CMSC 132: Object-Oriented Programming II

Binary Trees
Trees

University

Engineering  Education  Sciences

Physics  Computer Science
Trees
Trees

- A tree is a node with a value and zero or more children.
- No Cycle

**Properties**
- Number of nodes
- Height
- Root Node
- Leaves
- Interior nodes
- Ancestor
- Descendant
- Siblings
- Subtrees
Binary Tree

- Each internal node has at most two children (degree of two)
- The children of a node are an ordered pair
- We call the children of an internal node left child and right child
- Applications:
  - arithmetic expressions
  - decision processes
  - searching
A full binary tree is a tree in which every node other than the leaves has two children.

A full (perfect) binary tree of a given height $k$ has $2^{k+1}-1$ nodes.
Complete Binary Trees

A **complete binary tree** is a **binary tree** in which every level, except possibly the last, is completely filled, and all nodes are as far left as possible.

Complete binary tree

Full binary tree
Binary Tree Traversal

Traversal: Process of visiting each node in a tree, exactly once

- **preOrder:**

- **inOrder:**

- **postOrder:**

- **levelOrder:**
Binary Tree Traversal

preOrder: root, left, right
A B D G H C E F I J

inOrder: left, root, right
B G D H A E C I F J

postOrder: left, right, root
G H D B E I J F C A

Level Order: BFS
A B C D E F G H I J
Quiz 1:

What is the preOrder traversal of this binary tree?

A. 5 2 8 0 1 3
B. 5 2 1 0 3 8
C. 5 2 0 1 8 3
D. 5 2 0 1 3 8
Quiz 1:

What is the **preOrder** traversal of this binary tree?

A. 5 2 8 0 1 3  
B. 5 2 1 0 3 8  
C. 5 2 0 1 8 3  
D. 5 2 0 1 3 8
Quiz 2:

What is the inOrder traversal of this binary tree?

A. 0 1 2 3 8 5
B. 0 2 1 5 8 3
C. 0 2 1 5 3 8
D. 5 2 0 1 3 8
What is the **inOrder** traversal of this binary tree?

A. 0 1 2 3 8 5  
B. 0 2 1 5 8 3  
C. 0 2 1 5 3 8  
D. 5 2 0 1 3 8
Quiz 3:

What is the postOrder traversal of this binary tree?

A. 0 1 2 3 8 5
B. 0 2 1 5 8 3
C. 0 1 2 5 3 8
D. 5 2 0 1 3 8
Quiz 3:

What is the *postOrder* traversal of this binary tree?

A. 0 1 2 3 8 5
B. 0 2 1 5 8 3
C. 0 1 2 5 3 8
D. 5 2 0 1 3 8
Binary Tree Traversal

preOrder: \textcolor{red}{5 \ 2 \ 0 \ 1 \ 8 \ 3}

inOrder: \textcolor{red}{0 \ 2 \ 1 \ 5 \ 8 \ 3}

postOrder: \textcolor{red}{0 \ 1 \ 2 \ 3 \ 8 \ 5}

Level Order: \textcolor{red}{5 \ 2 \ 8 \ 0 \ 1 \ 3}
Arithmetic Expression Trees

Arithmetic Expression:

\[ A + (B \times (C / D)) \]

Tree for the above expression:

Used in most compilers

No parenthesis need to evaluate

Calculate by traversing tree
Traversing Trees

- Preorder: Root, then Children
  - \( + A * B / C D \)

- Postorder: Children, then Root
  - \( A B C D / * + \)

- Inorder: Left child, Root, Right child
  - \( A + B * C / D \)
Build a Binary Tree

Build a Binary Tree from given inOrder, postOrder

inOrder: 9, 5, 1, 7, 2, 12, 8, 4, 3, 11
postOrder: 9, 1, 2, 12, 7, 5, 3, 11, 4, 8

root

\begin{itemize}
\item 8
\item 9, 5, 1, 7, 2, 12
\item 4, 3, 11
\end{itemize}
Build a Binary Tree

Build a Binary Tree from given inOrder, postOrder

inOrder: 9,5,1,7,2,12,8,4,3,11
postOrder: 9,1,2,12,7,5,3,11,4,8
Build a Binary Tree

Build a Binary Tree from given inOrder, postOrder

inOrder: 9,5,1,7,2,12,8,4,3,11
postOrder: 9,1,2,12,7,5,3,11,4,8

preorder: 8,5,9,7,1,12,2,4,11,3
levelOrder: 8,5,4,9,7,11,1,12,3,2
Build a Binary

Build Binary Tree from inOrder, preOrder

inOrder: DBHEIAFCG
preOrder:ABDEHICFG
Build a Binary

Build Binary Tree from inOrder, preOrder

inOrder: DBHEIAFCG
preOrder: ABDEHICFG

postOrder: DHIEBFGCA
Binary Tree Implementation

Height:
Size:
Diameter:
Mirror:
Path:
Least Common Ancestor (LCA):
class Node {
    private E key;
    private Node left, right;
    Node(E key) {
        this.key = key;
    }
}

Node left | E key | Node right
public class BinaryTree<E> {  
    private Node root;
    class Node {  
        private E key;  
        private Node left, right;
        Node(E key) {  
            this.key = key;
        }
    }
}
Binary Tree Implementation

Check out the Binary Tree code examples from github