CMSC 132: Object-Oriented Programming II

Minimum Spanning Trees
Minimum spanning tree

- **Given:** Undirected graph $G$ with positive edge weights (connected).
- **Definition:** A spanning tree of $G$ is a subgraph $T$ that is connected and acyclic.
- **Goal:** Find a min weight spanning tree.

Graph $G$
Minimum spanning tree

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Spanning Tree $T$: cost = $4+6+8+5+11+9+7 = 50$
MST of random graph

https://who.rocq.inria.fr/Nicolas.Broutin/gallery.html
Simplifying assumptions

- Simplifying assumptions.
  - Edge weights are distinct.
  - Graph is connected.
- Consequence. MST exists and is unique.
**MST Algorithms**

- Greedy Algorithms: Prim’s and Kruskal’s.

- Both Prim’s and Kruskal’s Algorithms work with undirected graphs

- Both work with weighted and unweighted graphs but are more interesting when edges are weighted

- Both are greedy algorithms that produce optimal solutions
Kruskal's algorithm

- Minimum-spanning-tree algorithm
  - Consider edges in *ascending order* of weight.
  - Add next edge to tree $T$ unless doing so would create a cycle.
  - If the graph is not connected, then it finds a minimum spanning forest
Consider an undirected, weight graph

Kruskal's algorithm Demo

Consider an undirected, weight graph
Kruskal's algorithm Demo

Add Edge (E,1,D)
Kruskal's algorithm Demo

Add Edge (H,2,G)
Add Edge (E,3,B)
Kruskal's algorithm Demo

Add Edge (F,4,B)
Kruskal's algorithm Demo

Add Edge (G,4,E)
Kruskal's algorithm Demo

Add Edge (F,5,C)
Kruskal's algorithm Demo

Add Edge (B,8,A)
Kruskal's algorithm Demo

Cost: 5+8+4+2+3+1+4.5 = 27.5
Kruskal’s Algorithm Demo
Kruskal’s Algorithm Demo
Prim's algorithm

- Builds the tree one vertex at a time
- Starts from an arbitrary starting vertex
- Each step adds the cheapest possible connection from the tree to another vertex.
Prim's algorithm Demo

Consider an undirected, weight graph
Consider an undirected, weight graph

Add Edge (E,1,D)
Prim's algorithm Demo

Consider an undirected, weight graph

Add Edge (D, 2, G)
Prim's algorithm Demo

Consider an undirected, weight graph

Add Edge (D,3,C)
Prim's algorithm Demo

Consider an undirected, weight graph

Add Edge (G,3,H)
Consider an undirected, weight graph

Add Edge (E,4,B)
Consider an undirected, weight graph

Add Edge (F,4,B)
Prim's algorithm Demo

Consider an undirected, weight graph

Add Edge (H,5,A)
Consider an undirected, weight graph

Cost: $6 + 5 + 3 + 3.5 + 2 + 4 + 1 = 24.5$
Prim’s Algorithm Demo
Prim’s Algorithm Demo

Diagram: A graph with labeled edges illustrating the steps of Prim’s algorithm.