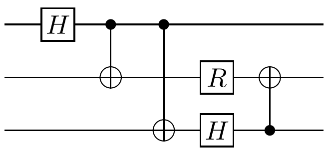
Problem Set #3

Quantum Error Correction
Instructor: Daniel Gottesman

Due Tues., Feb. 6, 2007

Problem #1. A Clifford Group Circuit

For this problem, consider the following circuit:



Here, $R = R_{\pi/4}$, which maps $|0\rangle \to |0\rangle$ and $|1\rangle \to i|1\rangle$, and H is the Hadamard transform.

a) Find the action of this circuit on the six $\overline{X}$ and $\overline{Z}$ logical basis operators.

b) Suppose the first and third qubits both start the circuit in the state $|0\rangle$. What is the overall effect of the circuit if we add a measurement of Z_1 at the end? What if instead we measure Z_2? What about Z_3? What if we measure all three sequentially? (Z_1, Z_2, and Z_3 refer to the Z operator on the first, second, and third qubits respectively.)

Problem #2. Equivalence of Stabilizer Entangled States

a) Consider the stabilizer state with stabilizer generators

$$\begin{array}{ccc} Y \otimes Y & \otimes & X \otimes X \\ Z \otimes Z & \otimes & Y \otimes Z \\ I \otimes X & \otimes & Y \otimes X \\ Z \otimes X & \otimes & I \otimes X \end{array}$$

Suppose the first two qubits are held by Alice and the second two qubits are held by Bob. Show that there are local Clifford group operations on Alice's side and on Bob's side that convert their state into two EPR pairs.

b) Show in general, for any number of qubits n_A and n_B held by Alice and Bob, that if Alice and Bob together have a stabilizer state, there are local Clifford group operations on Alice's side and on Bob's side such that the joint state becomes some number of EPR pairs plus qubits in the state $|0\rangle$ (which could be held by either person). (Hint: Consider the stabilizer of a number of EPR pairs and try converting Alice's side as much as possible to that form through Clifford operations and new choices of generators.)

Problem #3. Number of Stabilizer Codes

a) Suppose we have a stabilizer S on n qubits with r generators. Ignoring overall phase, how many Pauli operators are there that commute with S but are not in S? Suppose we pick an ordered sequence $M_1, \ldots, M_r$ of independent commuting Pauli operators, again ignoring global phases. How many ways are there to do this?

b) Part a does not tell you the total number of stabilizer codes with r generators since there are many ways to choose generators for a particular stabilizer code. Suppose we have a stabilizer S on n qubits with r generators. How many ways are there to pick an ordered set of generators $M_1, \ldots, M_r$?

c) Find the total number $N_{n,k}$ of stabilizer codes on n qubits with $n-k$ generators. Remember to include phases now. Show that $\log N_{n,k}$ is polynomial in n and k and determine the dominant term.