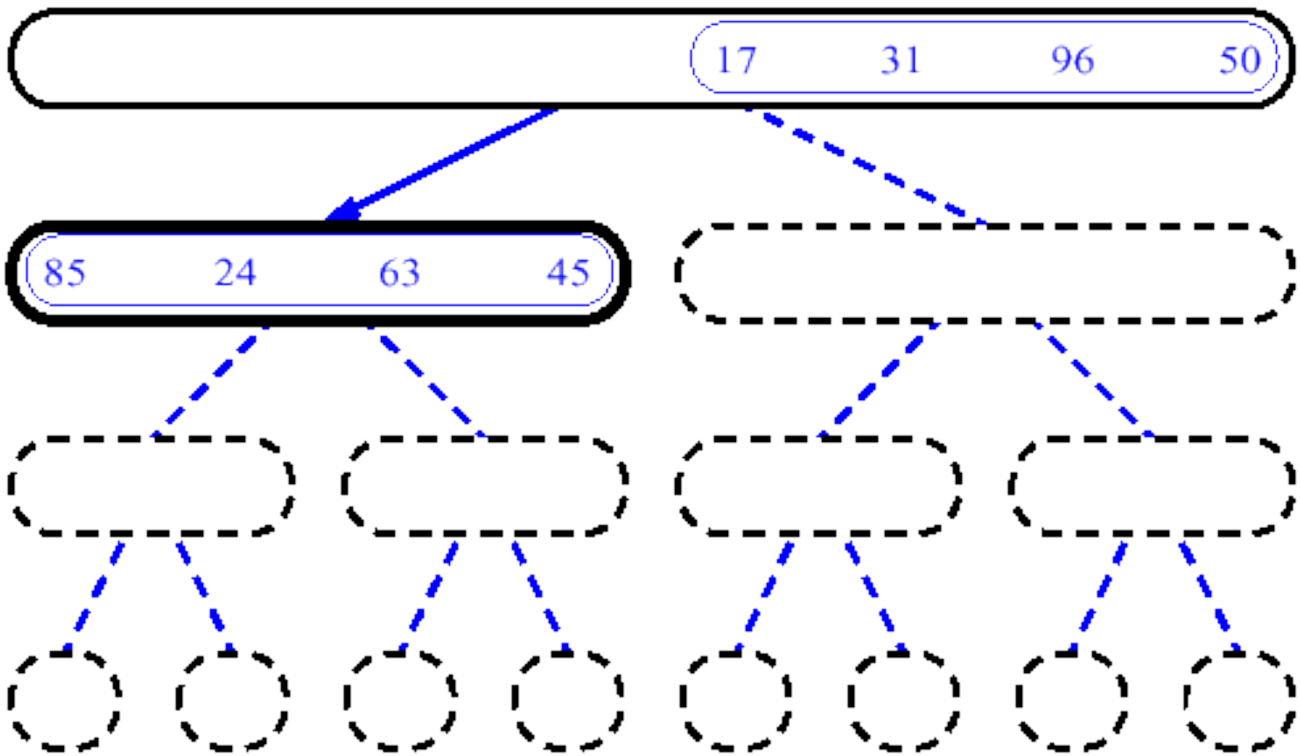

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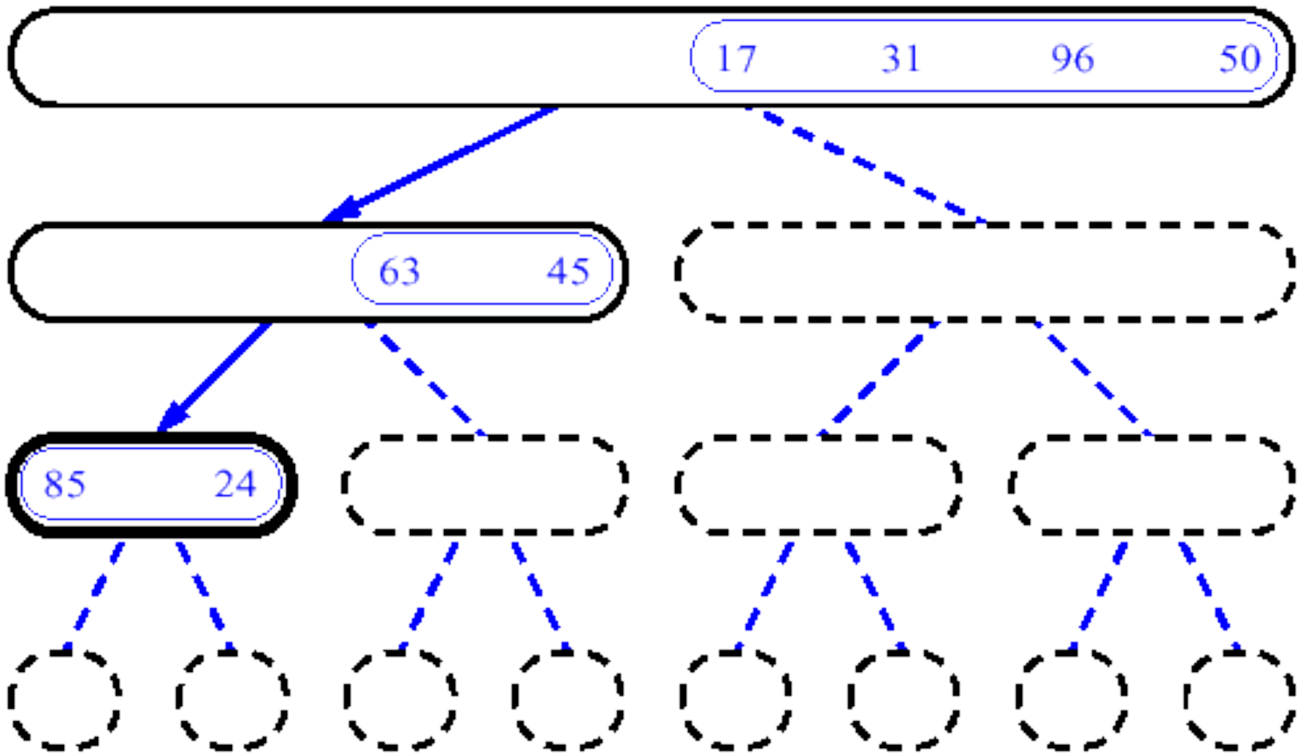

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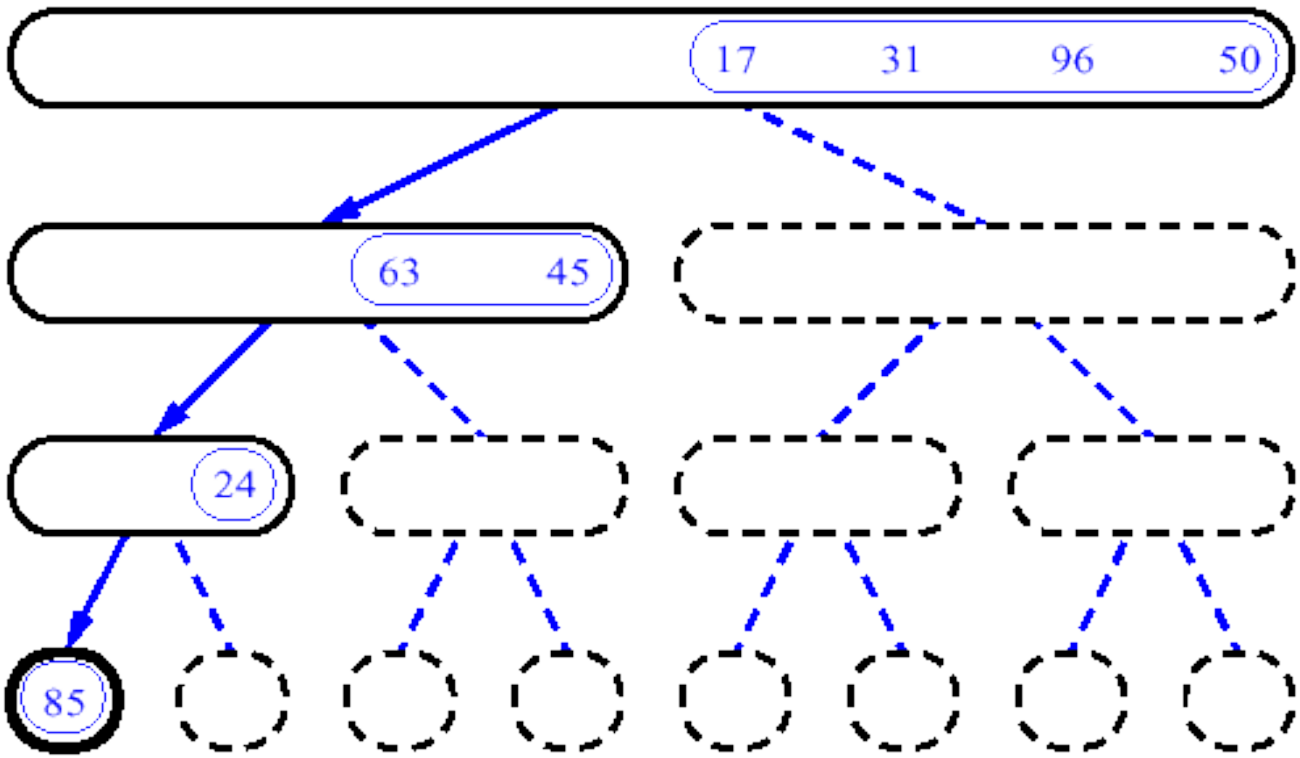

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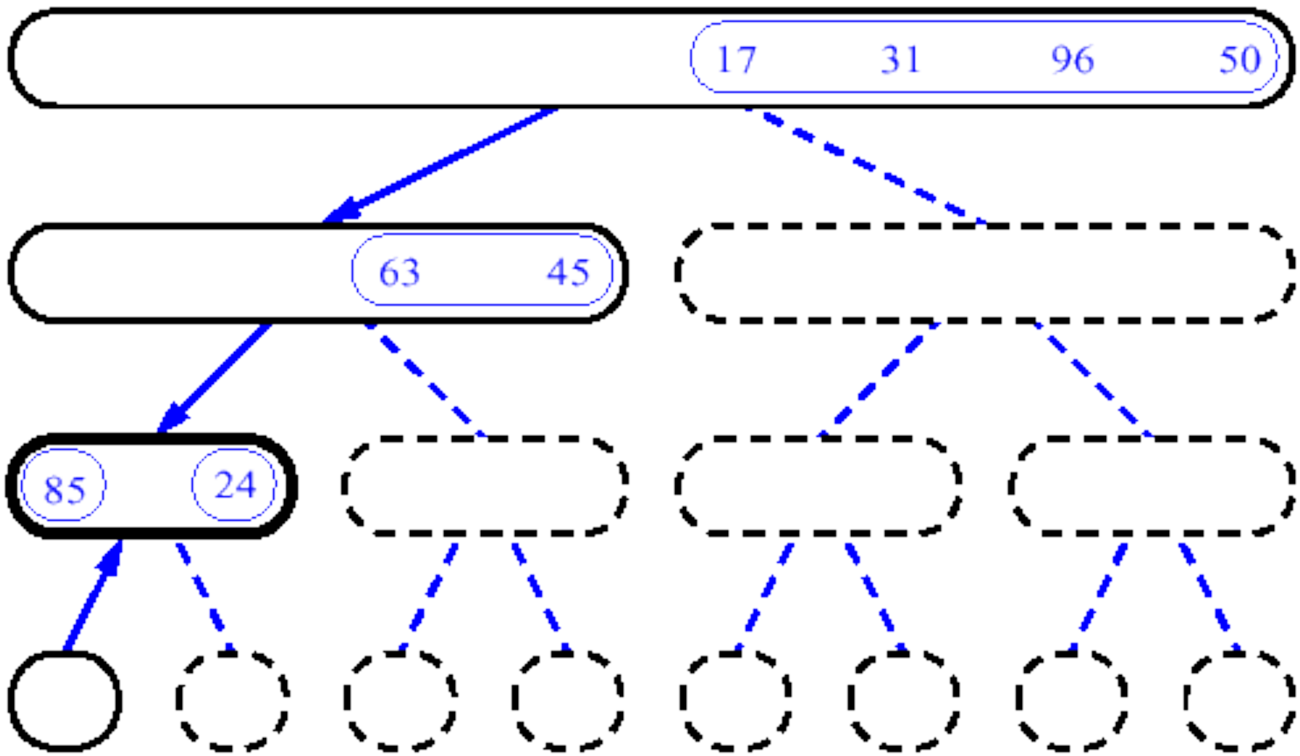
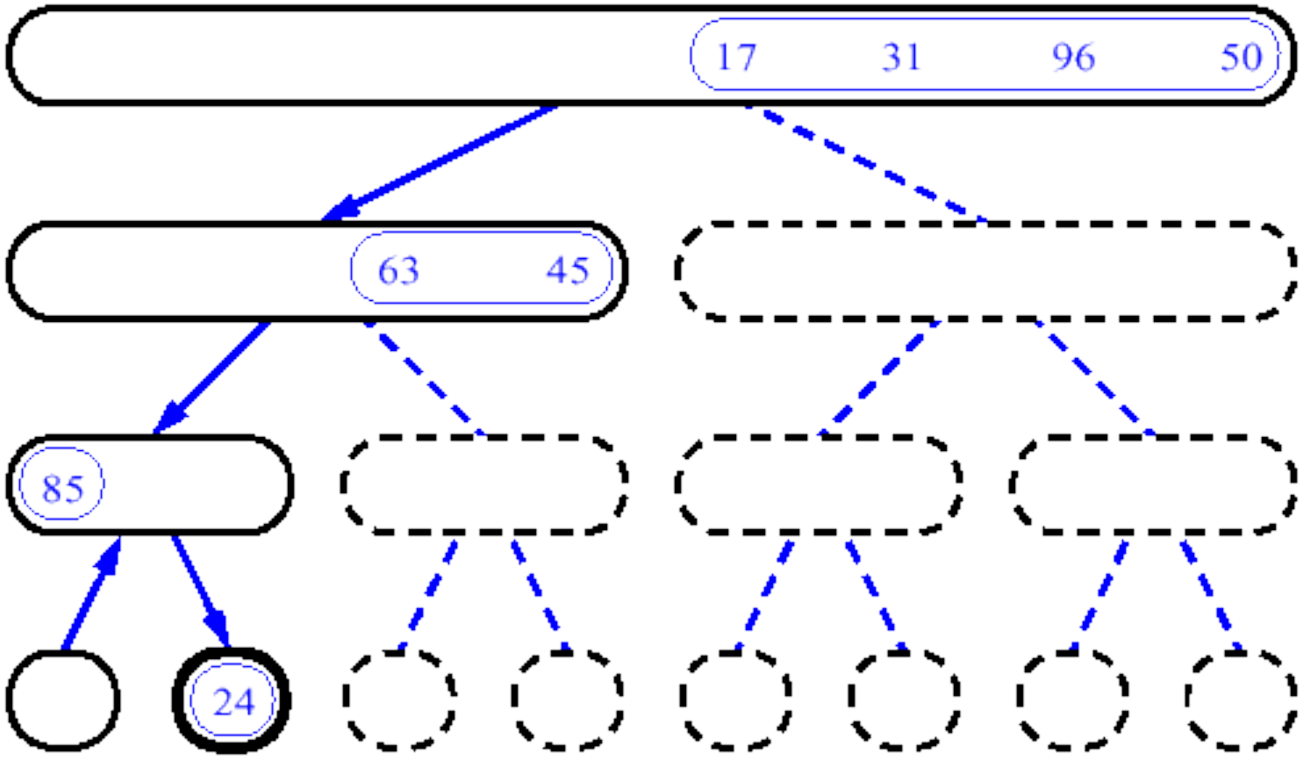


Merge sort (Example)



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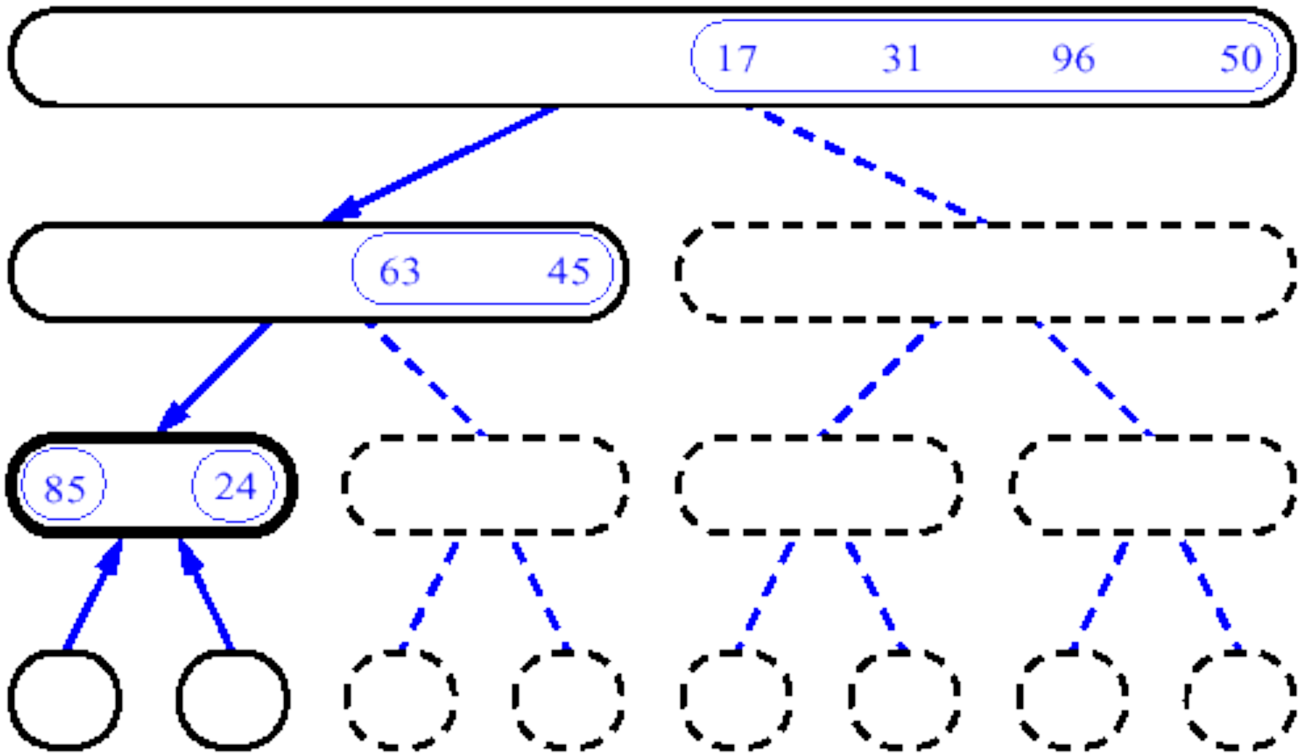

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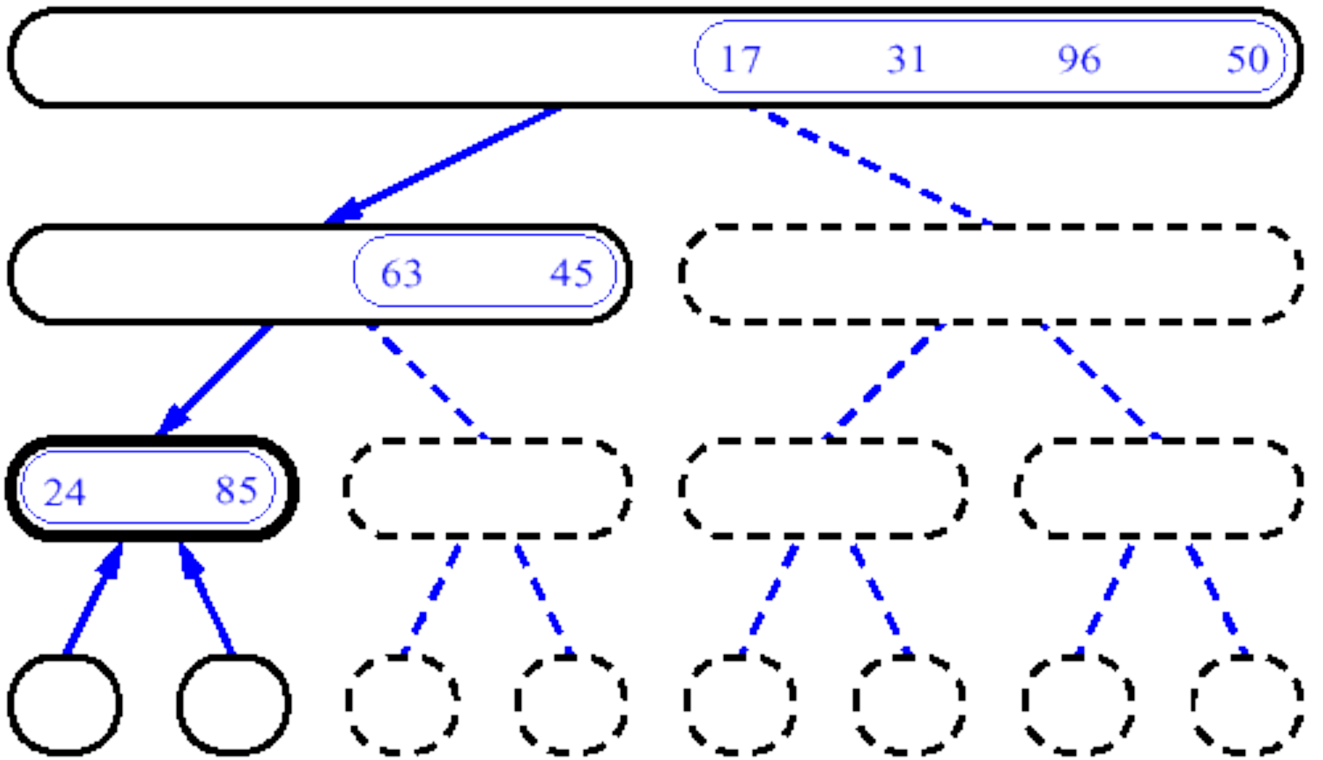
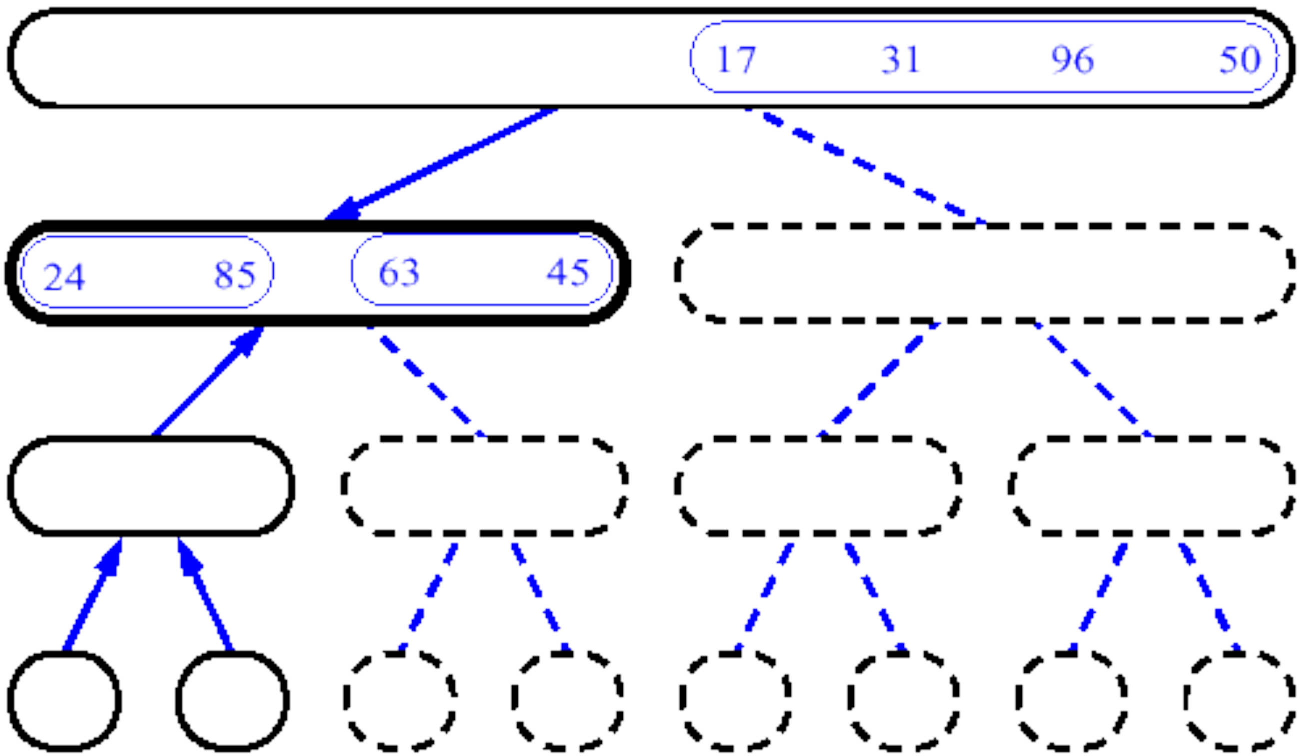
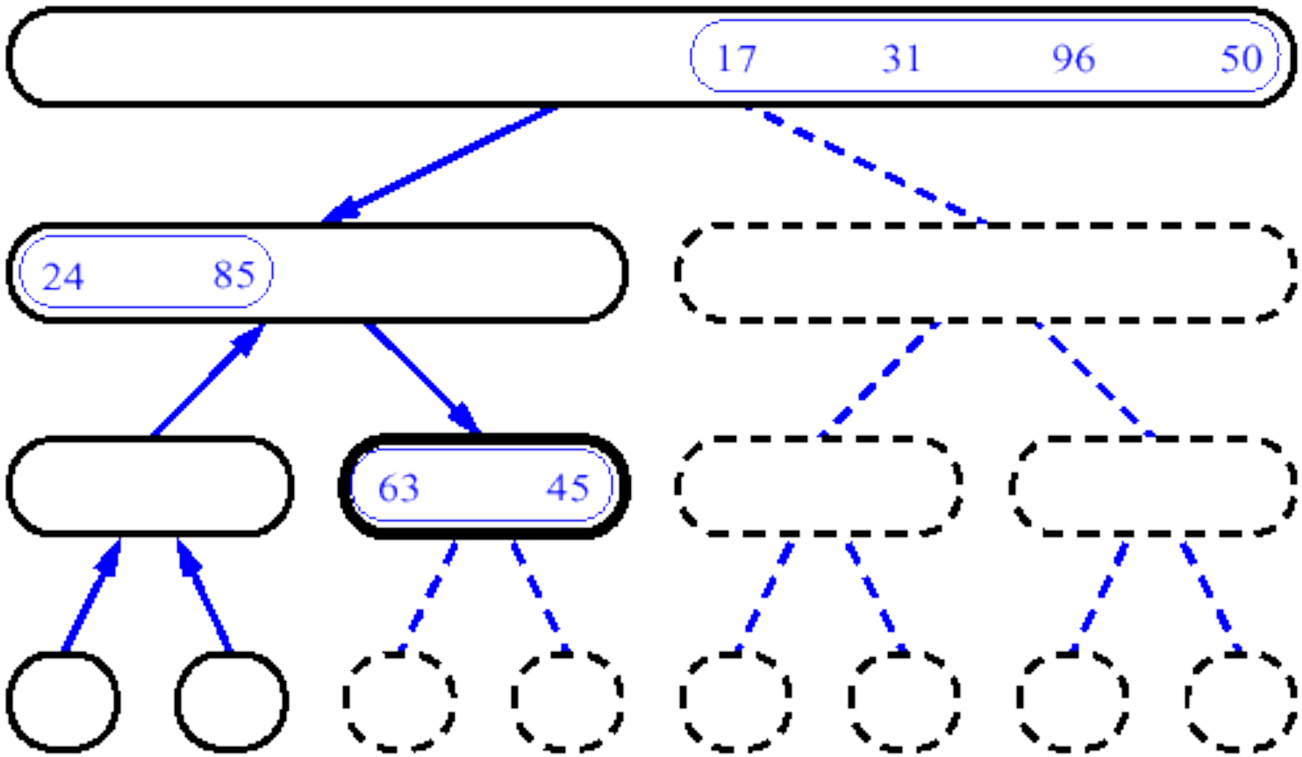


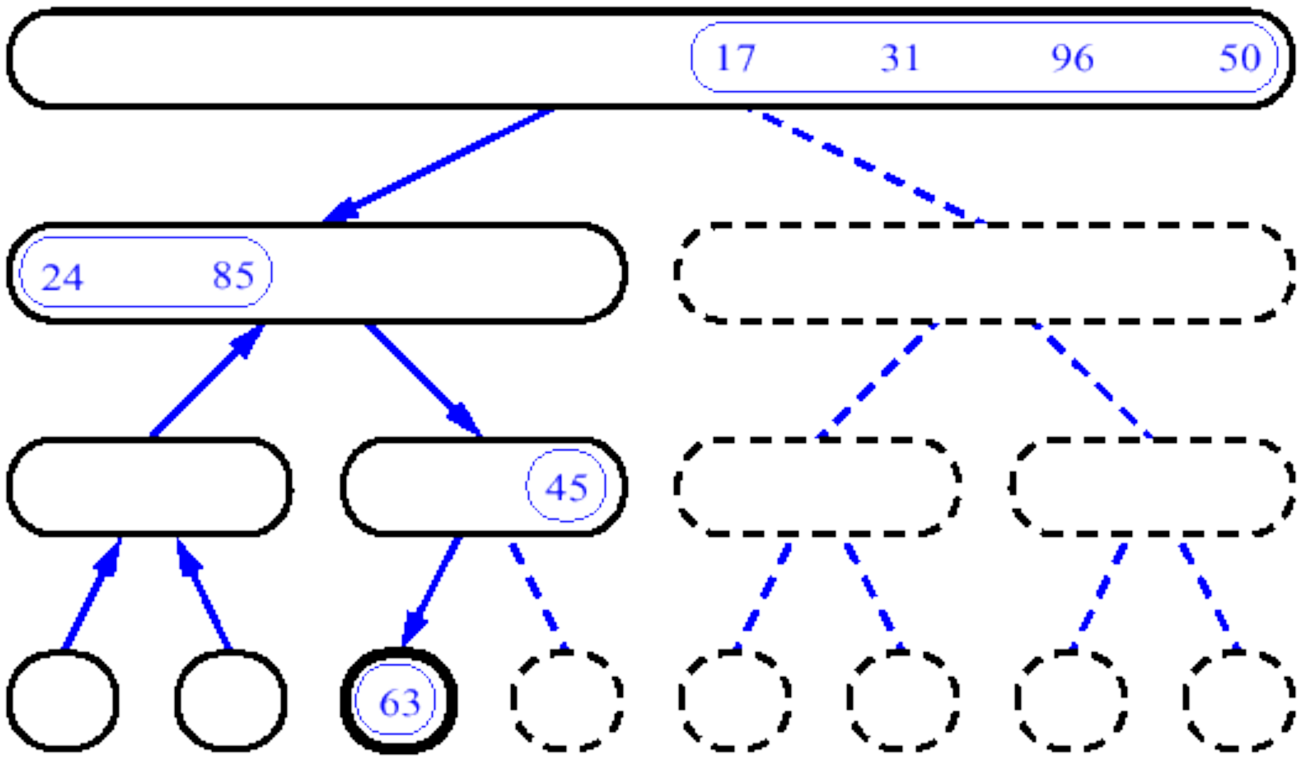
Merge sort (Example)



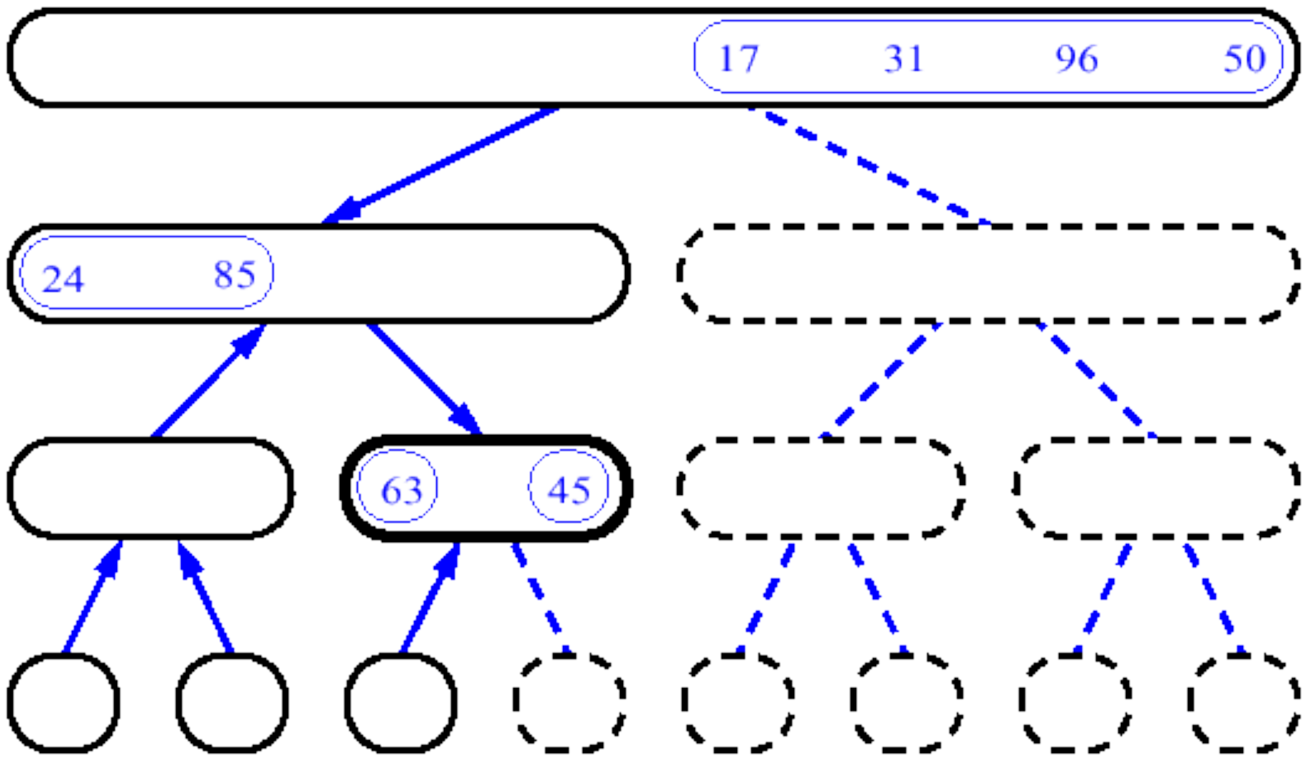
Merge sort (Example)



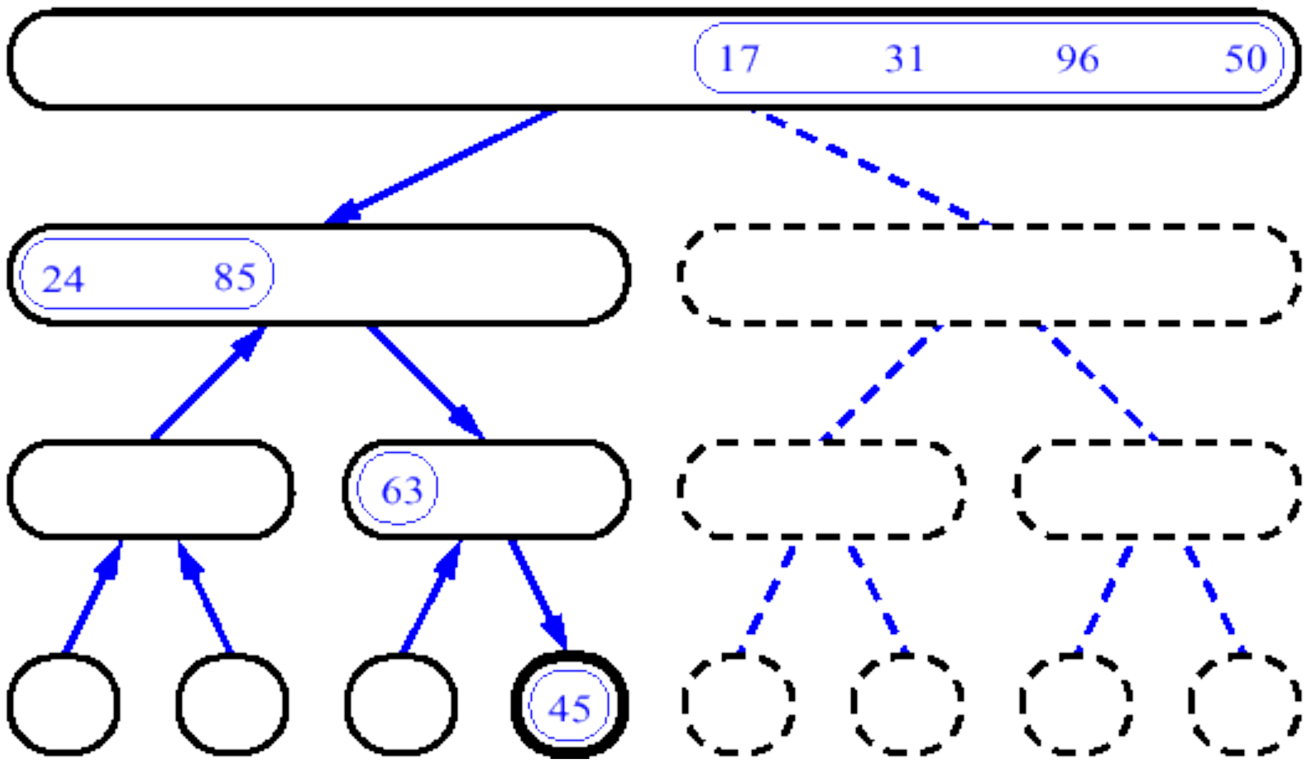
Merge sort (Example)



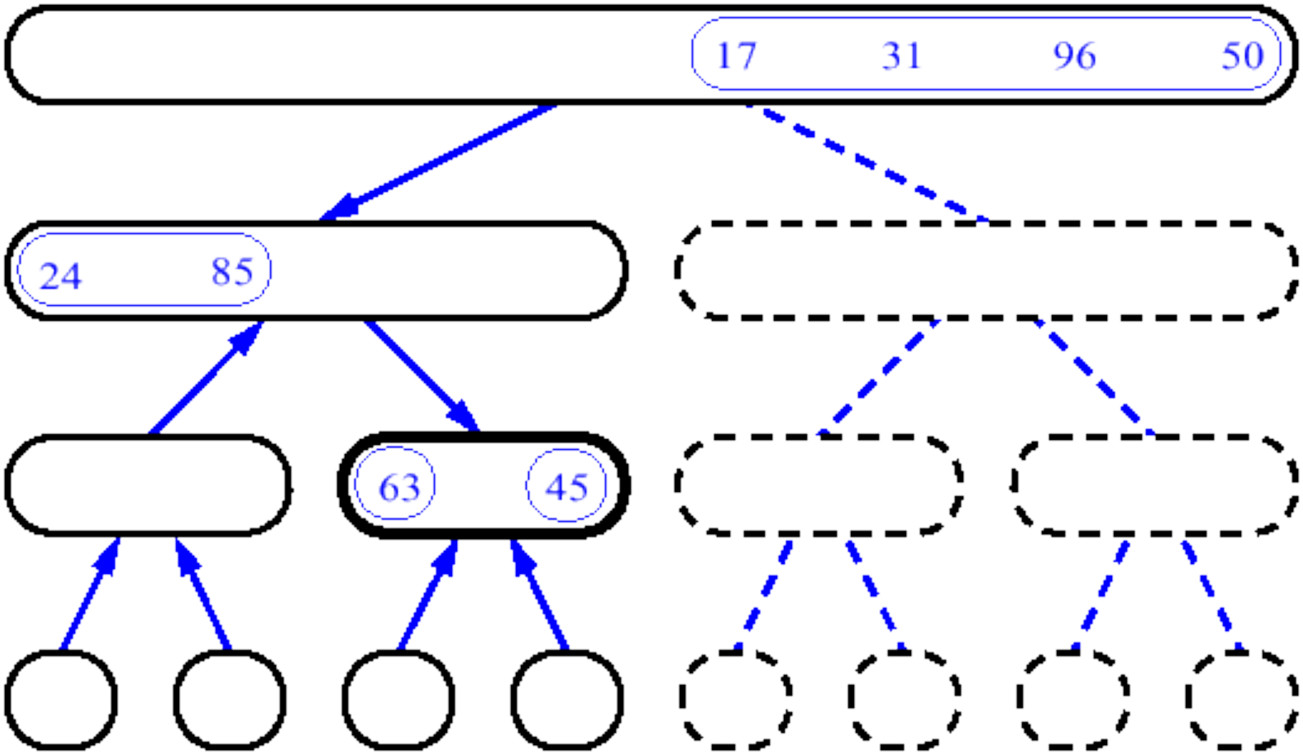
Merge sort (Example)



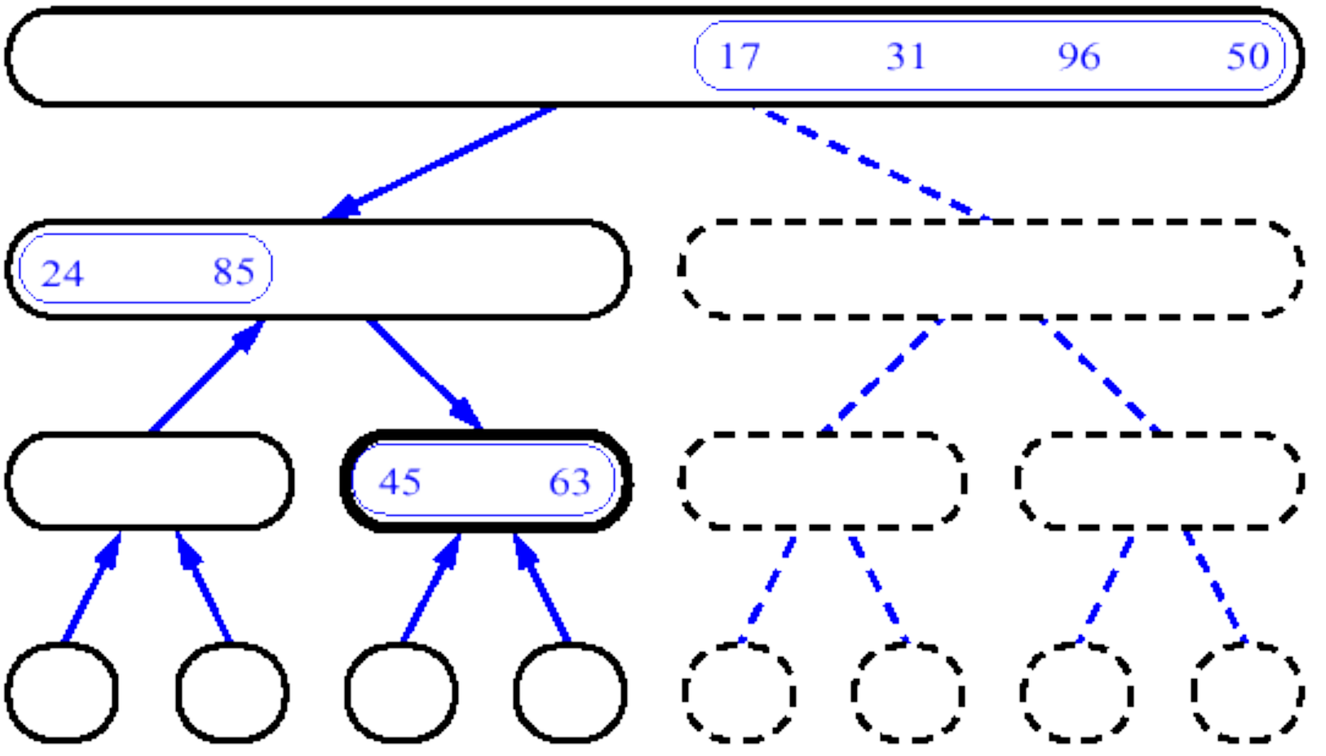
Merge sort (Example)



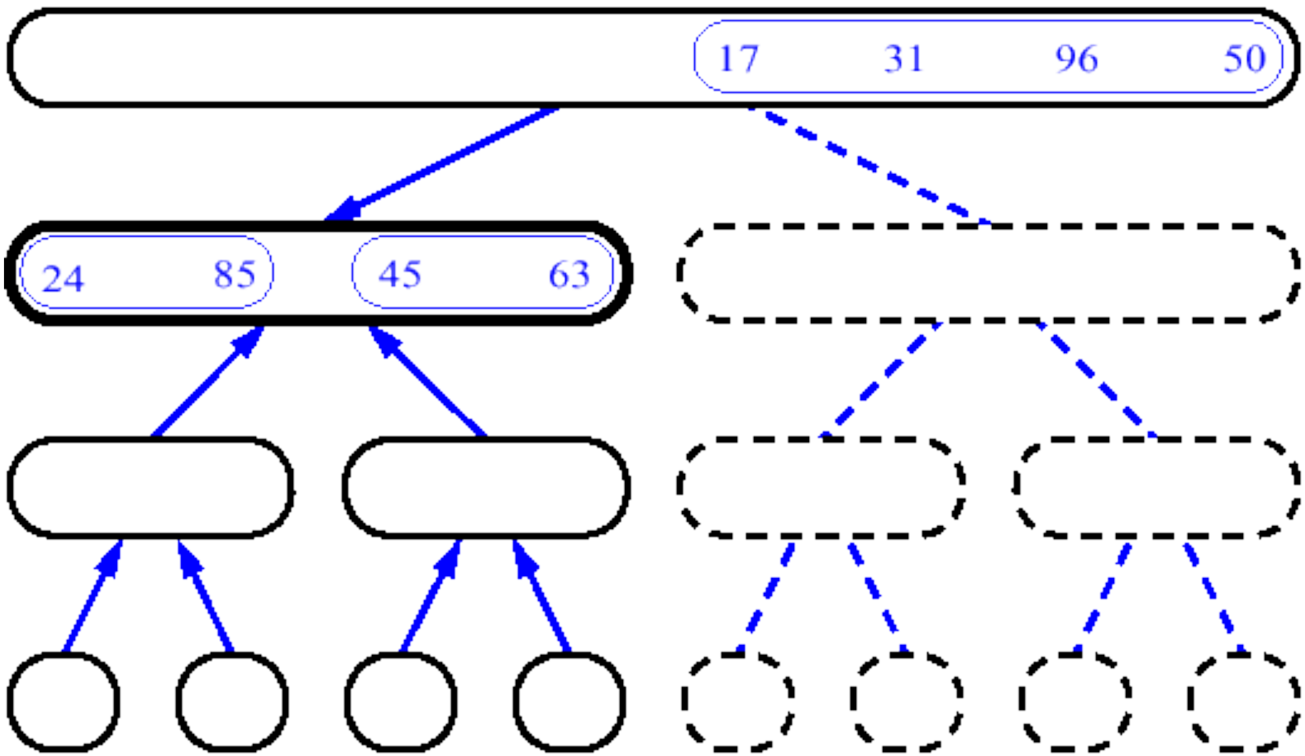
Merge sort (Example)



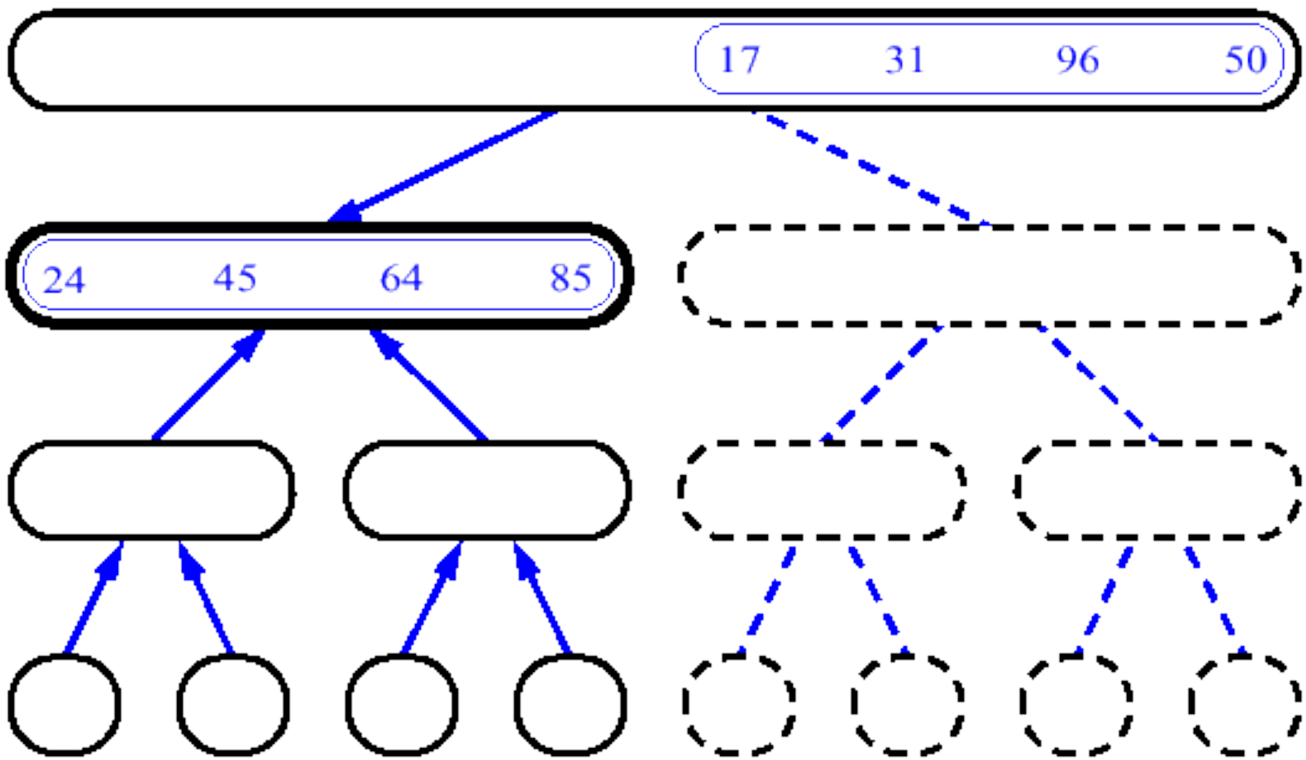
Merge sort (Example)



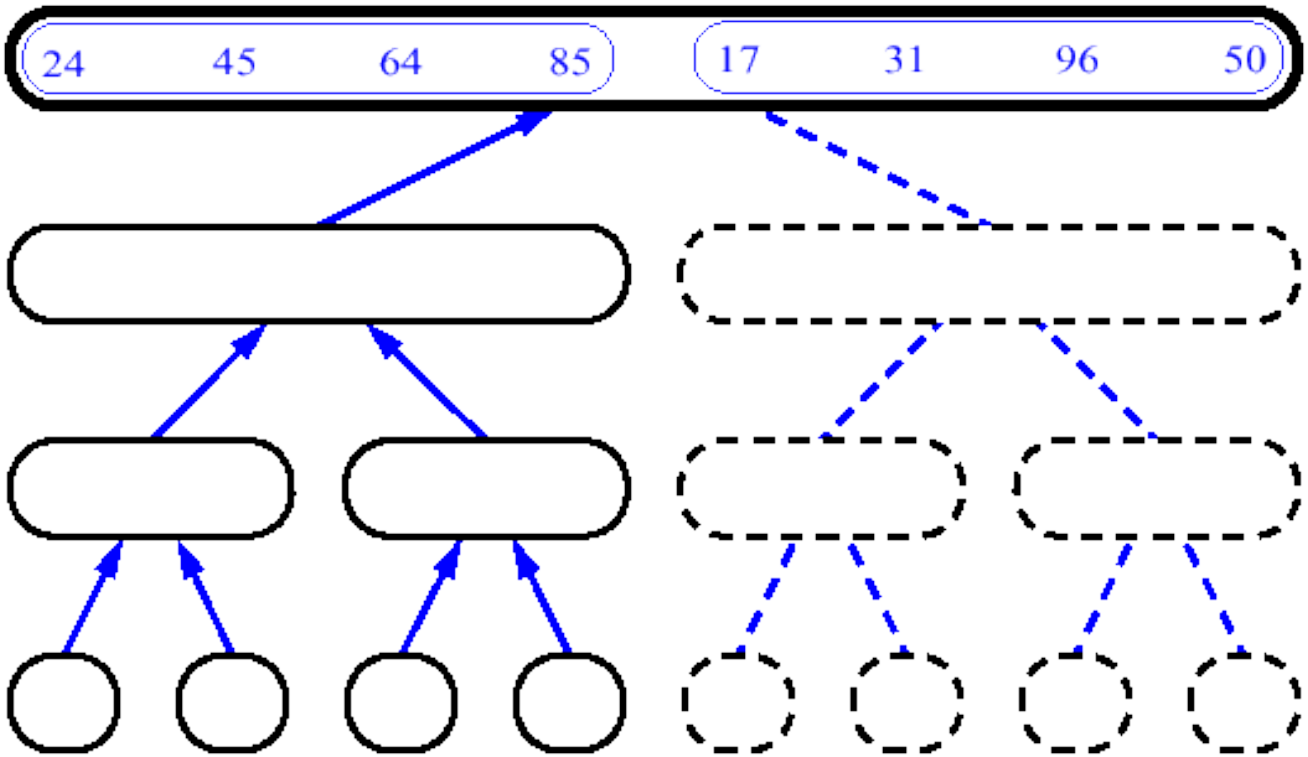
Merge sort (Example)



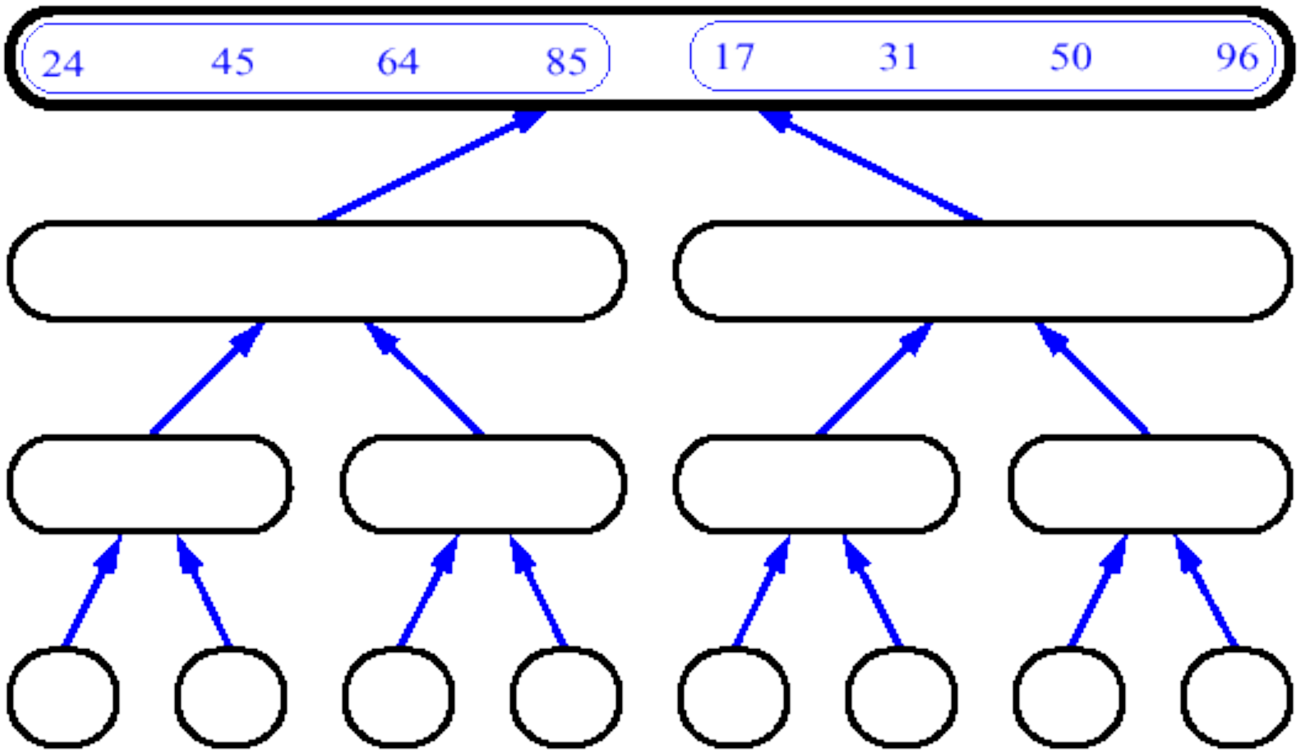
Merge sort (Example)



Merge sort (Example)



Merge sort (Example)



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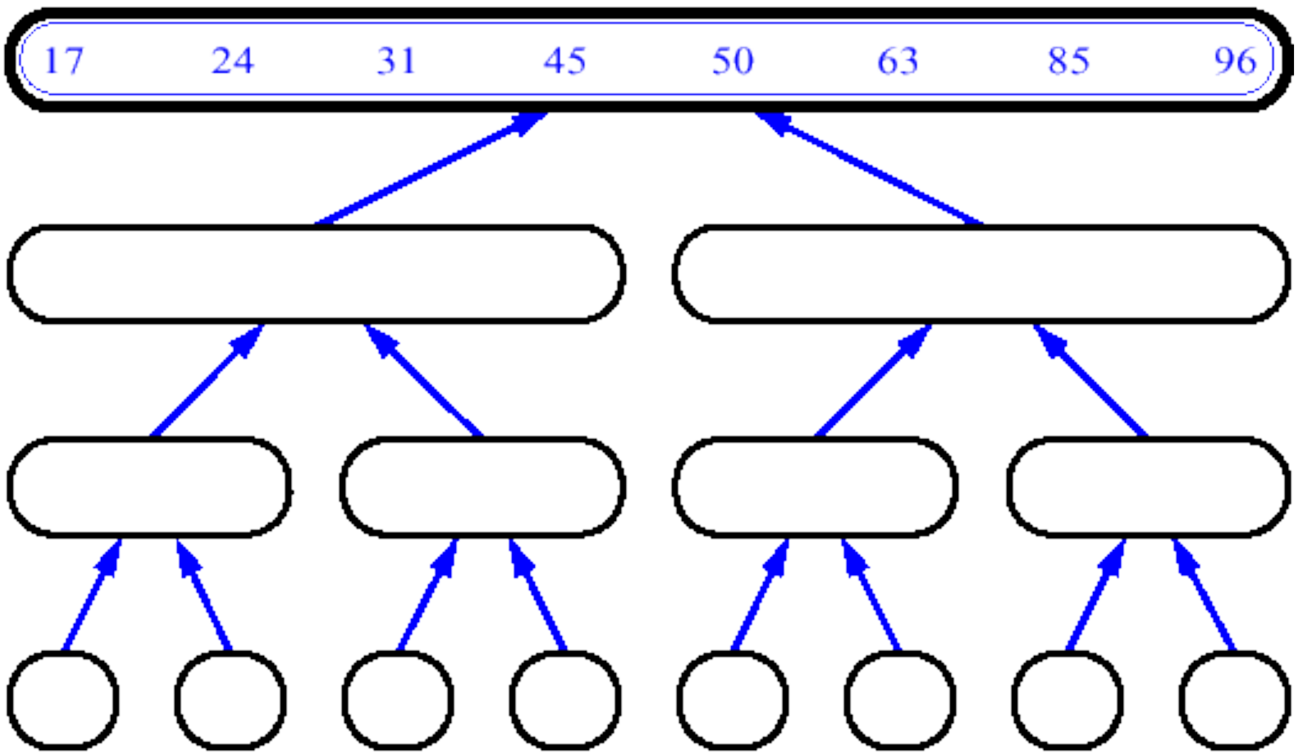


Analysis of merge sort

$T(n)$	**MERGE-SORT** $A[1 \,.\,.\, n]$
$\Theta(1)$	1. If $n = 1$, done.
$2T(n/2)$	2. Recursively sort $A[\,1 \,.\,.\, \lceil n/2 \rceil\,]$ and $A[\,\lceil n/2 \rceil + 1 \,.\,.\, n\,]$.
$\Theta(n)$	***3.*** ***"Merge"*** the 2 sorted lists

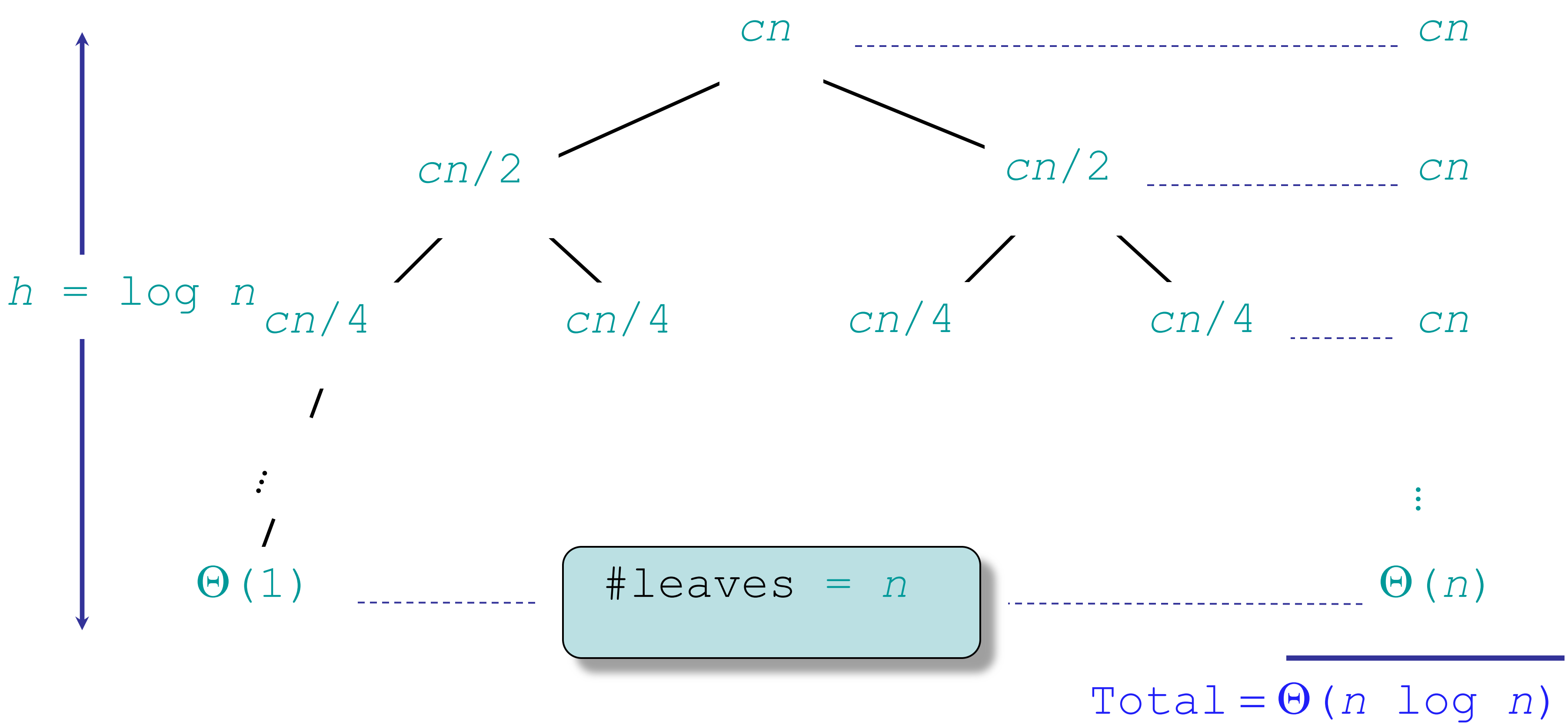
Analyzing merge sort

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1; \\ 2T(n/2) + \Theta(n) & \text{if } n > 1. \end{cases}$$

$$T(n) = \Theta(n \lg n) \qquad (n > 1)$$

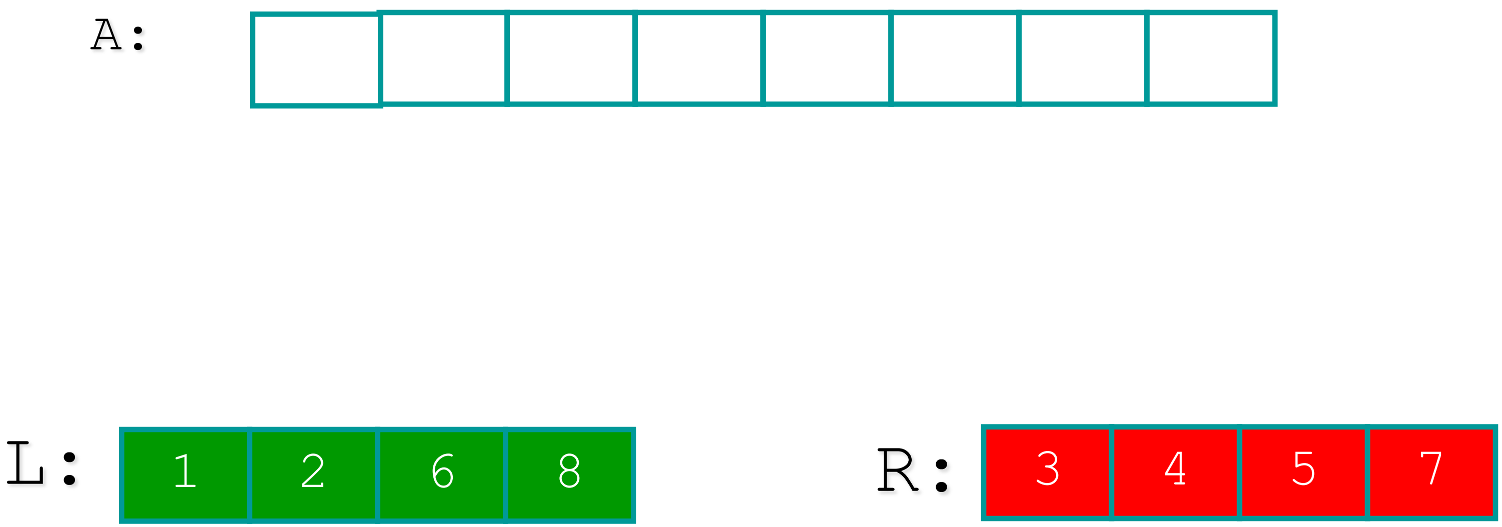
Recursion tree

Solve $T(n) = 2T(n/2) + cn$, where $c > 0$ is constant.



Memory Requirement

Needs additional *n* locations because it is difficult to merge two sorted sets in place



Merge Sort Conclusion

- Merge Sort: $O(n \log n)$
  - asymptotically beats insertion sort in the worst case
  - In practice, merge sort beats insertion sort for $n > 30$ or so
- Space requirement:
  - $O(n)$, not in-place

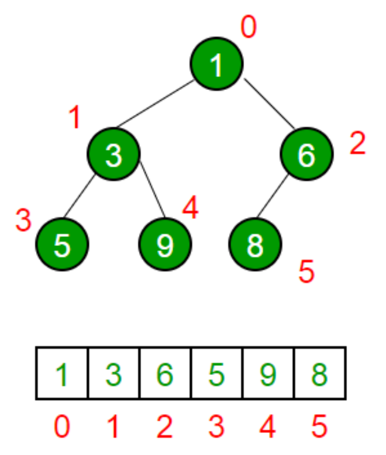
HEAPSORT

Heapsort

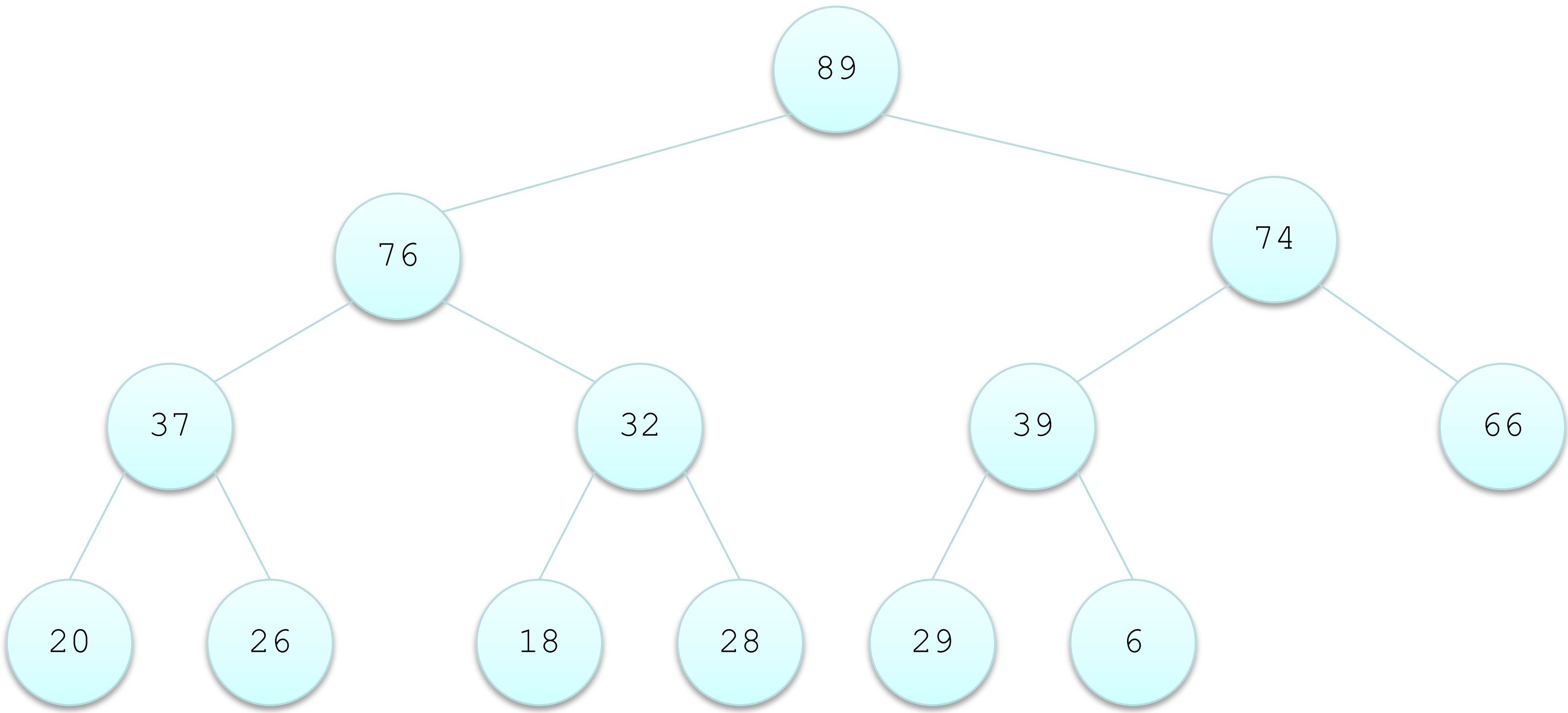
- Merge sort time is O(*n* log *n*) but still requires, temporarily, *n* extra storage locations
- *Heapsort* does not require any additional storage
- As its name implies, heapsort uses a heap to store the array

Heapsort Algorithm

1. Insert each value from the array to be sorted into a priority queue (min-heap).
2. Swap the first element of the list with the final element. Decrease the considered range of the list by one.
3. Call the sink() function on the list to sink the new first element to its appropriate index in the heap.
4. Go to step (2) unless the considered range of the list is one element.

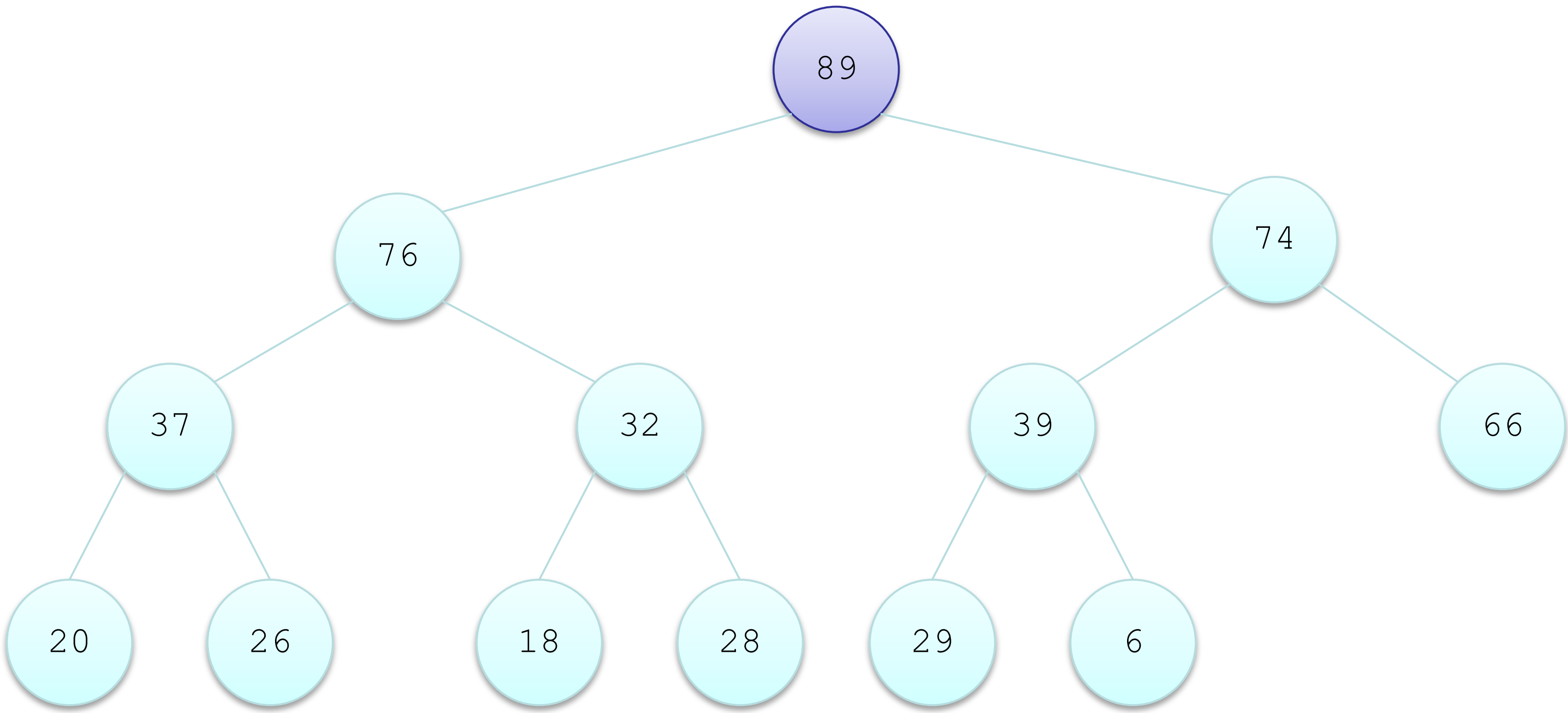


Trace of Heapsort

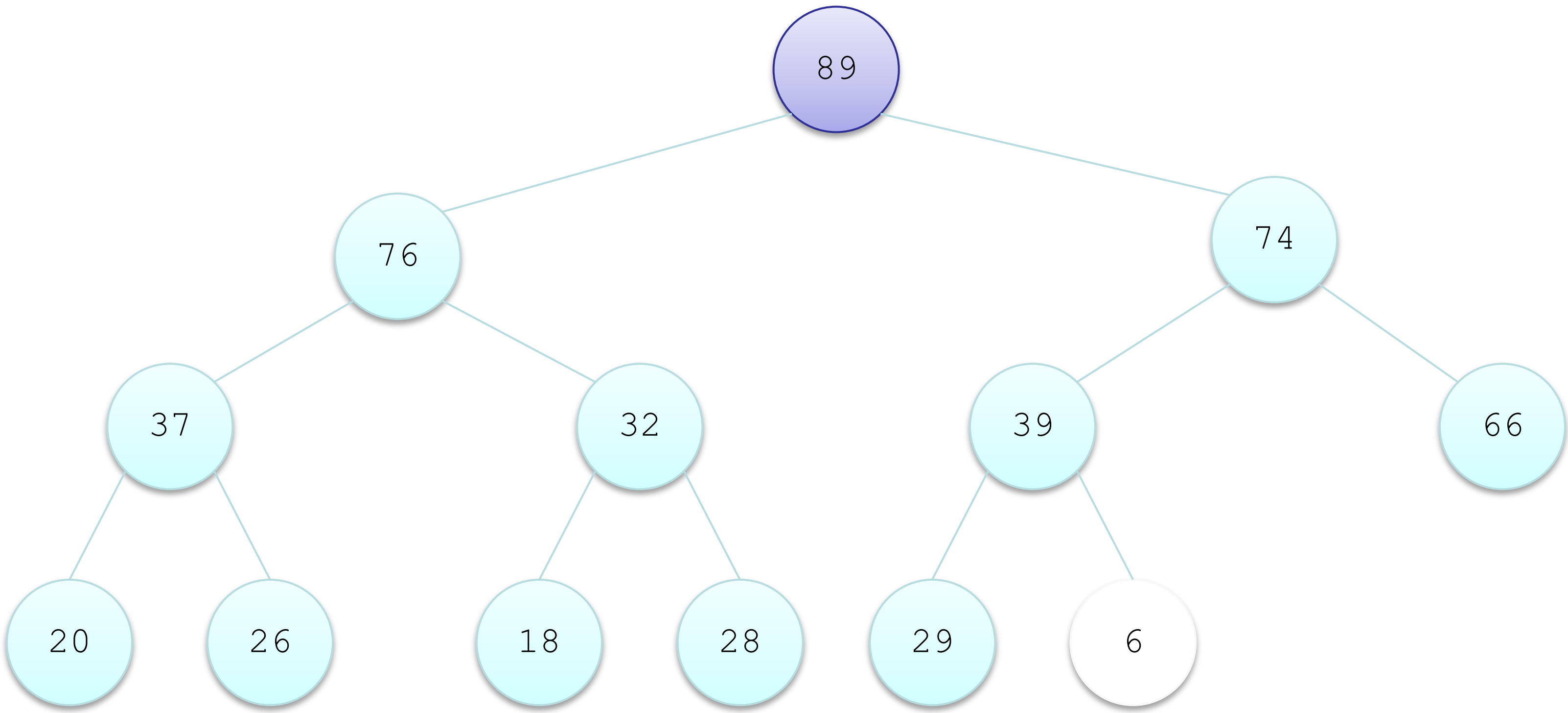


[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
89	76	74	37	32	39	66	20	26	18	28	29	6

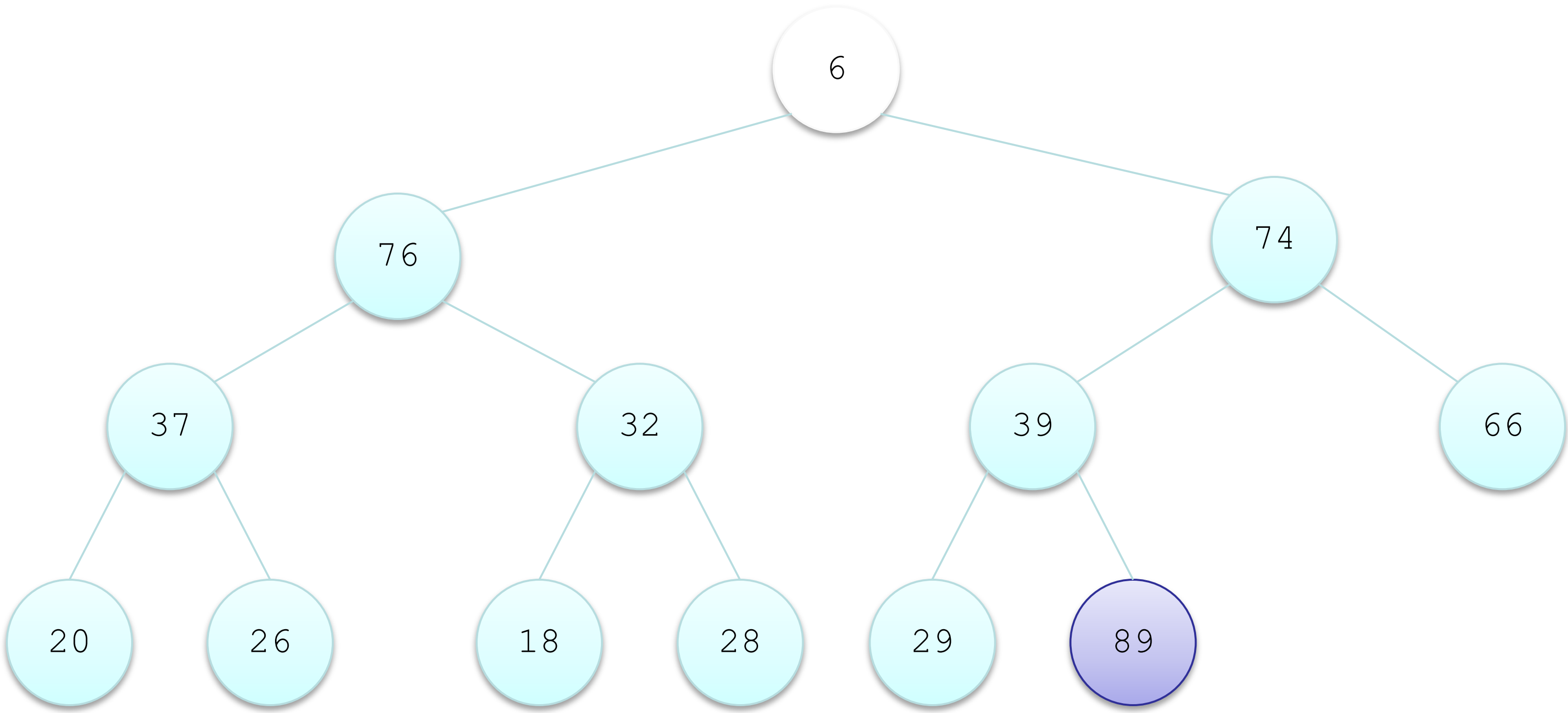
Trace of Heapsort (cont.)



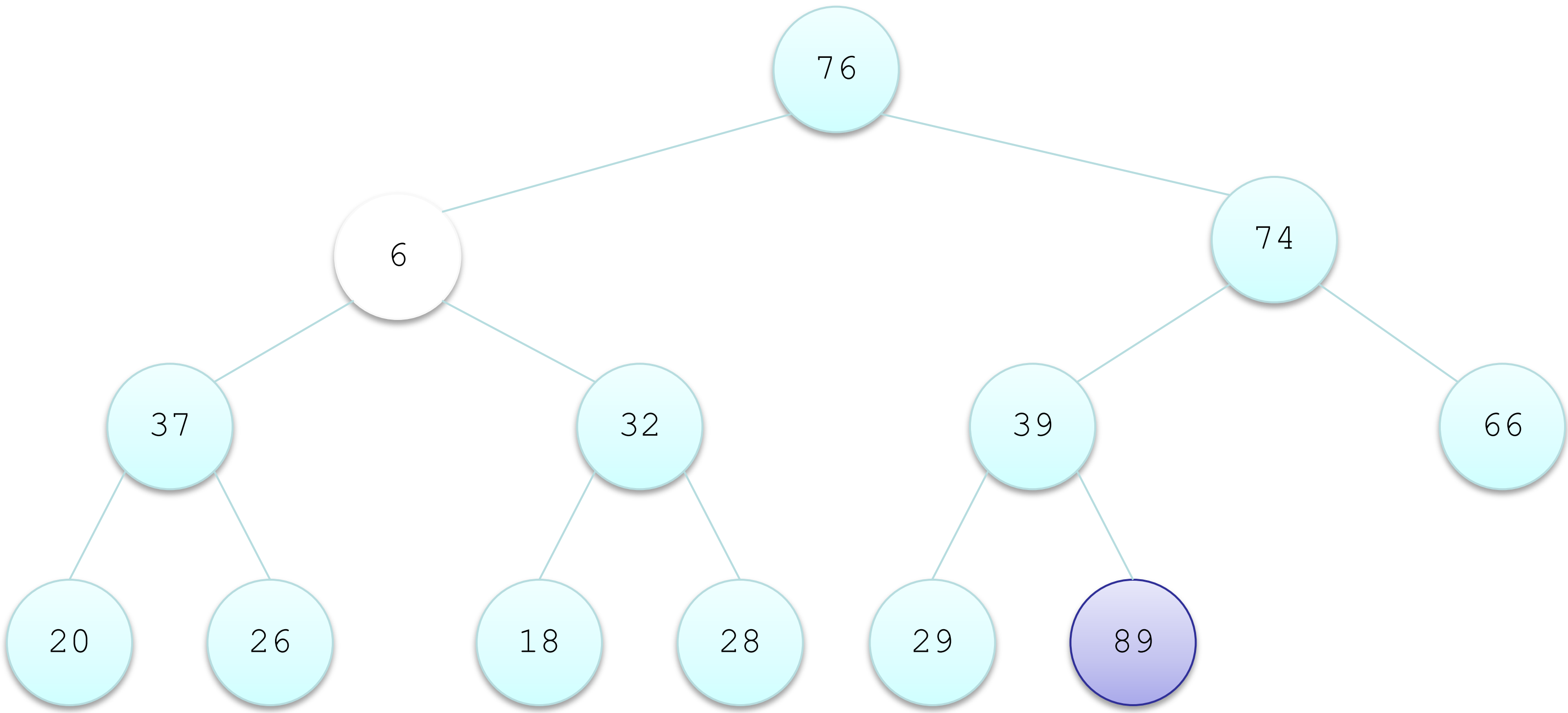
Trace of Heapsort (cont.)



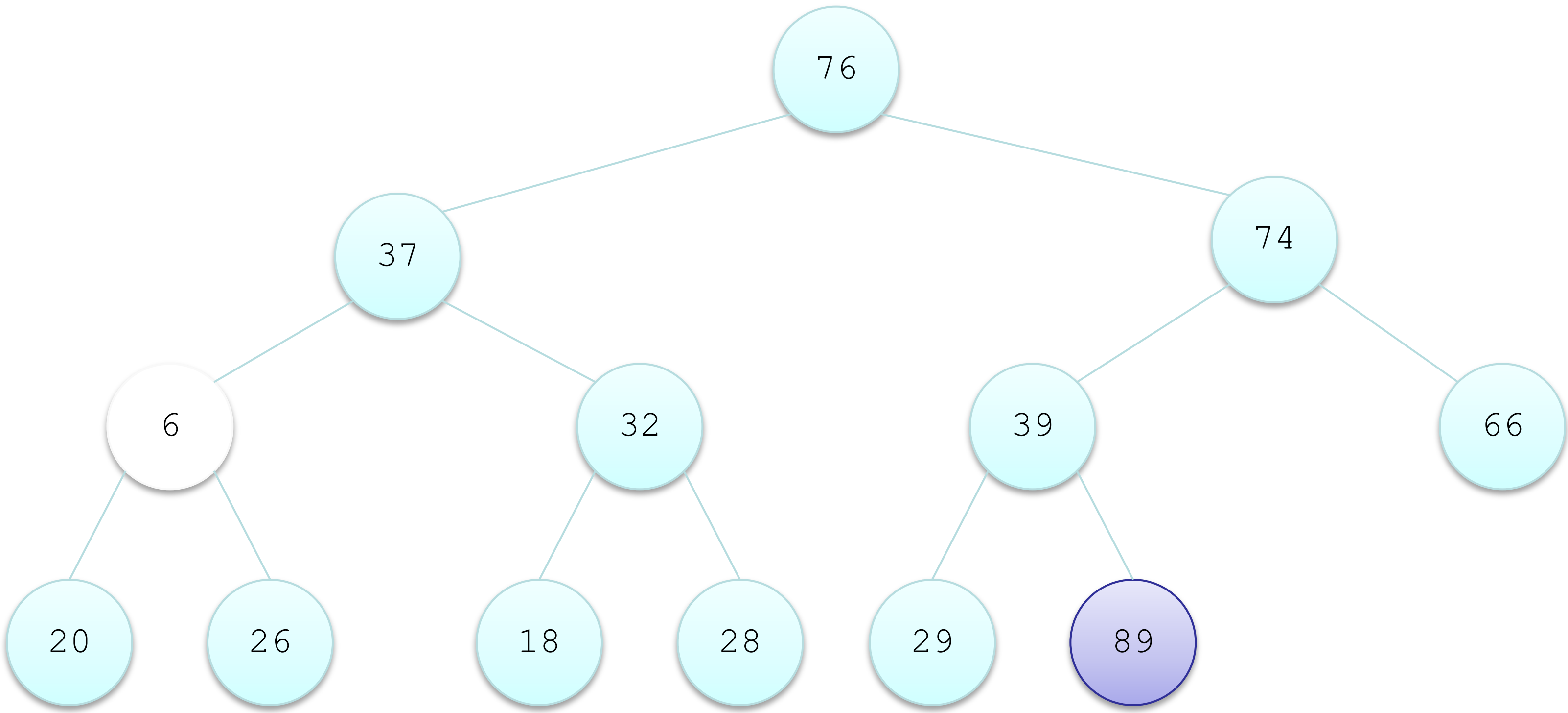
Trace of Heapsort (cont.)



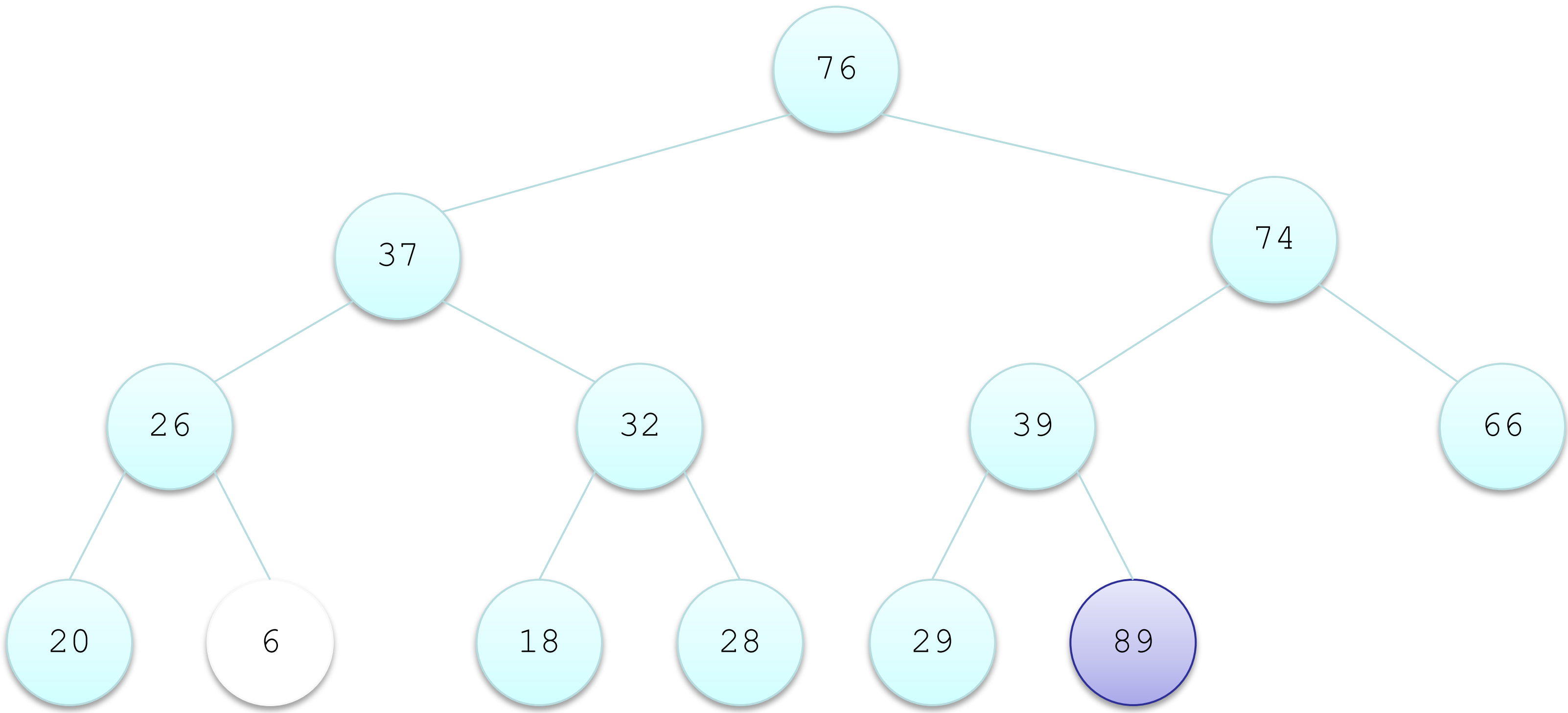
Trace of Heapsort (cont.)



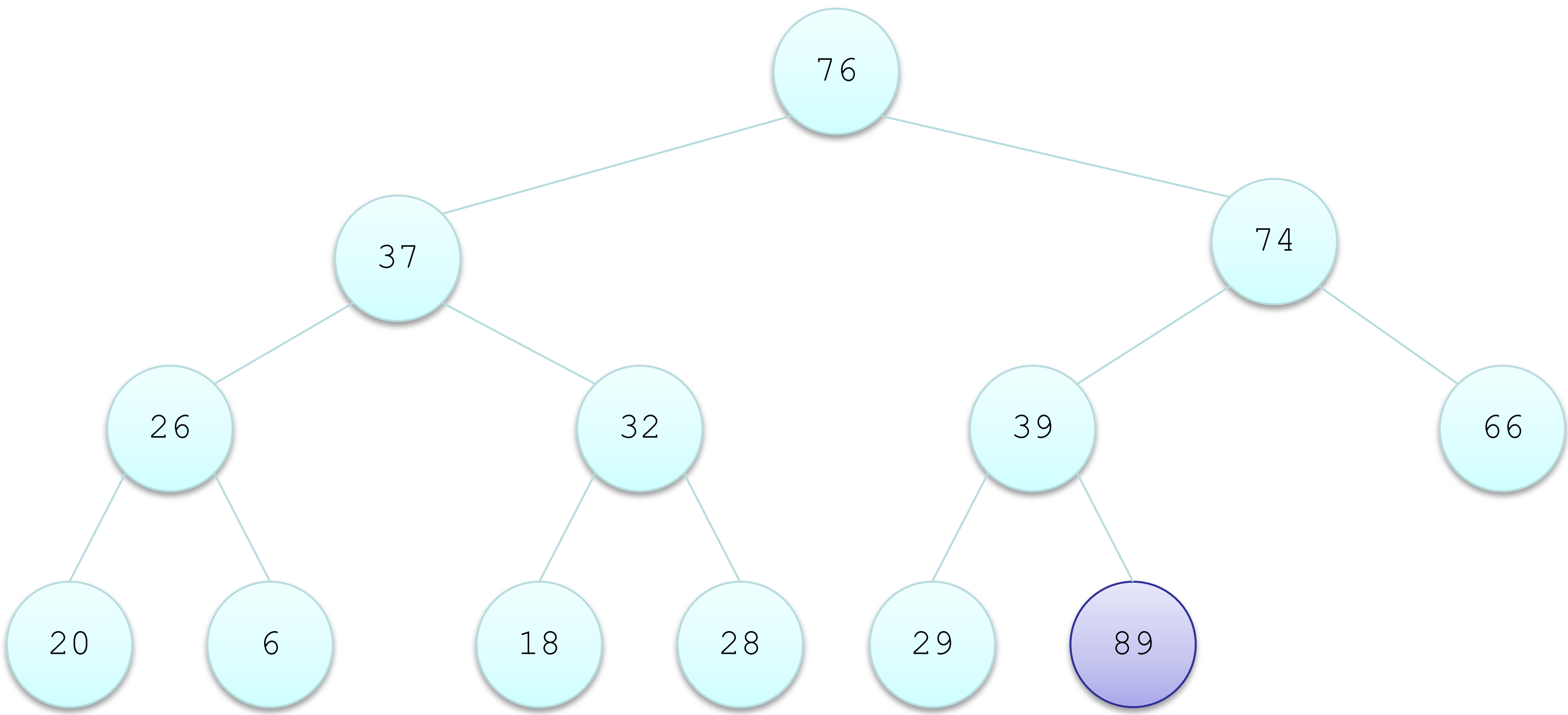
Trace of Heapsort (cont.)



Trace of Heapsort (cont.)



Trace of Heapsort (cont.)



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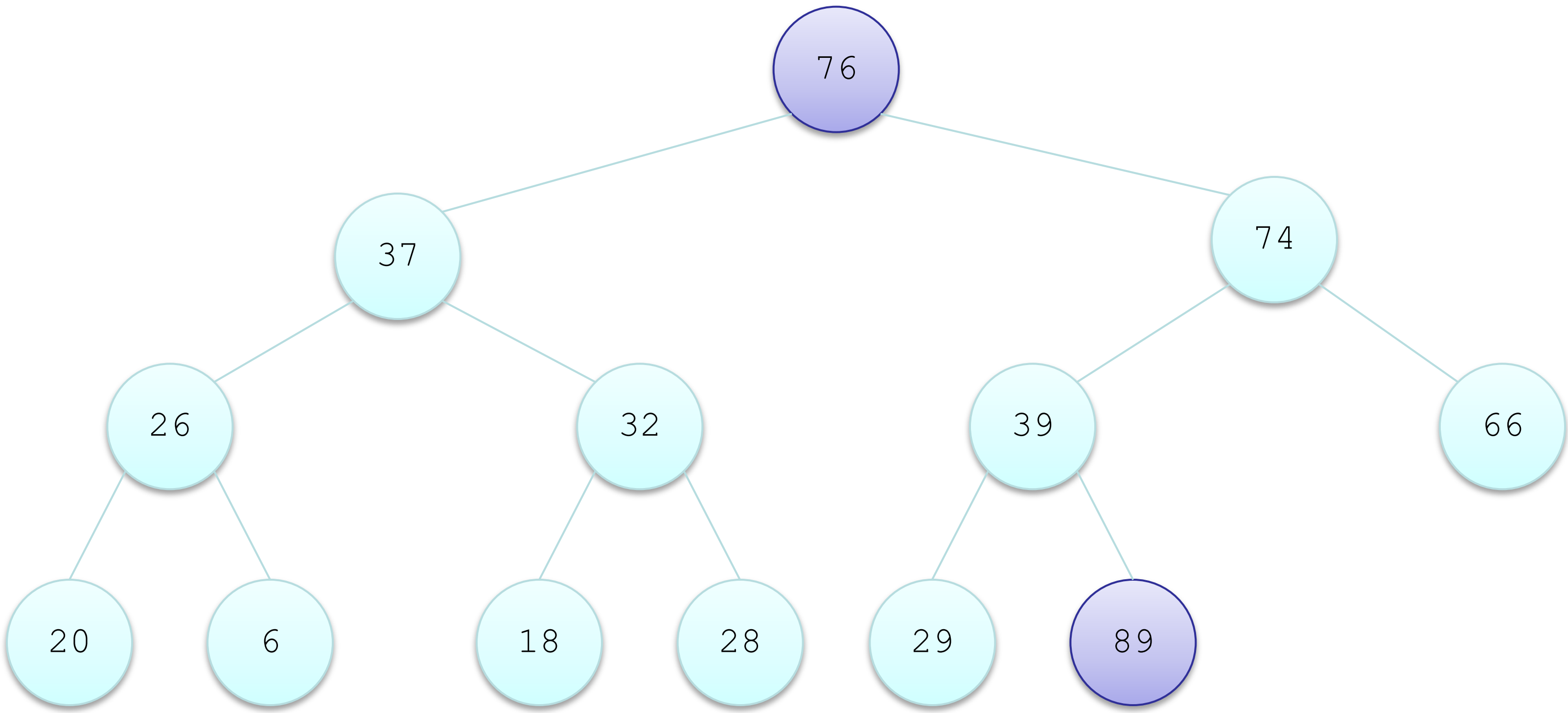
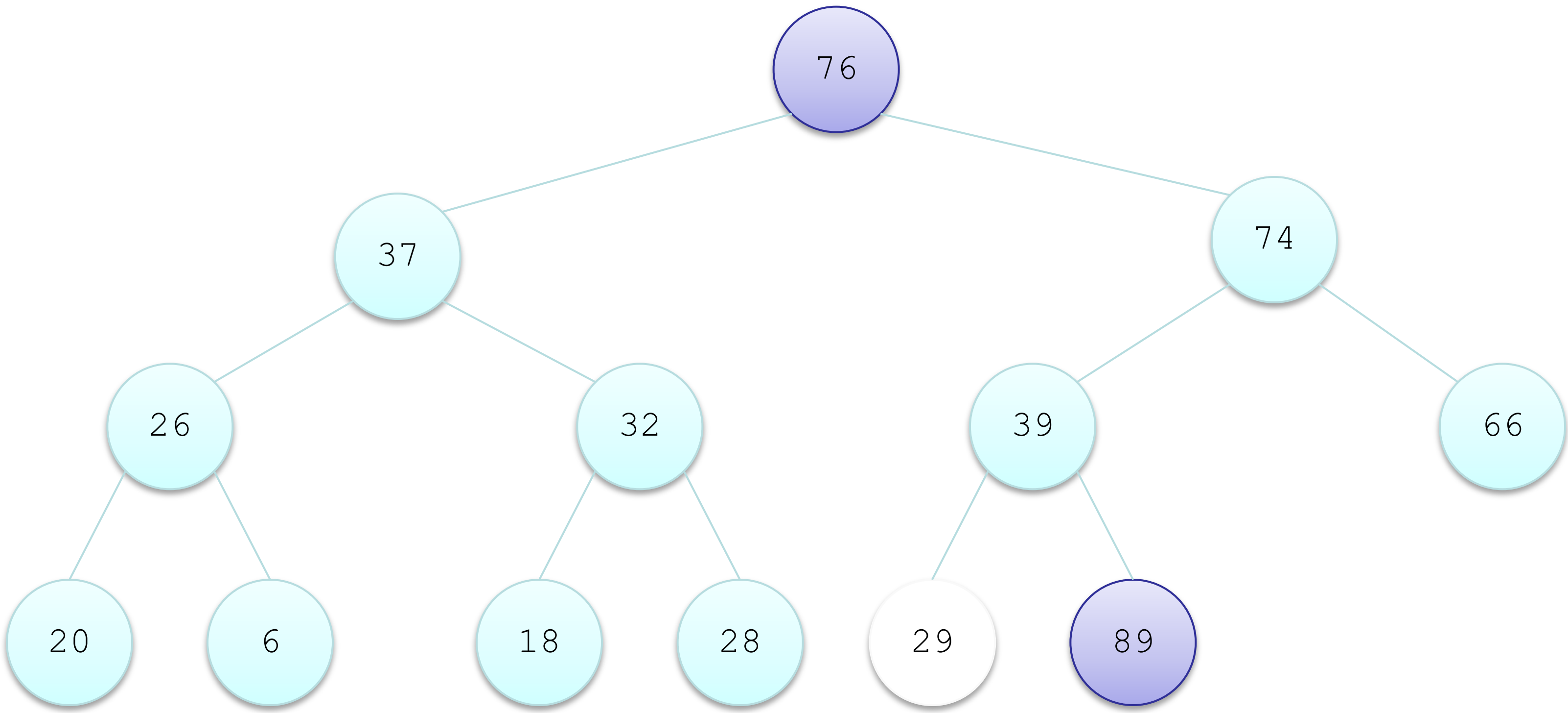
76	37	74	26	32	39	66	20	6	18	28	29	89

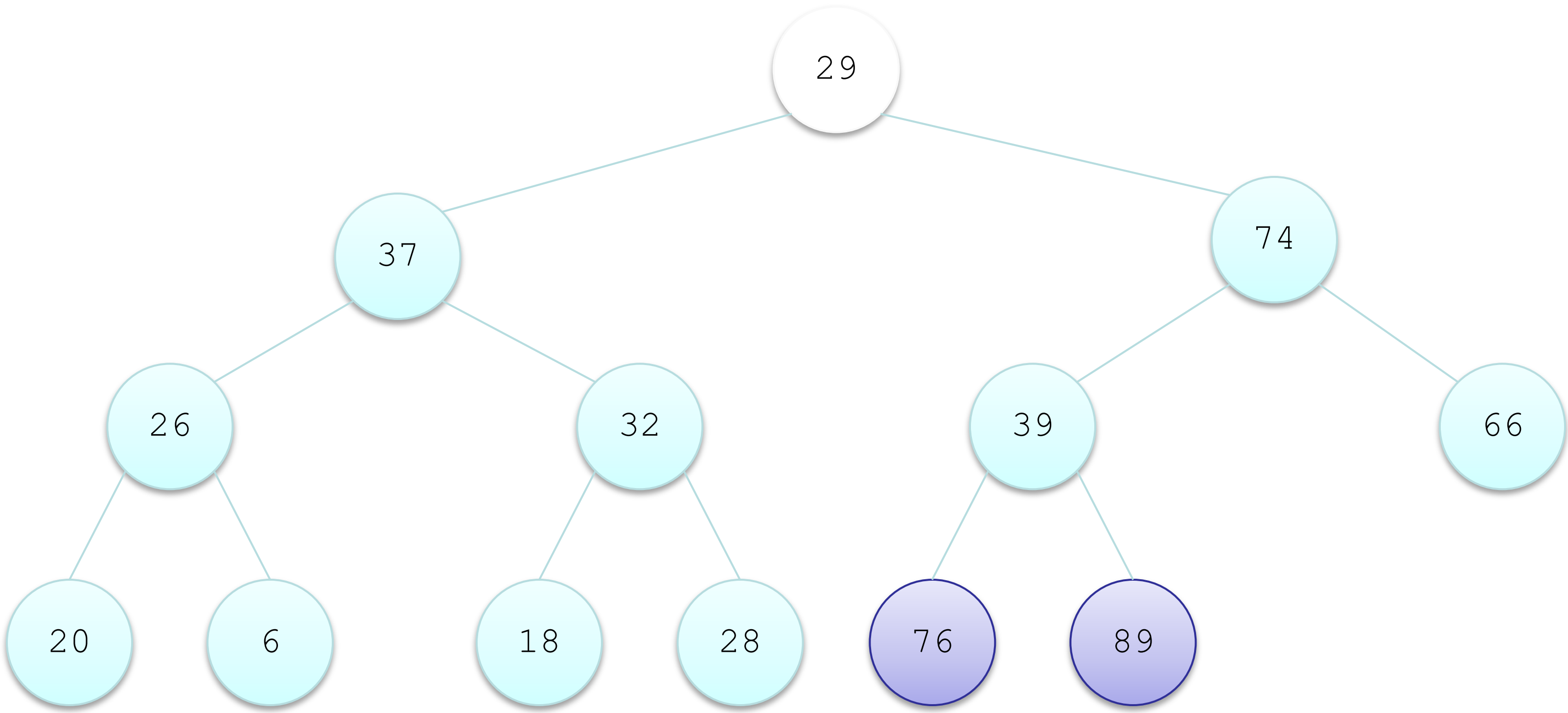
Trace of Heapsort (cont.)



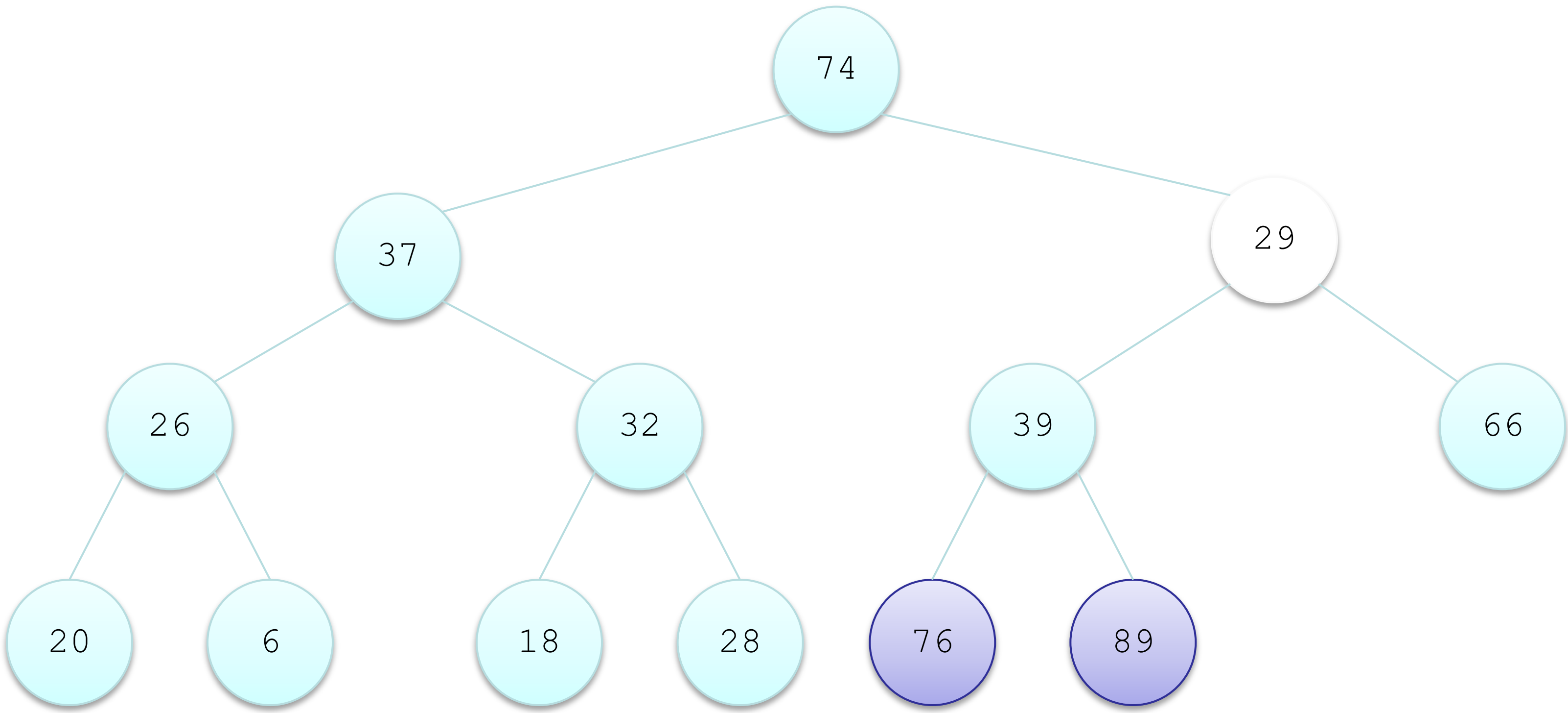
Trace of Heapsort (cont.)



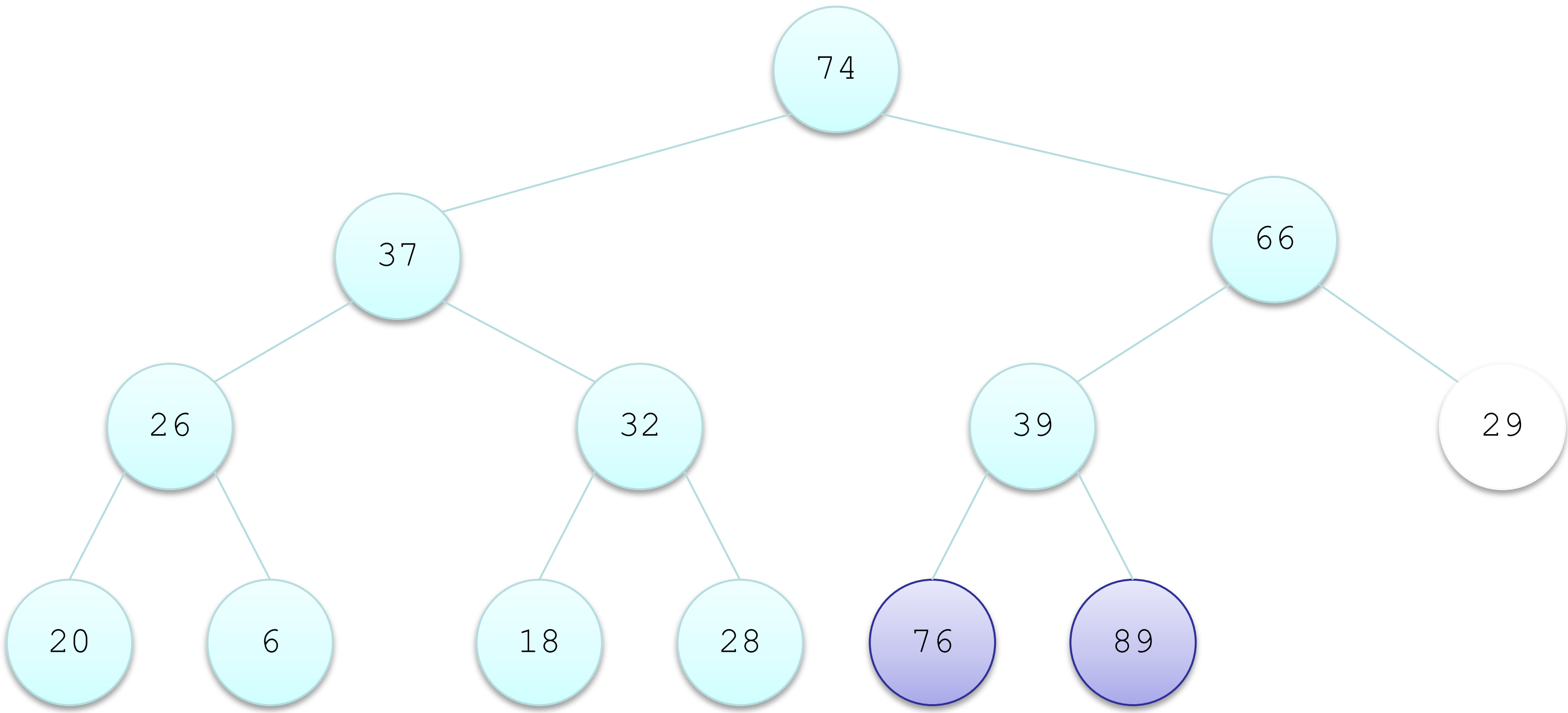
Trace of Heapsort (cont.)



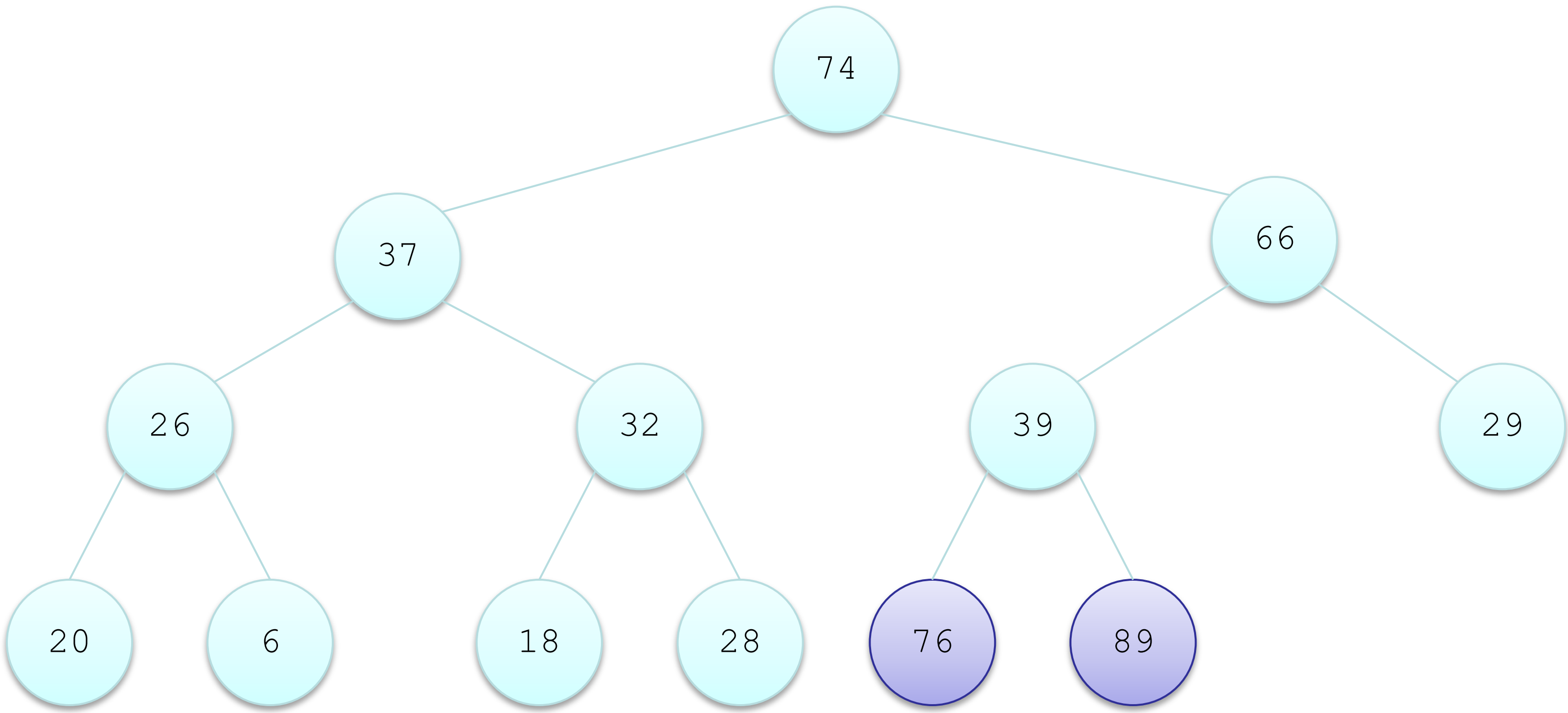
Trace of Heapsort (cont.)



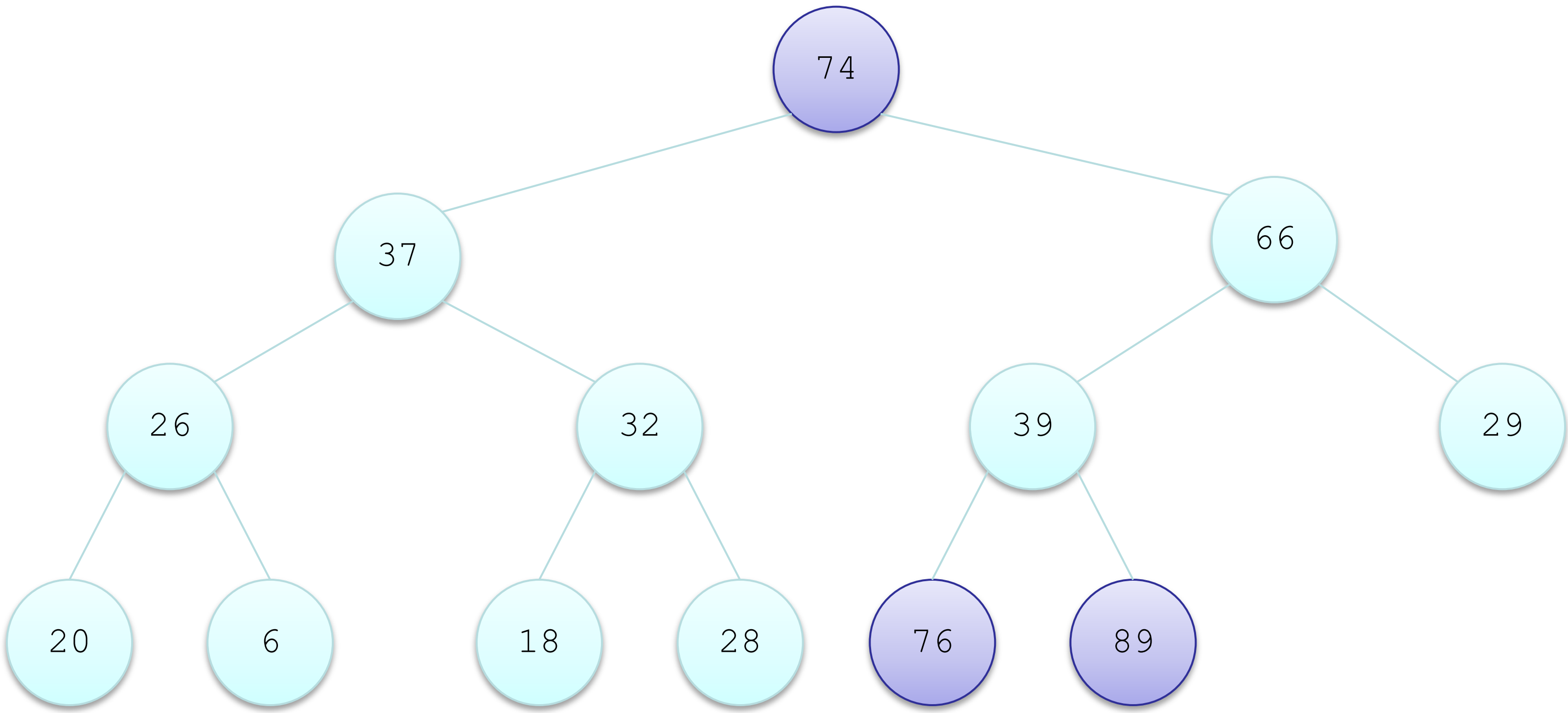
Trace of Heapsort (cont.)



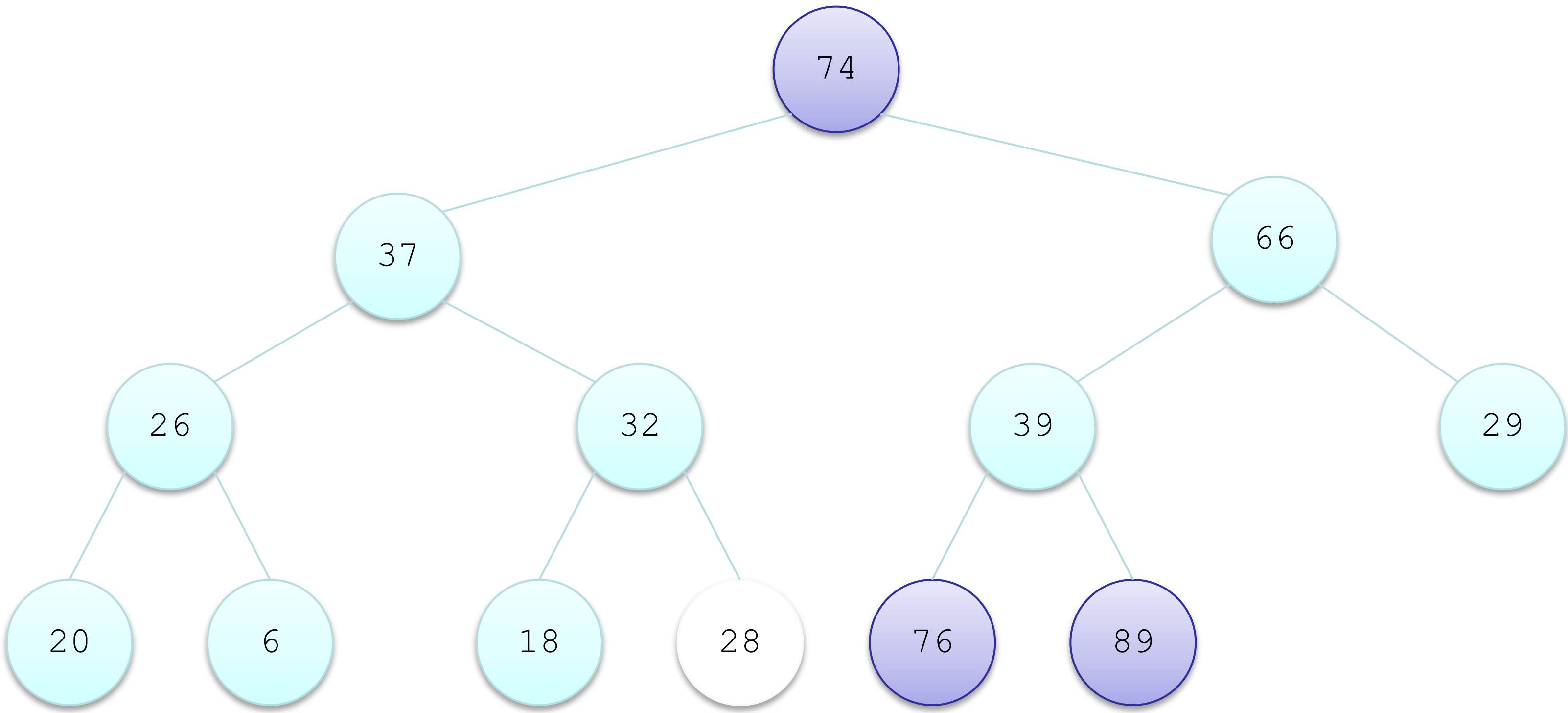
Trace of Heapsort (cont.)



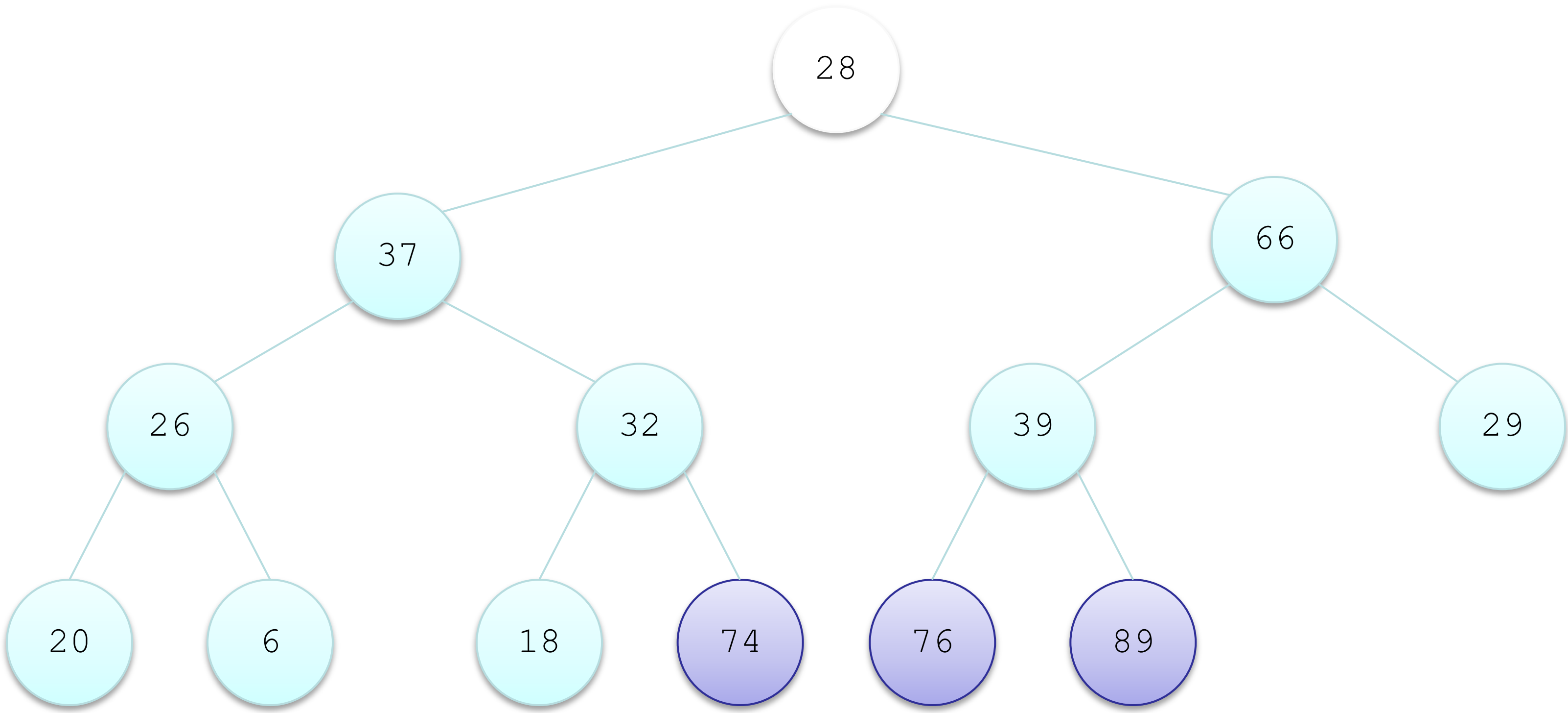
Trace of Heapsort (cont.)



Trace of Heapsort (cont.)

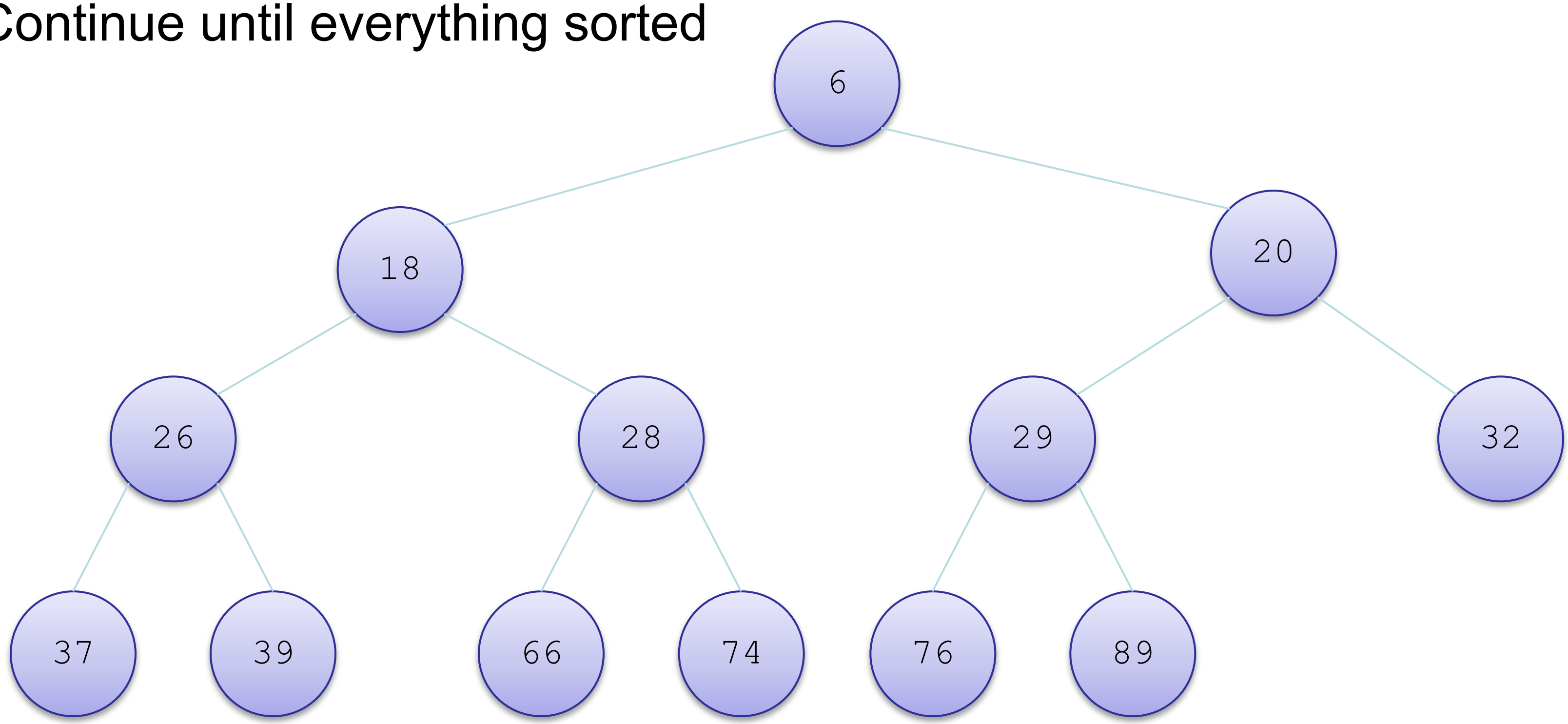


Trace of Heapsort (cont.)



Trace of Heapsort (cont.)

Continue until everything sorted



Revising the Heapsort Algorithm

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
89	76	74	37	32	39	66	20	26	18	28	29	6

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
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[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
74	37	66	26	32	39	29	20	6	18	28	76	89

⋮

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
6	18	20	26	28	29	32	37	39	66	74	76	89

- Each element removed will be placed at the end of the array.
- The heap part of the array decreases by one element

Analysis of Heapsort

- Because a heap is a complete binary tree, it has log *n* levels
- Building a heap of size *n* requires finding the correct location for an item in a heap with log *n* levels
- Each insert (or remove) is O(log *n*)
- With *n* items, building a heap is O(*n* log *n*)
- No extra storage is needed

Quicksort

- Developed in 1962
- Quicksort selects a specific value called a pivot and rearranges the array into two parts (called *partioning*)
  - all the elements in the left subarray are less than or equal to the pivot
  - all the elements in the right subarray are larger than the pivot
  - The pivot is placed between the two subarrays
- The process is repeated until the array is sorted

Merge sort vs Quick Sort

Merge sort

13	89	46	22	57	76	98	34	66	83

split

13	89	46	22	57

76	98	34	66	83

sort recursively

13	22	46	57	89

76	98	34	66	83

merge

13	22	34	46	57	66	76	83	89	98

Merge sort vs Quick Sort

13	89	46	22	57	76	98	34	66	83

Split (smart, extra work here)

<= 57

13	46	22	57	34

> 57

89	76	98	66	83

sort recursively

13	22	34	46	57

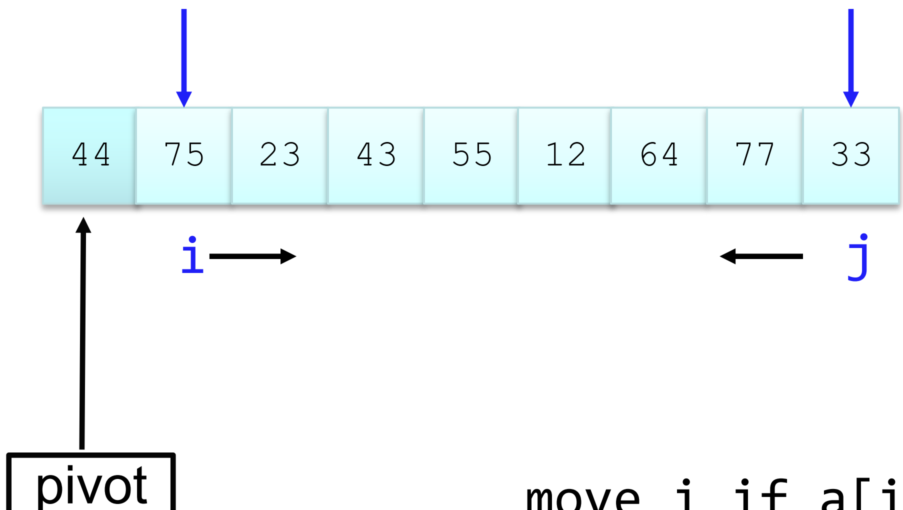
66	76	83	89	98

Merge is not necessary

Trace of Quicksort

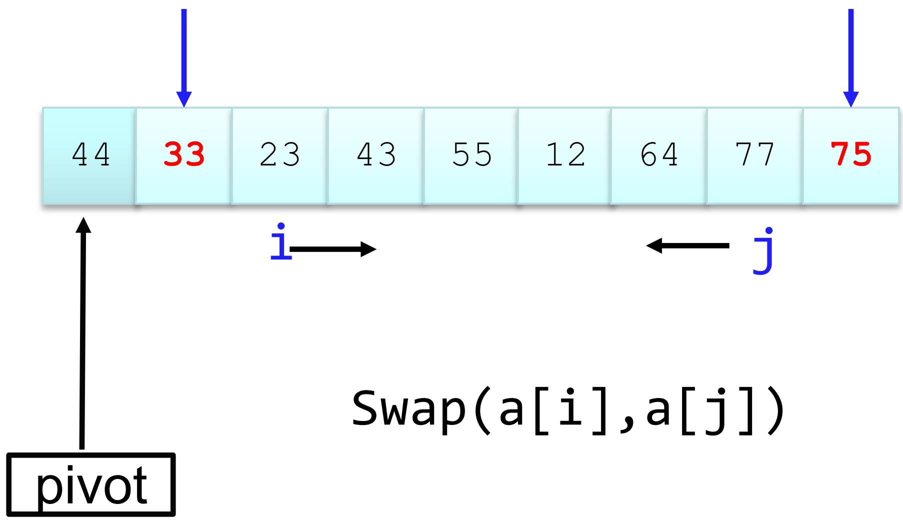
44	75	23	43	55	12	64	77	33

Trace of Quicksort (cont.)



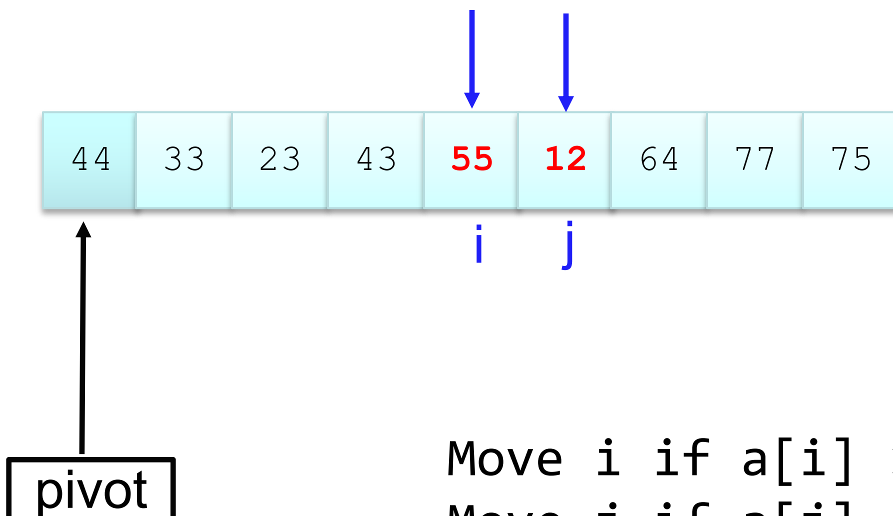

```
move i if a[i] > pivot
move j if a[j] < pivot
```

Trace of Quicksort (cont.)



Swap(a[i],a[j])

Trace of Quicksort (cont.)




```
Move i if a[i] > pivot
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```
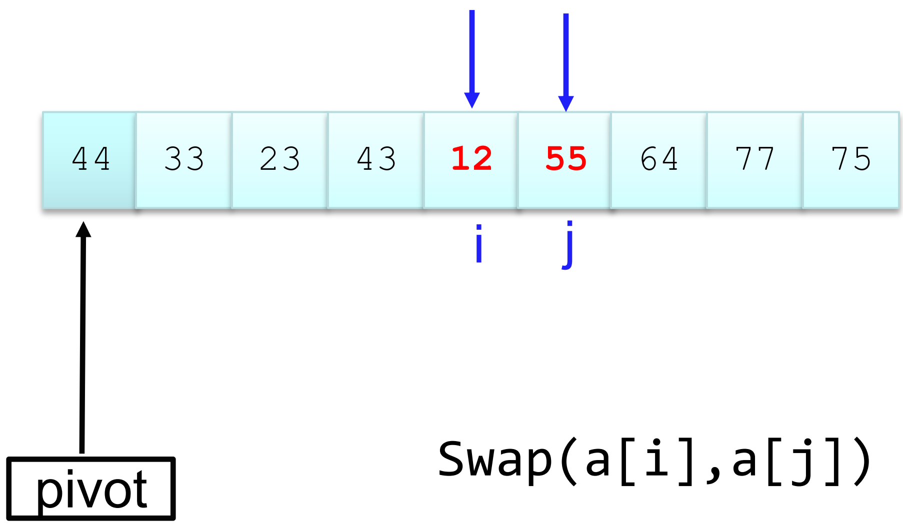
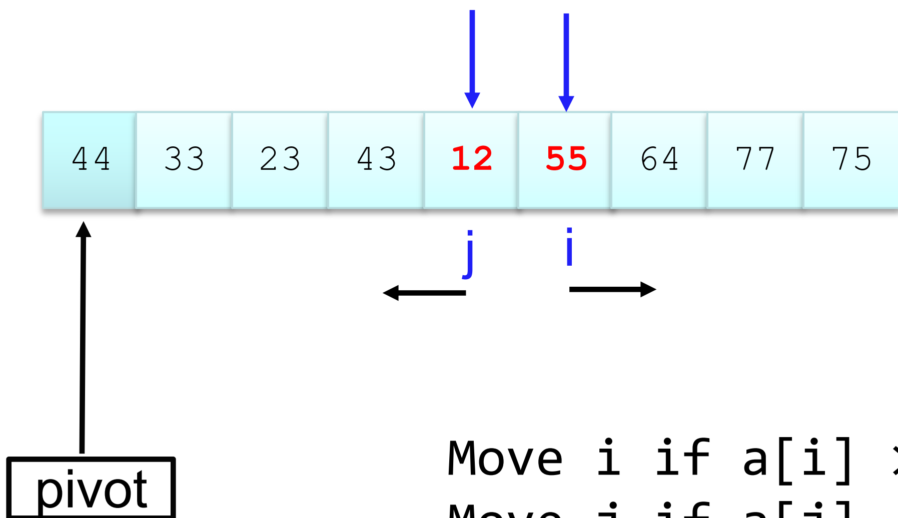

Trace of Quicksort (cont.)



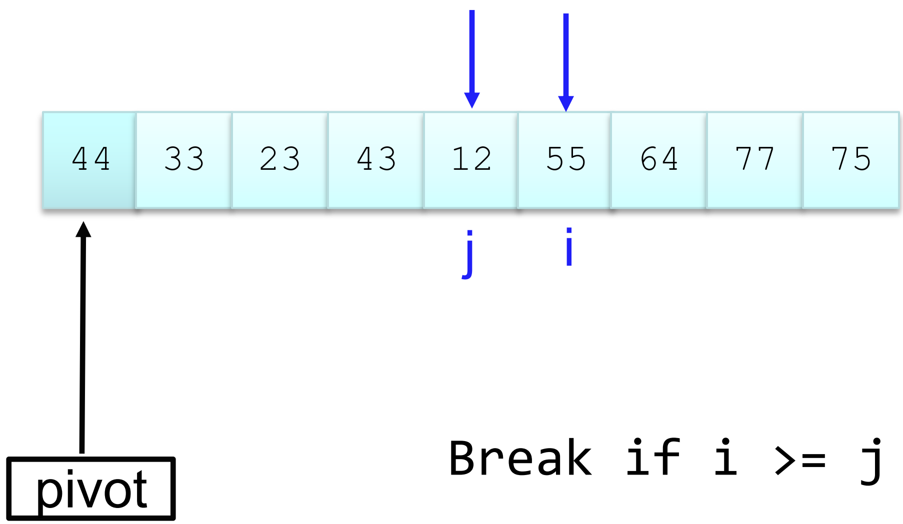

```
Swap(a[i],a[j])
```

Trace of Quicksort (cont.)



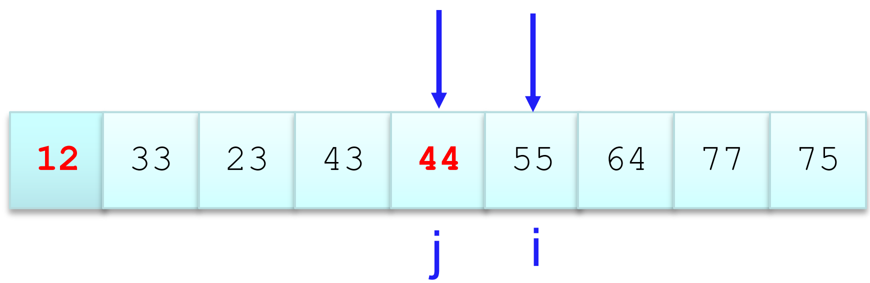

```
Move i if a[i] > pivot
Move j if a[j] < pivot
```

Trace of Quicksort (cont.)




```
Break if i >= j
```

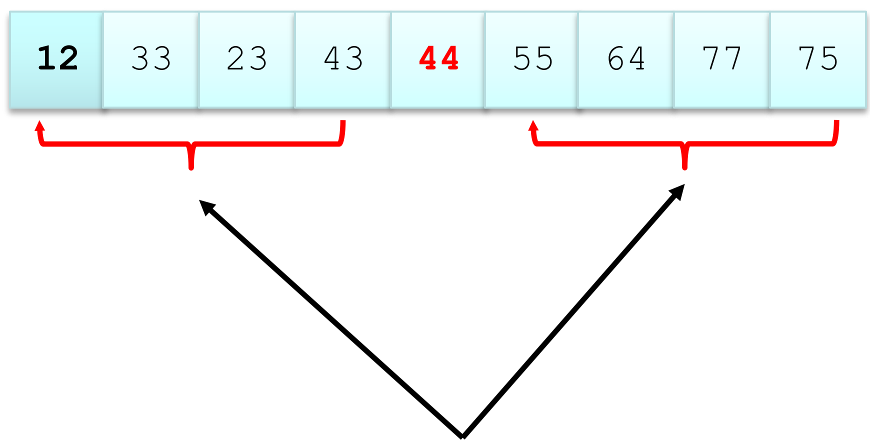
Trace of Quicksort (cont.)




```
Swap(pivot,a[j]
```

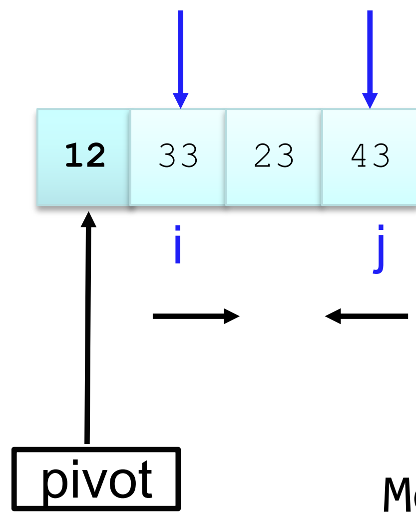
One iteration is done

Trace of Quicksort (cont.)



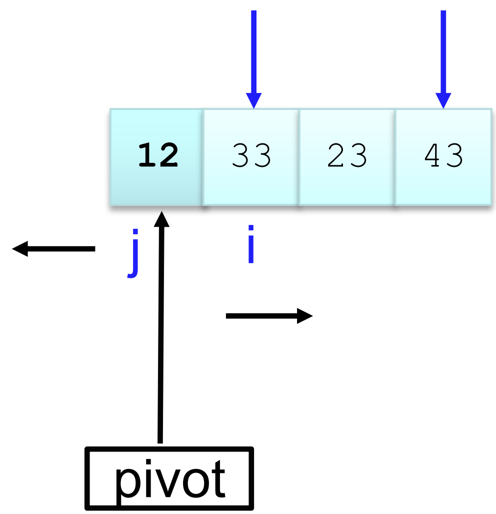
Recursively sort first and second subarray

Trace of Quicksort (cont.)



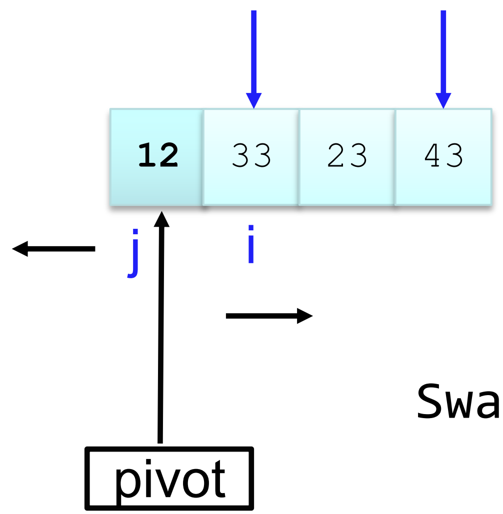

```
Move i if a[i] > pivot
Move j if a[j] < pivot
```

Trace of Quicksort (cont.)




```
Break if i >= j
```

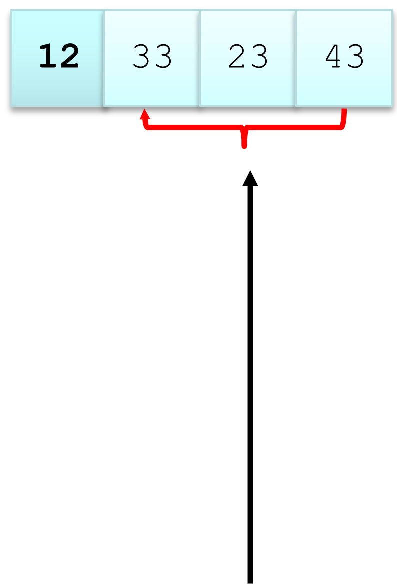
Trace of Quicksort (cont.)



`Swap(pivot,a[j]`

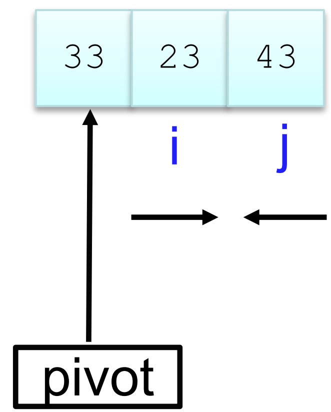
Second iteration for left half is done

Trace of Quicksort (cont.)



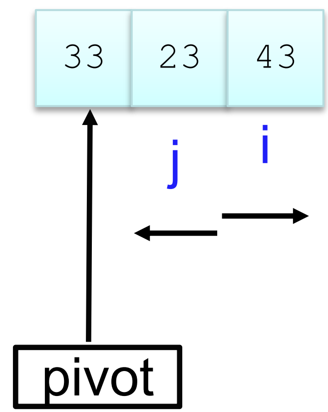
Recursively sort second subarray

Trace of Quicksort (cont.)



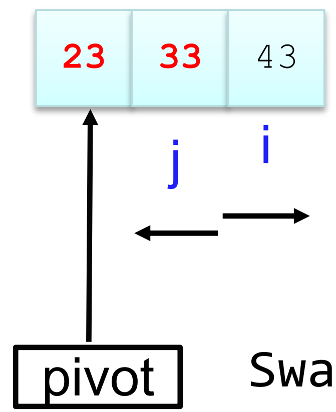

```
Move i if a[i] > pivot
Move j if a[j] < pivot
```

Trace of Quicksort (cont.)



`Break if i >= j`

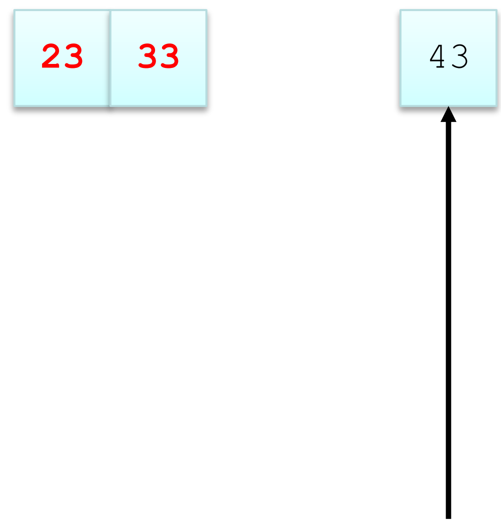
Trace of Quicksort (cont.)



Swap(pivot,a[j]

Another iteration is done

Trace of Quicksort (cont.)



Subarray to sort

Trace of Quicksort (cont.)

12	23	33	43	**44**	55	64	77	75

Sorted

Recursively sort the second subarray

Quick Sort Algorithm

```
/* quicksort the subarray from a[lo] to a[hi] */

void sort(Comparable[] a, int lo, int hi) {
   if (hi <= lo) return;
   int j = partition(a, lo, hi);
   sort(a, lo, j-1);
   sort(a, j+1, hi);
}
```

Partition

```
// partition the subarray a[lo..hi] so that a[lo..j-1] <= a[j] <=
a[j+1..hi]
// and return the index j.
int partition(Comparable[] a, int lo, int hi) {
   int i = lo;
   int j = hi + 1;
   Comparable v = a[lo];
   while (true) {
       // find item on lo to swap
       while (less(a[++i], v))
           if (i == hi) break;
      /* find item on hi to swap */
      while (less(v, a[--j]))
        if (j == lo) break;
      // check if pointers cross
      if (i >= j) break;
      exch(a, i, j);
    }
    // put partitioning item v at a[j]
    exch(a, lo, j);
  // now, a[lo .. j-1] <= a[j] <= a[j+1 .. hi]
  return j;
}
```

Analysis of Quicksort

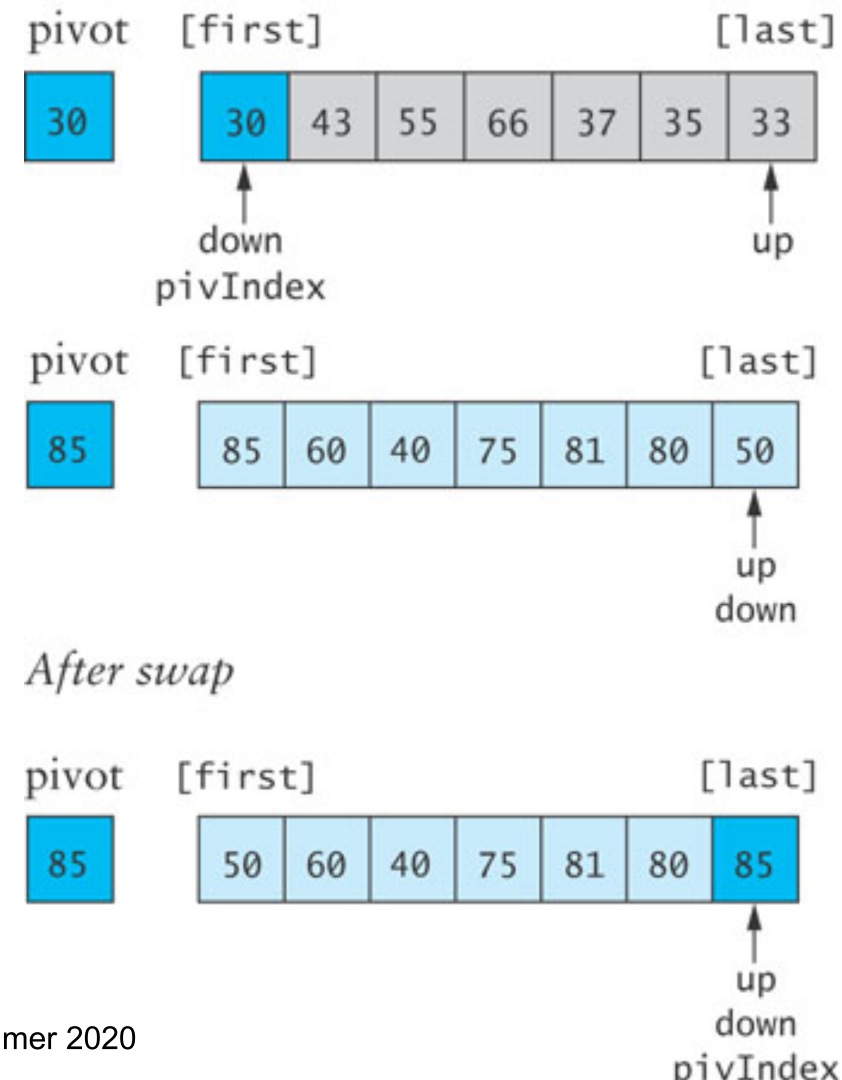
- If the pivot value is a random value selected from the current subarray,
  - then statistically half of the items in the subarray will be less than the pivot and half will be greater
- If both subarrays have the same number of elements (best case), there will be log *n* levels of recursion
- At each recursion level, the partitioning process involves moving every element to its correct position—*n* moves
- Quicksort is O(*n* log *n*), just like merge sort

Analysis of Quicksort (cont.)

- A quicksort will give very poor behavior if, each time the array is partitioned, a subarray is empty.
- In that case, the sort will be $O(n^2)$

- Under these circumstances, the overhead of recursive calls and the extra run-time stack storage required by these calls makes this version of quicksort a poor performer relative to the quadratic sorts.

If `Pivot` is the largest or smallest value



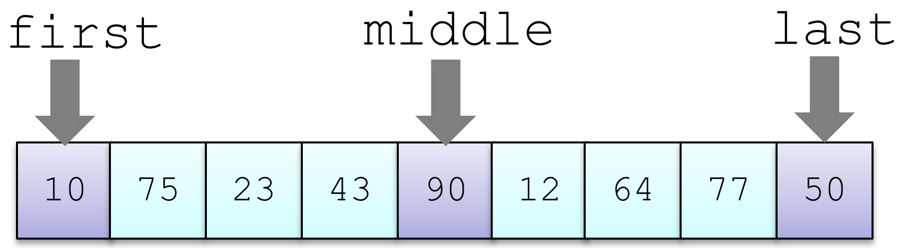
Revised Partition Algorithm

- A better solution is to pick the pivot value in a way that is less likely to lead to a bad split
  - Use three references: `first, middle, last`
  - Select the median of these items as the pivot

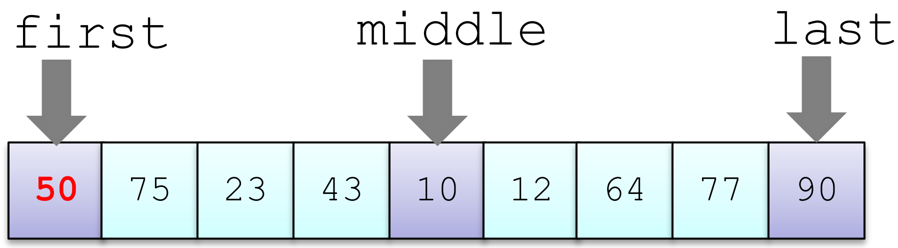
Trace of Revised Partitioning

10	75	23	43	90	12	64	77	50

Trace of Revised Partitioning (cont.)



Trace of Revised Partitioning (cont.)



Make the middle number pivot

Sorting Algorithm Comparison

Name	**Best**	**Averag e**	**Worst**	**Memory**	**Stable**
Bubble Sort	*n*	n^2	n^2	1	yes
Selection Sort	n^2	n^2	n^2	1	no
Insertion Sort	n	n^2	n^2	1	yes
Merge Sort	*nlogn*	*nlogn*	*nlogn*	*n*	yes
Quick Sort	*nlogn*	*nlogn*	n^2	*log n*	no
Heap Sort	*nlogn*	*nlogn*	*nlogn*	1	no