A Markov Reward Model for Software Reliability

YoungMin Kwon and Gul Agha
Open Systems Laboratory
Department of Computer Science
University of Illinois at Urbana-Champaign

Motivation

- Evaluating software with many processes/threads
  - State explosion problem
    - 100 3-state system => 3^{100} states
- Statistical abstraction of state
  - State (*pmf*): probability that a randomly picked thread is in certain module.
    - e.g.: 90% of the threads are in ‘A’ state
  - Abstract enough to handle state
  - Detailed enough to evaluate reliability

Overview

- Model the software as Markov reward model
  - Each module represents a state
  - Module reliabilities are *rewards*
  - Transition probabilities between modules are obtained by *operational profiling*
- System Reliability Estimation
  - Evaluated by *Probabilistic Model Checking*
  - Helps focus testing on particular modules that may increase the reliability of the entire system
    - Modules are not equally important

Markov Reward Model for Software Reliability

Markov model

- Model the program by a DTMC X = (S,M)
  - S is the set modules in the program and M represents the transition probabilities between modules.
- Reliability of a module:
  - Probability that a module does not produce a fault when a control is passed to it.
  - NHPP: a simple reliability model

$$R(x \mid t) = e^{-a\,(e^{-bt} - e^{-b(t+x)})}$$

R(x|t) is the probability that a module does not have a failure during the time interval t to t + x.

Markov model

DTMC model extension for reliability checking

- *Add fail state f.*
- Transition probability matrix M is extended as follows

$$\mathbf{M}' = \begin{bmatrix} r_1 M_{11} & r_2 M_{12} & \cdots & r_n M_{1n} & 0 \\ r_1 M_{21} & r_2 M_{22} & \cdots & r_n M_{2n} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_1 M_{n1} & r_2 M_{n2} & \cdots & r_n M_{nn} & 0 \\ 1 - r_1 & 1 - r_2 & \cdots & 1 - r_n & 1 \end{bmatrix}.$$

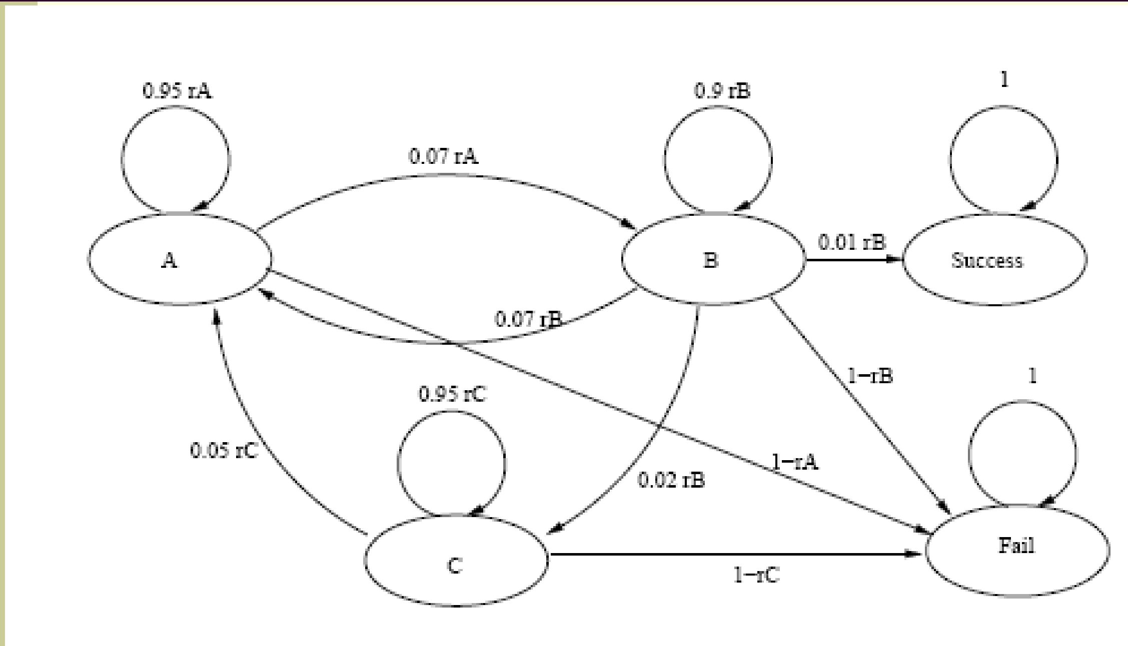
Reliability

- Reliability of a program is the probability that a program eventually arrives at the final success state:

$$r = M_{n*} \lim_{t\to\infty} \sum_{i=1}^{t} M_*^i \cdot \mathbf{x}(0)$$

where M_* is a sub-matrix of M that comprises the first $n-1$ rows and the first $n-1$ columns of M, M_{n*} is a n^{th} row vector of M with first $n-1$ elements, and $x(0)$ is an initial probability mass function of $X(0)$

Example

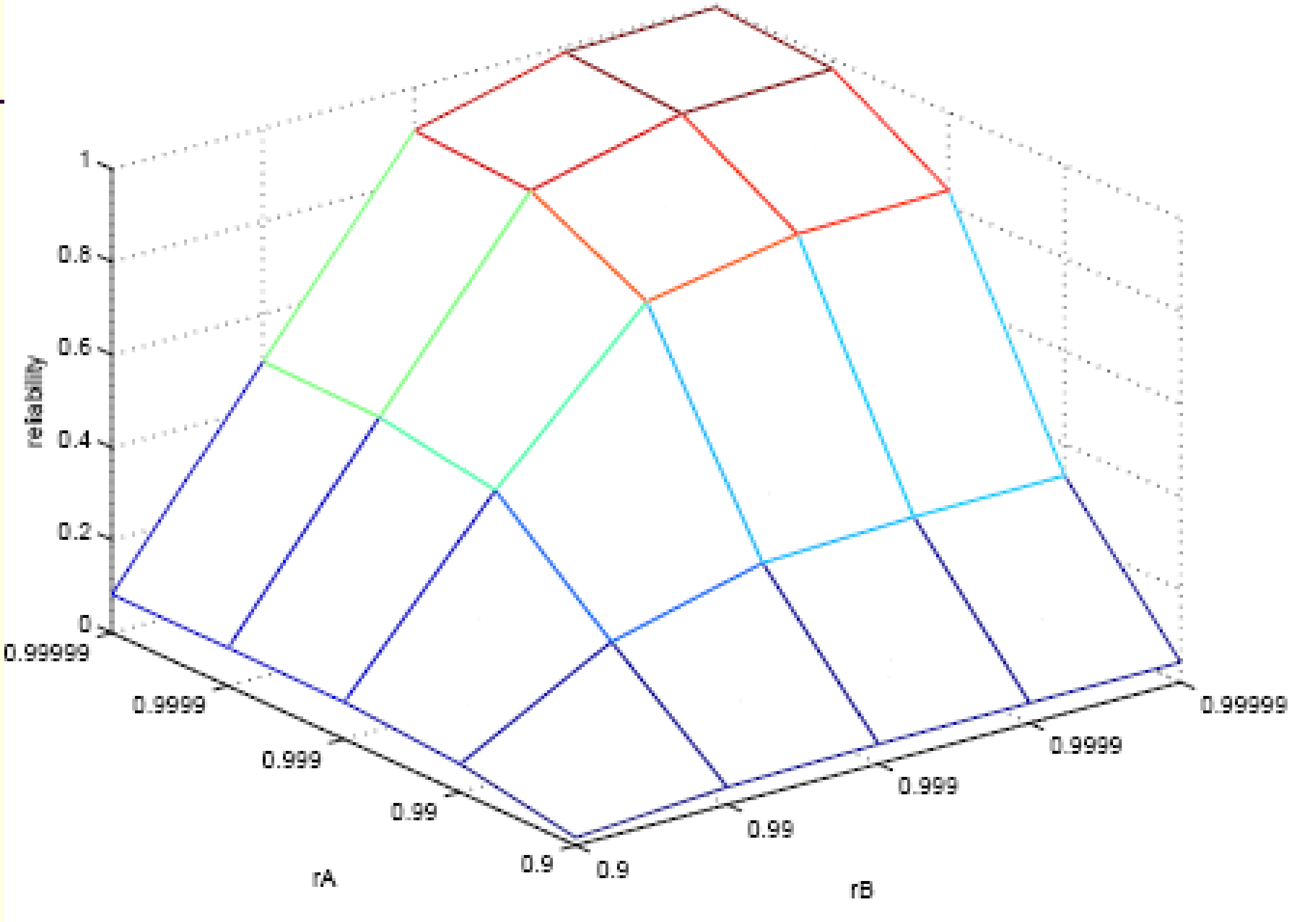


$$\mathbf{M} = \begin{bmatrix} .95r_A & .07r_B & .05r_C & .00 & .00 \\ .05r_A & .90r_B & .00r_C & .00 & .00 \\ .00 & .02r_B & .95r_C & .00 & .00 \\ .00 & .01r_B & .00 & 1.0 & .00 \\ 1-r_A & 1-r_B & 1-r_C & .00 & 1.0 \end{bmatrix}.$$

A module reliability diagram, where r_A ,r_B and r_c are the reliabilities of the modules A,B and C.

Reliability of the program is modeled by a DTMC X =(S,M) where S={A, B, C, success, fail} and M as above.

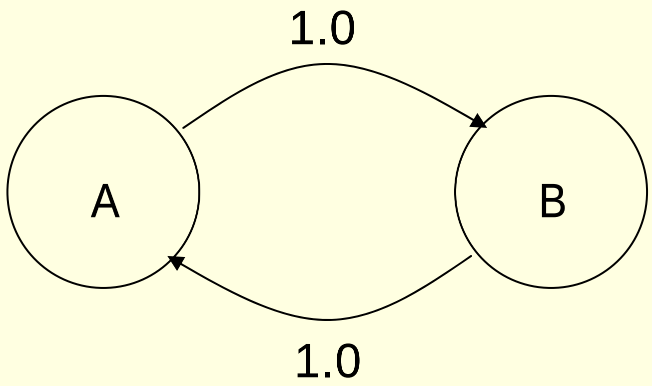
Example…



Reliability of a program as a function of module Reliabilities r_A and r_B with $r_c = 1 - 10^{-5}$

Transition probabilities significantly affect the overall reliability of the program.

Probabilistic Model Checking



- With PCTL like logics, ‘P[X=A] > .3 is always true’ is always false regardless of initial *pmfs*
- However if 50 out of 100 threads are in A state and the others are in B state ⇨ 50% of the threads are always in A state.
- iLTL can specify this situation because it works on *pmf transitions*

iLTL Formula

- Syntax

$$
\begin{array}{rcl}
\psi & ::= & \mathrm{T} \mid \mathrm{F} \mid ineq \mid \\
 & & \neg\psi \mid \psi \vee \phi \mid \psi \wedge \phi \mid \\
 & & \mathtt{X}\psi \mid \psi\ \mathtt{U}\ \phi \mid \psi\ \mathtt{R}\ \phi \\
ineq & ::= & \sum_{i=1}^{n} a_i \cdot \mathrm{P}[X = s_i] < b
\end{array}
$$

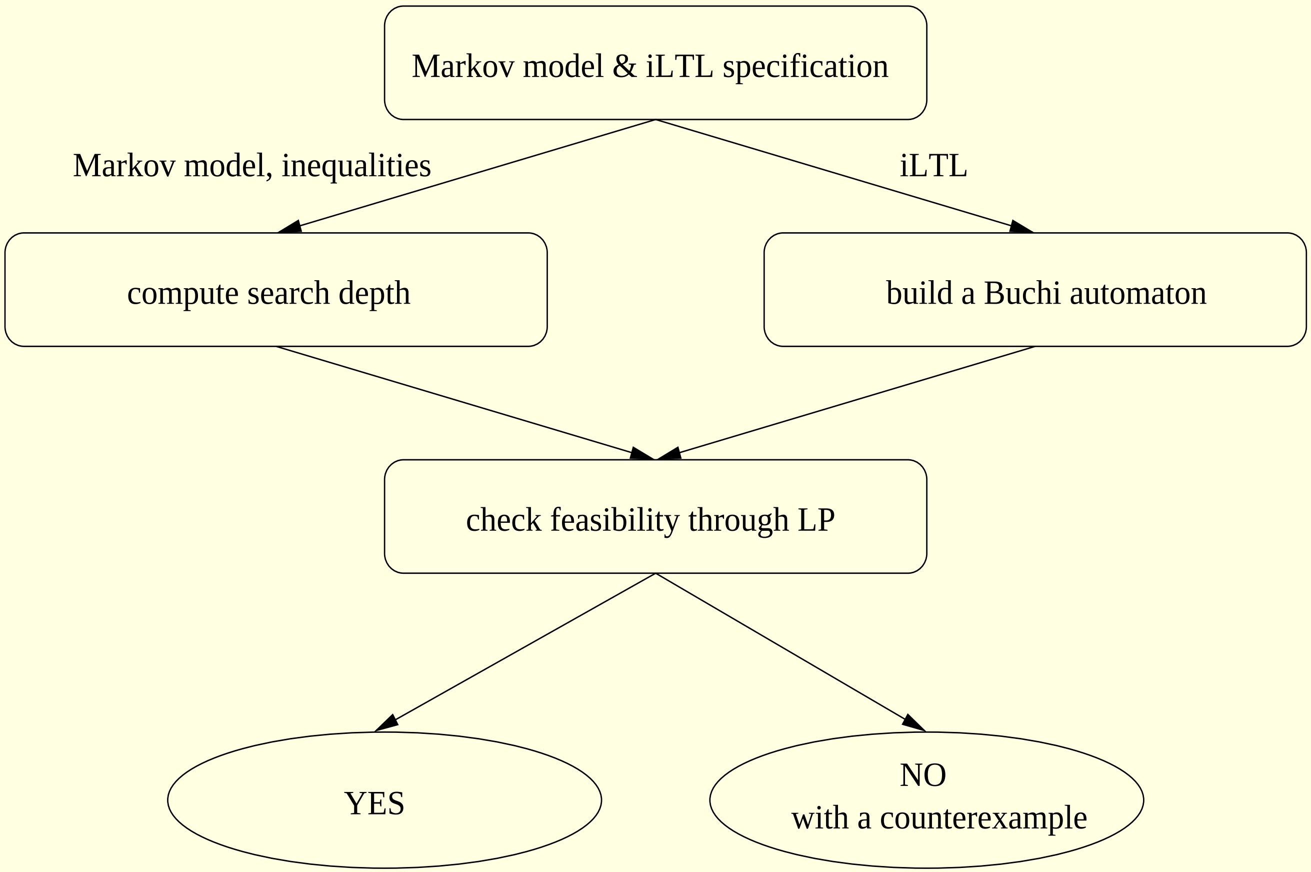
iLTL Formula
:atomic propositions

- At least 10% more nodes are in READY state than in IDLE state
  - P[X=READY] > P[X=IDLE] + 0.1
  - P[X=READY] - P[X=IDLE] > 0.1
- Expected Queue length is less than 2
  - 1*P[Q=S1] + 2*P[Q=S2] + 3*P[Q=S3] < 2
- Availability of a system X is 10% larger than that of a system Y
  - P[X=READY] > P[Y=READY] + 0.1

Main Theorem

- If
  - Markov matrix M is diagonalizable
  - Absolute value of second largest eigenvalue of M is strictly less than 1
  - For all inequalities of an iLTL formula Ψ, the steady state expected value of LHS is not equal to its RHS
- Then
  - There is a bound N after which all inequalities of Ψ become constants

Model Checking Algorithm



iLTL Model Checking of Software Reliability

Program properties related to reliability that can be evaluated using iLTL are

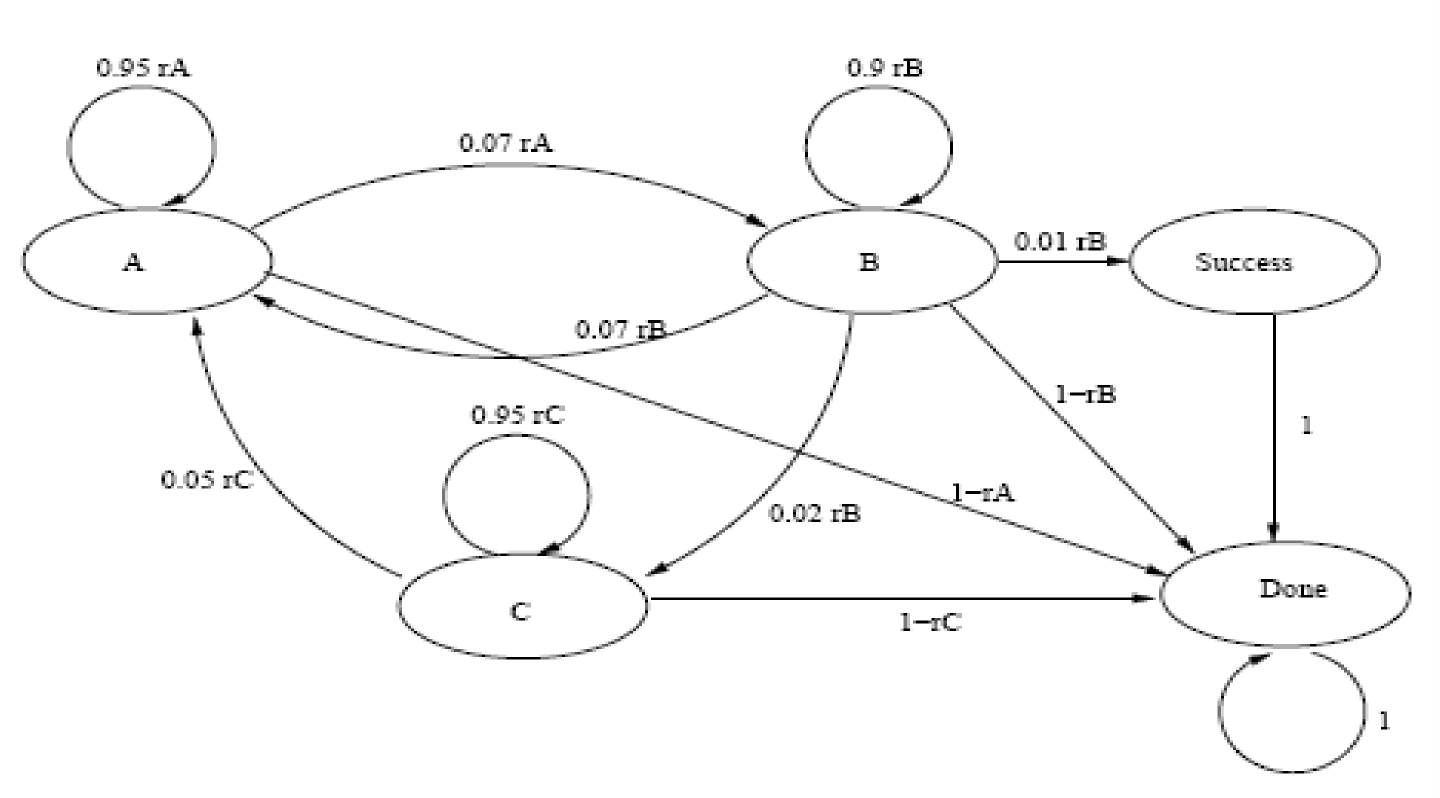
- Find the configuration of a system (represented by pmf) that will make the system most unreliable.
- Reliability of a system given its configuration.
- Effects on the reliability of the program if different executions constraints are enforced on the program.
- System parameter adjustment through comparison between systems with different parameters.

Model not appropriate….

- iLTL model checking algorithm cannot be directly applied on the previous markov model because the model violated the eigenvalue constraints of the theorem.
- Transformation is required.

Add Success State

- Fail state is replaced by done state and the self loop transition of success state is removed
- Transition from success to done with a probability one is added and success state is made transient.



Modification…

- Modified DTMC model is X = (S,M) where S = { A, B, C, success, done} and

$$\mathbf{M} = \begin{bmatrix} .95r_A & .07r_B & .05r_C & .00 & .00 \\ .05r_A & .90r_B & .00r_C & .00 & .00 \\ .00 & .02r_B & .95r_C & .00 & .00 \\ .00 & .01r_B & .00 & .00 & .00 \\ 1-r_A & 1-r_B & 1-r_C & 1.0 & 1.0 \end{bmatrix}.$$

- The reliability of the program is the accumulated sum of the probabilities that the success state is visited. It is given by

$$r = \sum_{t=0}^{\infty} P\{ X(t)=\text{success}\}$$
$$= \sum_{t=0}^{\infty} [0, 0, 0,1, 0] \cdot x(t)$$
$$= \sum_{t=0}^{\infty} [0, 0, 0, 1, 0] \cdot M^t \cdot x(0)$$

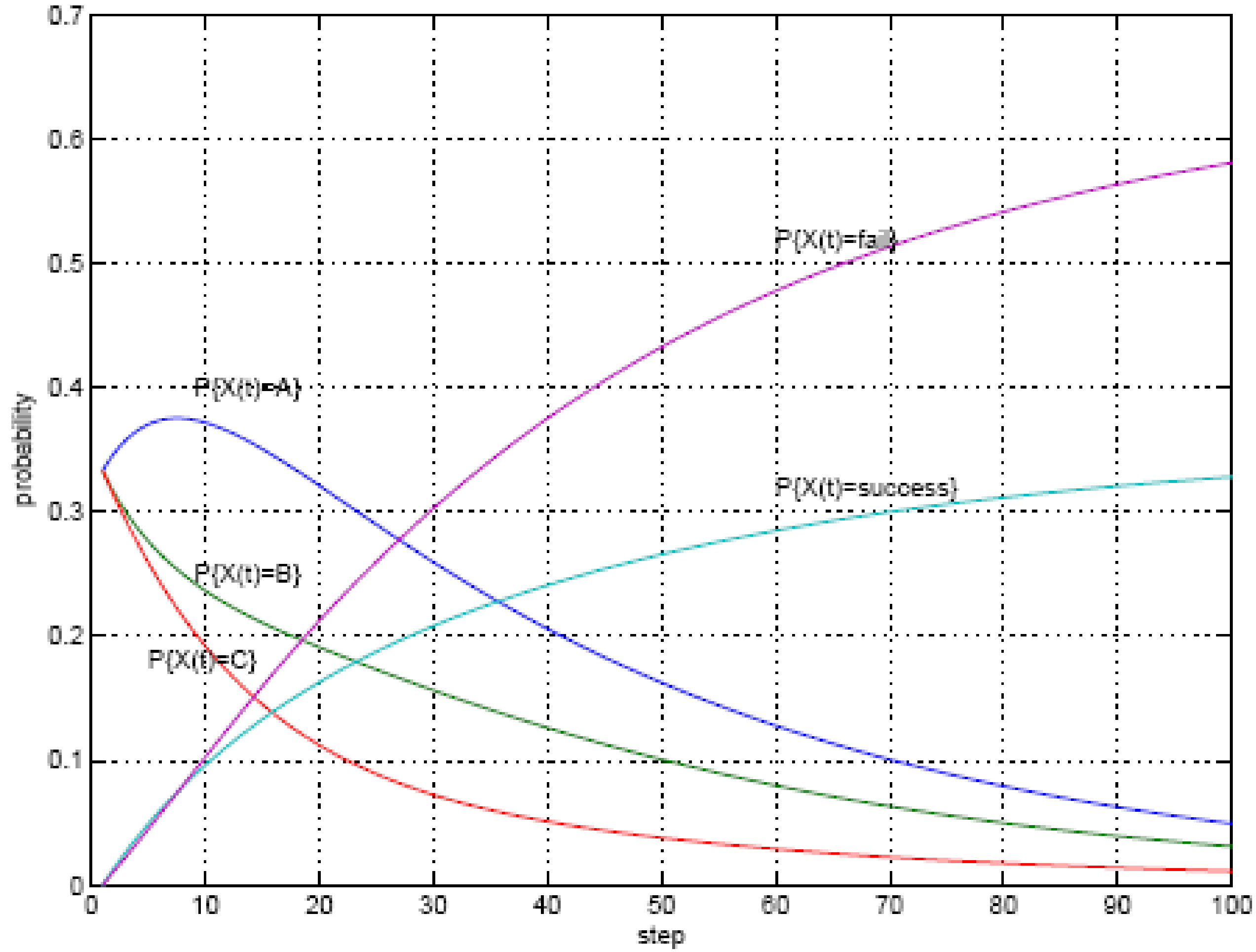


Figure shows how the probabilities of each states change over time and how the reliability of the program (P{X(t) = success}) is accumulated with the module reliabilities r_A, r_B and r_C and initial pmf x(0) = [1/3,1/3,1/3,0,0]

iLTL checker

```
model:
  Markov chain pgm
    has states :
      { A, B, C, S, D},
    transits by :
      [ .9215, .0699, .05,    .0,  .0;
        .0485, .8991, .0,     .0,  .0;
        .0,    .02,   .9191, .0,  .0;
        .0,    .01,   .03,    .0,  .0;
        .03,   .001,  .001,   1.0,1.0        ]
 specification:
  a : .2149*P{pgm=A} + .3478*P{pgm=B}  + .5036*P{pgm=C} < .7,
  b : .2149*P{pgm=A} + .3478*P{pgm=B} + .5036*P{pgm=C} < .5,
  c : .2149*P{pgm=A} + .3478*P{pgm=B} + .5036*P{pgm=C} < .3,
  d : P{pgm=S} + P{pgm=D} > .0,
  e : P{pgm=A} > P{pgm=C} + .3
  a                        # 1)
  #(b /\ ~ d) -> ~ e      2)
  #(b /\ ~ d) -> <>~ e   3)
```

An iltl checker description of the modified reliability model

Results

- The specification "a" checks whether the reliability of the program pgm is less than 0.7

  Depth 22

  Result T

  Here 22 indicates the required search depth for the formula.

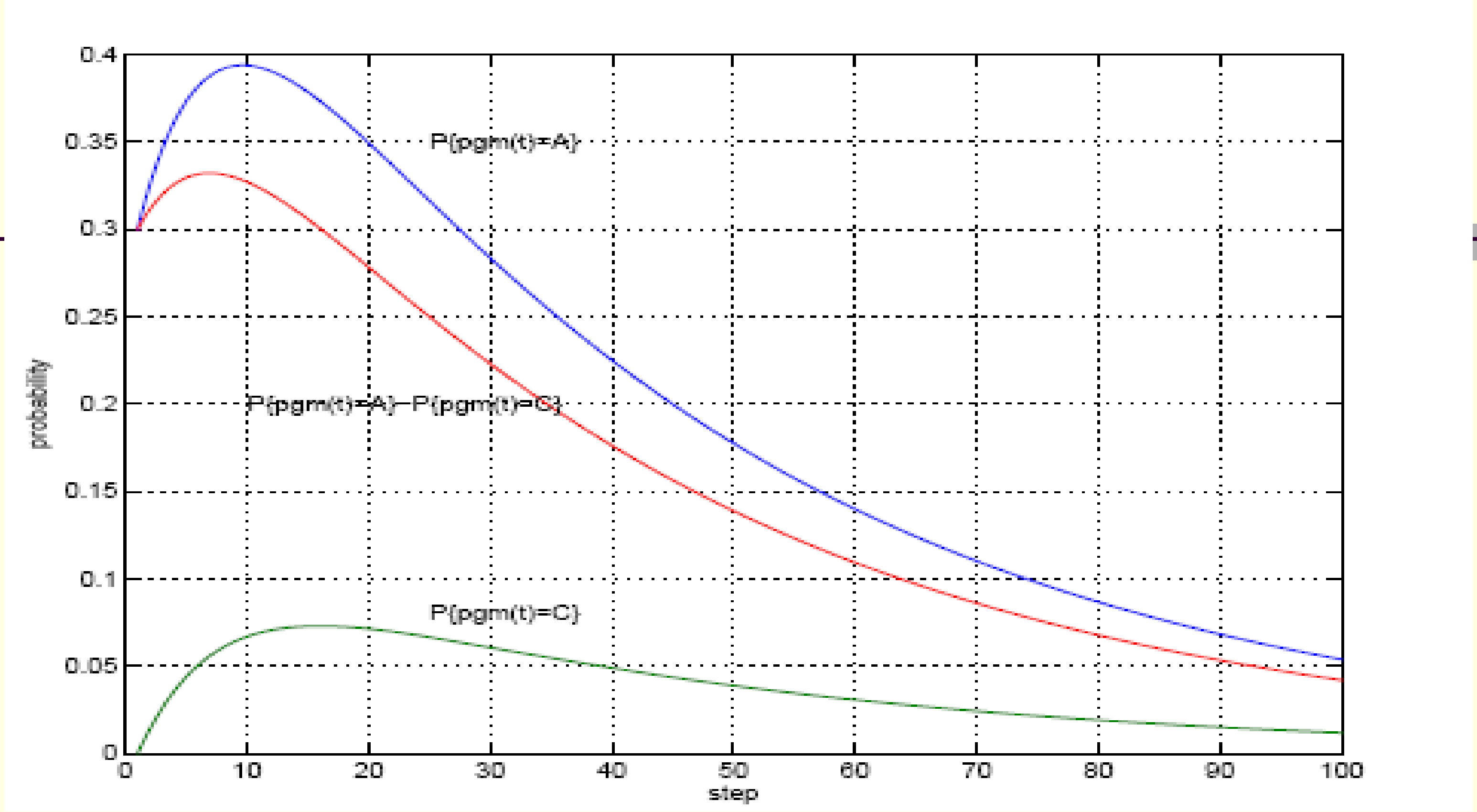
- The second example ($b \rightarrow \sim e$) checks whether the fact that the reliability of pgm is less than 0.5 implies not e

  Depth 78

  Result F

  Counterexample: *pmf* (pgm(0)) : [.3 .7 .0 .0 .0]

  Result is false and it provides with the counter example of X(0).



- Figure explain the second and third examples. From step 1 to 15, the probability difference is larger than 0.3. However, eventually after step 15 the difference becomes less than .3

Future Work

- Develop a distributed algorithm for iLTL to speed up model checking
  - iLTL model checking is a feasibility checking of Disjunctive Normal Form of (in)equality constraints: each conjunctive set of constraints can be checked independently.
- Once search depth N is computed, bounded model checking techniques can be introduced