## CMSC132 Summer 2016 Midterm 2

## Multiple Choice ( 12 points)

Identify the choice that best completes the statement or answers the question.
$\qquad$ 1. (2 points) The preorder traversal sequence of a binary search tree is $30,40,10,15,25,27,39,23,35,42$. Which one of the following is the postorder traversal sequence of the same tree?
a. $23,27,25,15,10,35,42,40,39,30$
b. $23,27,15,25,10,35,42,39,40,30$
c. $23,27,25,15,10,35,42,39,40,30$
d. $23,27,25,15,10,30,35,42,39,40$
$\qquad$ 2. (2 points) What is the worst case time complexity for search, insert and delete operations in a general Binary Search Tree?
a. $\mathrm{O}(\mathrm{n})$ for all
c. $\mathrm{O}(\operatorname{Logn})$ for all
b. $\mathrm{O}(\mathrm{Logn})$ for search and insert, and $\mathrm{O}(\mathrm{n})$ for delete and delete
$\qquad$ 3. (2 points) In an array based implementation of a Binary Heap, the children of a node at position $k$ are at positions:
a. 2 k and $2 \mathrm{k}-1$
b. 2 k and $2 \mathrm{k}+1$
c. $2 \mathrm{k}+1$ and $2 \mathrm{k}+2$
d. $2(k+1)$ and $2(k+1)+1$
$\qquad$ 4. (2 points) Suppose the numbers 7, 5, 1, 8, 3, 0, 9, 4, 2, are inserted in that order into an initially empty binary search tree. The binary search tree uses the usual ordering on natural numbers. What is the in-order traversal sequence of the resultant tree?
a. 751032489
b. 012345789
c. 024315987
d. 984230157
$\qquad$ 5. (2 points) What is the asymptotic run-time complexity for the recursive Towers of Hanoi method?
a. logarithmic
c. linear
b. quadratic
d. exponential
6. (2 points) The following numbers are inserted into an empty binary search tree in the given order: $10,11,3$, $5,15,12,16,20$. What is the height of the binary search tree (the height is the maximum distance (number of edges) of a leaf node from the root, height of a tree with one node is 0 .)?
a. 6
b. 4
c. 3
d. 2

## Short Answer (48 points)

7. (3 points) What is the worst case time complexity for the following operations in a BST?

Search: $O(\quad)$
Insert: $O(\quad)$
Delete: $O(\quad)$
8. (3 points) What is the difference between a binary search tree and a binary heap? Answer:
9. (3 points) Given the preOrder and inOrder traversal of a binary tree, construct the tree. preorder: $7,10,4,3,1,2,8,11$
inorder: $4,10,3,1,7,11,8,2$
Answer:
10. (3 points) Explain the difference between binary search tree and red black tree? Answer:
11. (3 points) If the given preOrder traversal for a binary search tree is $\{10,5,1,7,40,50\}$, construct the binary search tree.
Answer:
12. (3 points) What is the time coplexity (Big O) of finding any given key in a binary heap? Explain your answer.
Answer:
$\qquad$
13. (6 points) (3 pts) Consider the min heap below, shown in the form of an array. Is this a valid min heap?If yes, draw the binary heap.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | 8 | 10 | 28 | 16 | 12 | 30 | 36 | 32 | 20 | 34 | 38 | 30 |

(3 pts) Show the contents of the heap after the value 15 is inserted.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

$\qquad$
14. (6 points) Given a node class BinaryTree, containing child-references left and right, an info member, and a method "check", implemented as follows:

```
public static boolean check(BinaryTree t1, BinaryTree t2)
{
    boolean result;
    if ((t1 == null)&&(t2 == null)){
        result = true;
    }
    else if ((t1 != null) && (t2 != null) && (t1.value == t2.value)){
            boolean left = check(t1.left, t2.right);
            boolean right = check(t1.right, t2.left);
            result = left && right;
    } else {
            result = false;
    }
    return result;
}
```

Pay close attention to the parameters!
a) What does check return for these trees?
t1 9
1 1
123
t2
$\stackrel{9}{9}$
$/ 12$
12

## Answer:

b) What does check return for these trees?
t1

t2

| $\mathbf{8}$ |  |
| :---: | :---: |
| $/$ | 1 |
| 6 | 11 |
| $/$ | $/$ |
| 4 | 5 |

Answer: $\qquad$
c) What does "check" do in general? That is - what does it return for any two binary trees t 1 and t 2 ?
$\qquad$
15. (3 points) Suppose that you do binary search for the key 74 in the following sorted array of size 15 :

101125313639535556646875788287
Give the sequence of keys in the array that are compared with 74
Answer:
16. (3 points) If the key in every node of a binary tree is greater than the key in its left child and less than the key in its right child, then is it a binary search tree? Give an example binary tree that shows this is NOT true.
Answer:
$\qquad$

17. (5 points)

For the binary tree above,
A) Write the preOrder traversal
B) Write the inOrder traversal
C) Write the postOrder traversal:
D) level order traversal
E)Number of leaves
18. ( 4 points) Construct a left-leaning red-black tree when the following elements are inserted in this order: 13 , $4,2,8,19,5,11$. Show your steps for each number. Use dashed line for red edge.

Name: $\qquad$
19. (3 points) Draw the Binary Search Tree after you delete key 7.


## Name:

$\qquad$

## Coding Problems (40 points)

20. (6 points) Write an iterative method "E max(Node r)", which receives a Binary Search Tree and returns the minimum key in the tree. For example, below tree returns 3


Answer:
$\qquad$
21. (8 points) Write a function "Integer lca(Integer n1, Integer n2)", which returns the lowest common ancestor of nodes with keys n 1 and n 2 in a given binary search tree. Arguments to lca always exist in the BST.


For example: lca $(1,7)$ returns 3. $\operatorname{lca}(4,13)$ return 8 Answer:
$\qquad$
22.
(5 points)

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
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Given a 5 * 5 array board
Color[][] board $=$ new Color[5][5];
that represents the above board. Each cell is filled with BLACK or GRAY. Calling board[row][col] returns WHITE or GRAY. Row and column index of the upper left cell is $(0,0)$. Complete the function "clear", which recursively converts GRAY cells to WHITE cells, when the row and column of a GRAY cell are given as arguments. For example: clear( 1,2 ) clears all GRAY cells.
public void clear(int row, int col) \{
if (row < $0|\mid$ row $>=5| \mid$ col $<0| |$ col $>=5$ ) return;
\}
\}
23. (8 points) Write code for a method int assign(Node t) that puts the number of nodes in the node's subtree into the data member of each node of a binary tree $\mathbf{t}$. It also returns the number of nodes in $\mathbf{t}$. For example, for the tree below, after a call to the assign method, the data member of each leaf node should contain 1, the data member of the left child of the root should contain 3, etc. The method would return 7.

24. (6 points) Max-heap is a binary heap, in which the value of each node is greater than or equal to the value of its children, with the maximum-value element at the root. Build a max-heap using the input $5,9,20,10$, 7,12 , step by step.
Answer:
$\qquad$
25. (7 points) Write a method "void printLevel(Node $r$, int $N$ )", which receives a binary tree and an integer $N$, and prints all keys in level N .


For example: For above tree, "printLevel(root, 3)" prints A,D,I

