

# CMSC 330: Organization of Programming Languages

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## Closures (Implementing Higher Order Functions)

# Returning Functions as Results

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- ▶ In OCaml you can **pass functions as arguments**
  - to `map`, `fold`, etc.
- ▶ and you can **return functions as results**

```
# let pick_fn n =  
    let plus_three x = x + 3 in  
    let plus_four x = x + 4 in  
    if n > 0 then plus_three else plus_four  
val pick_fn : int -> (int->int) = <fun>
```

- ▶ Here, `pick_fn` takes an `int` argument, and returns a function

```
# let g = pick_fn 2;;  
val g : int -> int = <fun>  
# g 4;;    (* evaluates to 7 *)
```

# Multi-argument Functions

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- ▶ Consider a rewriting of the prior code (above)

```
let pick_fn n =  
  if n > 0 then (fun x -> x+3) else (fun x -> x+4)
```

- ▶ Here's another version

```
let pick_fn n =  
  (fun x -> if n > 0 then x+3 else x+4)
```

- ▶ ... the shorthand for which is just

```
let pick_fn n x =  
  if n > 0 then x+3 else x+4
```

*I.e., a multi-argument  
function!*

# Currying

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- ▶ **Multi-argument functions** not a separate concept
  - Can encode one as a *function that takes a single argument and returns a function that takes the rest*
- ▶ This encoding is called **currying** the function
  - Named after the logician Haskell B. Curry
  - But Schönfinkel and Frege discovered it
    - So maybe it should be called **Schönfinkelizing** or **Fregging**

# Curried Functions In OCaml

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- ▶ OCaml syntax defaults to currying. E.g.,

```
let add x y = x + y
```

- is identical to all of the following:

```
let add = (fun x -> (fun y -> x + y))  
let add = (fun x y -> x + y)  
let add x = (fun y -> x+y)
```

- ▶ Thus:

- `add` has type `int -> (int -> int)`
- `add 3` has type `int -> int`
  - ▶ `add 3` is a function that adds 3 to its argument
- `(add 3) 4 = 7`

# Syntax Conventions for Currying

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- ▶ Because currying is so common, OCaml uses the following conventions:
  - `->` associates from the right
    - Thus `int -> int -> int` is the same as
      - `int -> (int -> int)`
  - function application associates from the left
    - Thus `add 3 4` is the same as
      - `(add 3) 4`

# Quiz 1: Which f definition is equivalent?

---

```
let f a b = a / b;;
```

A. `let f b = fun a -> a / b;;`

B. `let f = fun a -> (fun b -> a / b);;`

C. `let f = fun a | b -> a / b;;`

D. `let f (a, b) = a / b;;`

# Quiz 1: Which f definition is equivalent?

---

```
let f a b = a / b;;
```

A. `let f b = fun a -> a / b;;`

B. `let f = fun a -> (fun b -> a / b);;`

C. `let f = fun a | b -> a / b;;`

D. `let f (a, b) = a / b;;`



# Multiple Arguments, Partial Application

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- ▶ Another way you could encode support for multiple arguments is using tuples
  - `let f (a,b) = a / b (* int*int -> int *)`
  - `let f a b = a / b (* int -> int-> int *)`
- ▶ Is there a benefit to using currying instead?
  - Supports **partial application** – useful when you want to provide some arguments now, the rest later
  - `let add a b = a + b;;`
  - `let addthree = add 3;;`
  - `addthree 4;; (* evaluates to 7 *)`

## Quiz 2: What does this evaluate to?

---

```
let f a b = a * b in  
let g = f 2 in  
let a = 3 in  
g 4
```

- A. 8
- B. 6
- C. 2
- D. 3

## Quiz 2: What does this evaluate to?

---

```
let f a b = a * b in
let g = f 2 in
let a = 3 in
g 4      (* f 2 4 = 8 *)
```

A. 8

B. 6

C. 2

D. 3

# Currying is Standard In OCaml

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- ▶ Pretty much all functions are curried
  - Like the standard library `map`, `fold`, etc.
  - See `/usr/local/ocaml/lib/ocaml` on Grace
    - In particular, look at the file `list.ml` for standard list functions
    - Access these functions using `List.<fn name>`
    - E.g., `List.hd`, `List.length`, `List.map`
- ▶ OCaml works hard to make currying efficient
  - Because otherwise it would do a lot of useless allocation and destruction of `closures`
  - What are those, you ask? Let's see ...

# Closures

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# Remember our partial application example

---

```
let add = fun a -> fun b -> a + b;;
```

```
let addthree = add 3 in  
addthree 4
```

- ▶ Let's evaluate it the expression (using substitution)

```
let addthree = add 3 in addthree 4
```

→ let addthree = (fun a -> fun b -> a+b) 3 in ...

→ let addthree = (fun b -> 3+b) in addthree 4

→ (fun b -> 3+b) 4

→ 3+4 → 7

# Using Substitution “Remembered” the $a$ is 3

---

```
let add = fun a -> fun b -> a + b;;
```

```
let addthree = add 3 in  
addthree 4
```

- ▶ Let's evaluate it the expression (using substitution)

```
let addthree = add 3 in addthree 4
```

```
→ let addthree = (fun a -> fun b -> a+b) 3 in ...
```

```
→ let addthree = (fun b -> 3+b) in addthree 4
```

```
→ (fun b -> 3+b) 4
```

```
→ 3+4 → 7
```

# How to use a stack, not substitution?

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- ▶ Substitution replaces the occurrence of the variable with the value it is bound to (e.g., at a call)
  - Like changing the code in place!
- ▶ In reality, we use a **stack** to remember **variable-to-value mappings**

```
let addthree = add 3 in  
addthree 4
```

  - But: If calling `add 3` pushes 3 on the stack, what happens when the call returns? *How does `addthree` remember that it was constructed by a call with 3?*



# Closures “Remember”

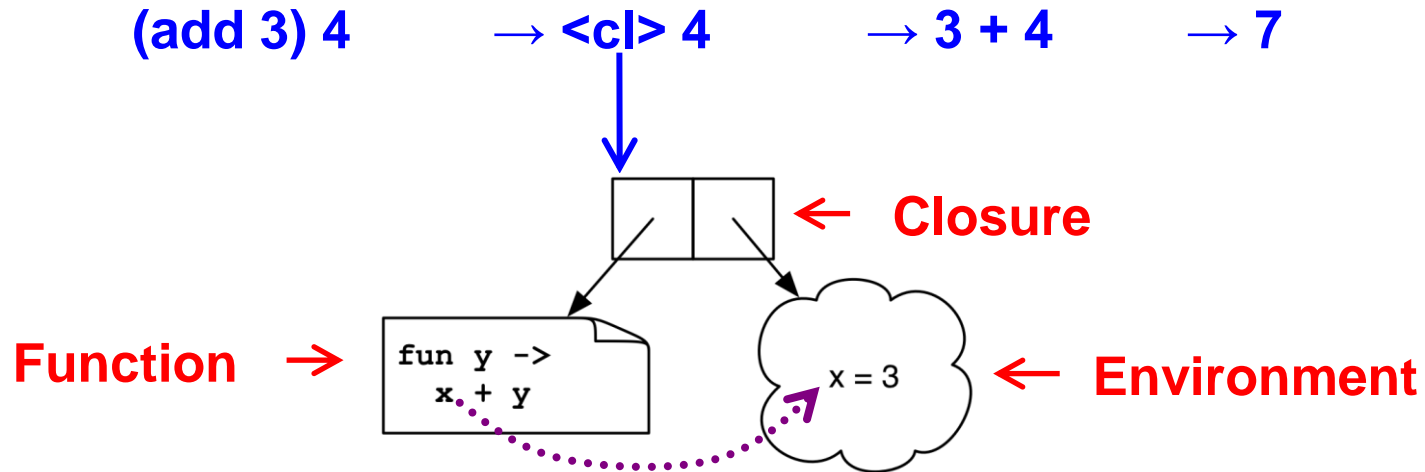
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- ▶ An **environment** is a mapping from variables to values
  - Like a stack frame
- ▶ A **closure** is a pair  $(f, e)$  consisting of function code  $f$  and an environment  $e$ 
  - Environment “captures” active bindings, **when closure is made**
  - These include “free variables” – these are mentioned in  $f$ ’s body but are not its formal parameters
- ▶ When you invoke a closure,  $f$  is evaluated using  $e$

# Example 1

---

```
let add x = (fun y -> x + y)
```

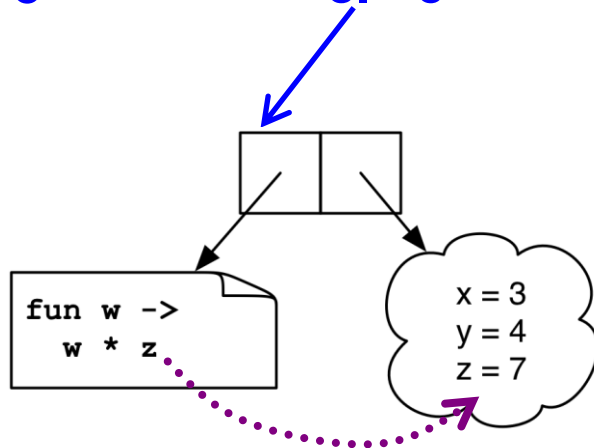


## Example 2

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```
let mult_sum (x, y) =  
  let z = x + y in  
  fun w -> w * z
```

$(\text{mult\_sum } (3, 4)) \ 5 \quad \rightarrow \ \langle \text{cl} \rangle \ 5 \quad \rightarrow \ 5 * 7 \quad \rightarrow \ 35$



## Quiz 3: What is x?

---

```
let a = 1;;  
let a = 0;;  
let b = 10;;  
let f () = a + b;;  
let b = 5;;  
let x = f ();;
```

A. 10

B. 1

C. 15

D. Error - variable name conflicts

## Quiz 3: What is x?

---

```
let a = 1;;  
let a = 0;;  
let b = 10;;  
let f = fun () -> a + b;;  
let b = 5;;  
let x = f ();;
```

**A. 10**

**B. 1**

**C. 15**

**D. Error - variable name conflicts**

## Quiz 4: What is z?

---

```
let f x = fun y -> x - y in
let g = f 2 in
let x = 3 in
let z = g 4 in
z;;
```

- A. 7
- B. -2
- C. -1
- D. Type Error – insufficient arguments

## Quiz 4: What is z?

---

```
let f x = fun y -> x - y in
let g = f 2 in
let x = 3 in
let z = g 4 in
z;;
```

- A. 7
- B. -2**
- C. -1
- D. Type Error – insufficient arguments

## Quiz 5: What does this evaluate to?

---

```
let f x = x+1 in
let g = f in
g (fun i -> i+1) 1
```

- A. Type Error
- B. 1
- C. 2
- D. 3



## Quiz 5: What does this evaluate to?

---

```
let f x = x+1 in
let g = f in
(g (fun i -> i+1)) 1
```

- A. **Type Error** – Too many arguments passed to g (application is *left associative*)
- B. 1
- C. 2
- D. 3

# Scope

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## ▶ **Dynamic scope**

- The body of a function is evaluated in the current dynamic environment at the time the function is **called**, not the environment that existed at the time the function was defined
  - Now basically considered a mistake

## ▶ **Lexical scope** (aka Static scope)

- The body of a function is evaluated in the old dynamic environment that existed at the time the function was **defined**, not the current environment when the function is called.
- **This is implemented by closures**

# Dynamic vs. Static Scope

---

```
let f a b = a * b in
let g = f 2 in
let a = 3 in
g 4
```

- A. **8**      Answer, if lexical/static scope
- B. **12**     Answer, if dynamic scope
- C. **2**
- D. **3**

# Higher-Order Functions in C

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- ▶ C supports **function pointers**

```
typedef int (*int_func)(int);
void app(int_func f, int *a, int n) {
    for (int i = 0; i < n; i++)
        a[i] = f(a[i]);
}
int add_one(int x) { return x + 1; }
int main() {
    int a[] = {5, 6, 7};
    app(add_one, a, 3);
}
```

# Higher-Order Functions in C (cont.)

---

- ▶ C does not support closures
  - Since no nested functions allowed
  - Unbound symbols always in global scope

```
int y = 1;
void app(int(*f)(int), n) {
    return f(n);
}
int add_y(int x) {
    return x + y;
}
int main() {
    app(add_y, 2);
}
```

## Higher-Order Functions in C (cont.)

---

- ▶ Cannot access non-local variables in C
- ▶ OCaml code

```
let add x y = x + y
```

- ▶ Equivalent code in C is illegal

```
int (* add(int x))(int) {  
    return add_y;  
}  
int add_y(int y) {  
    return x + y; /* error: x undefined */  
}
```

# Higher-Order Functions in C (cont.)

---

- ▶ OCaml code

```
let add x y = x + y
```

- ▶ Works if C supports nested functions

- Not in ISO C, but in gcc; **but** not allowed to return them

```
int (* add(int x))(int) {  
    int add_y(int y) {  
        return x + y;  
    }  
    return add_y; }  
}
```

- Does not allocate closure, so x popped from stack and add\_y will get garbage (potentially) when called

# Java 8 Supports Lambda Expressions

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- ▶ Ocaml's

```
fun (a, b) -> a + b
```

- ▶ Is like the following in Java 8

```
(a, b) -> a + b
```

- ▶ Java 8 supports closures, and variations on this syntax



# Java 8 Example

---

```
public class Calculator {  
    interface IntegerMath { int operation(int a, int b); }  
    public int operateBinary(int a, int b, IntegerMath op) {  
        return op.operation(a, b);  
    }  
    public static void main(String... args) {  
        Calculator myApp = new Calculator();  
        IntegerMath addition = (a, b) -> a + b;  
        IntegerMath subtraction = (a, b) -> a - b;  
        System.out.println("40 + 2 = " +  
            myApp.operateBinary(40, 2, addition));  
        System.out.println("20 - 10 = " +  
            myApp.operateBinary(20, 10, subtraction));  
    }  
}
```

← Lambda  
expressions