Problem Set #4

Quantum Error Correction
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Due Tues., Feb. 13, 2007

Problem #1. Using Bounds on QECCs

a) For this part of the problem, use only the quantum Gilbert-Varshamov bound, quantum Hamming bound, and quantum Singleton bound. For each of the following sets of parameters, say whether it must exist, may exist, or cannot exist, and if it can exist, decide whether it can be non-degenerate: [[15,5,3]], [[21,16,3]], [[9,2,5]].

b) Consider an encoding of a single qubit which gives 3 physical qubits to Alice, 2 physical qubits to Bob, and 1 physical qubit to Charlie. Is it possible to find such an encoding which corrects for the erasure of the qubits held by any single person? What about just one of Bob or Charlie? What about any two people? What about either Alice or both Bob and Charlie? In all cases, which of the possible erasures will take place is unknown at the time of encoding (e.g., for the last question, the encoder does not know whether Alice will be missing or if, instead, both Bob and Charlie will be missing). When the answer to one of these questions is "yes," give a possible encoding; when it is "no," give a proof.

Problem #2. Applying the Linear Programming Bounds

a) Find the weight enumerator for a $[[5, 1, 3]]$ code presented in class (either the one in stabilizer form or the GF(4) Hamming code version; they actually have the same weight enumerator). Use the quantum MacWilliams bound to find its dual weight enumerator.

b) Using the quantum MacWilliams identity, find the possible weight distributions for a $((4, 4, 2))$ code (i.e., one encoding 2 logical qubits in 4 physical qubits).