

Excerpt from SIGACT NEWS book review column
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Column Edited by William Gasarch

Joint review¹ of

Professor Stewart's Cabinet of Mathematical Curiosities

Author of Book: Ian Stewart

Basic Books, 310 pages, Softcover, \$17.50

and

Five Minute Mathematics

Author of Book: Ehrhard Behrends

AMS, 380 pages, Softcover, \$35.00

and

Aha Gotcha!- Aha Insight!

Author of Book: Martin Gardner

MAA, 180 pages, Hardcover, \$41.60

and

Origami, Eleusis, and the Soma Cube

Author of Book: Martin Gardner

Cambridge Press, 235 pages, Softcover, \$6.00

and

Hexaflexagons, Probability Paradoxes, and The Tower of Hanoi

Author of Book: Martin Gardner

Cambridge Press, 235 pages, Softcover, \$6.00

and

Group Theory in the Bedroom and Other Mathematical Diversions

Author of Book: Brian Hayes

Hill and Wang, 270 pages, Softcover, \$15.00

Author of Review: William Gasarch

1 Introduction

All of the books being reviewed are in the category *Mathematics for the layperson*. Hence the question to ask is not *Will I learn something I don't already know*. I suspect that 7/8 of my readers already know at least 3/4 of the material in these books. The question is *Would this book be a good gift for my mathematically-inclined great niece?*. This boils down to *are they well written?* and *is the choice of topics appropriate?*. Even if you know the material there may still be some pleasure in reading it to see how it would be presented to someone who does not. And there is indeed (at least for me) some new material of interest in all of them.

Three of the books have short articles and three of them have long articles. These are different types of books so I compare and contrast the first three to each other and the next three to each other. But to cut the suspense, these are all fine books.

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2 Review of Professor Stewart's Cabinet of Mathematical Curiosities

Professor Ian Stewart claims that when he was 14 he began keeping a notebook of math items he found interesting that were *not* being taught in school. He quickly needed to get another notebook, and eventually a cabinet of notebooks. Many years later this cabinet of notebooks became this book. I believe this story. He was writing to an audience of one. This works to his advantage—he didn't have to worry about whether the layperson will care about such-and-such. Often if you try to please only yourself you end up with a better book than one that is test marketed and focus grouped.

This book has 181 articles in 310 pages. Your great niece can tell you that most of the articles are short. Some are problems, some are stories, some are mathematics that is told to us without a problem. This flexibility frees the author to use which ever style is appropriate. For example, the 4-color theorem is worth talking about but not worth proving. The story of how Indiana almost passed legislation stating a value for π is included even though there is no problem involved. They follow this up with an amusing article about what would have happened had the law passed.

Some article are on topics familiar to most of my readers and appear in many other math books for the layperson: truth tellers and liars, counterfeit coins, Fibonacci numbers, and others. Some topics, while familiar to my readers, are not that common: why there is no Nobel Prize in Mathematics (the article debunks the usual myths but doesn't say why), the impossible of trisecting an angle with ruler and compass (with some of the math explained!), why $(-1)(-1) = 1$, and others.

There were some topics that I did not know before. I share two with my readers:

2.1 A Little Known Pythagorean Curiosity

If $a^2 + b^2 = c^2$ and $A^2 + B^2 = C^2$ then

$$(aA - bB)^2 + (aB + bA)^2 = (cC)^2.$$

Of more interest: Every Pythagorean triple can be "factored" into "prime" ones. They don't prove this, they just tell it to us.

2.2 Magic Square of Squares

A magic square is an $n \times n$ matrix of numbers such that every row, column, and both diagonals, have the same sum. These have been well studied. Is there such an object where all the entries themselves are squares?

The article tells us that for 3×3 there is an *almost* magic square of squares. Every row, column, and one diagonal add to the same sum. For 4×4 there is one (Euler knew this) and for 5×5 , 6×6 , and 7×7 Chritian Boyer found such. The 7×7 is particularly nice since it uses $0^2, 1^2, \dots, 48^2$.

Who is Chritian Boyer? The book does not say; however, a brief look at the web indicates that his discovery was made in 2005 and 2006. This points to one problem with this book that might bother you though not your great niece. There are few references. In the age of the web, this might not even bother you.

The article actually gave the magic squares.

3 Review of Five-Minute Mathematics

Professor Ehrhard Behrends writes a math column for the German newspaper *Die Welt*. This book is a collection of his columns translated into English. This book has exactly 100 articles in 370 pages. All of the articles tell you some math of interest. They are not math problems.

Much like *Professor Stewart's Cabinet* this is a mix of problems you have seen before that are common in such books (Barber's paradox, Birthday Paradox, Monty Hall Paradox, Hilbert's Hotel), some that you've seen before but are not that common (Crypto, Quantum Computing) and some that were new to me and might be to you (a chapter on Math Finance). But few of the chapters go into any depth; hence, even on topics that I did not know about I didn't learn much. Since he is writing for the masses he can't quite be as eclectic as Professor Stewart; nonetheless, this is a good book. For my great niece.

4 Review of Aha Gotcha!-Aha Insight!

Martin Gardner is well known for writing a Math Column in Scientific American from 1956 to 1981. He has several books consisting of collecting up columns and adding commentary. This *is not* one of those books (two of those books will be reviewed later in this column). He also wrote two books of short cute math items called *Aha! Gotcha* and *Aha! Insight*. The book under review is two-books-in-one.

There are 80 articles in 160 pages. The articles are even shorter than those numbers indicate since each article is accompanied by a cartoon. All of the articles claim to be about paradoxes. Whether that is true depends on how you define *paradox*. For example, I would not classify either (1) Hilbert's Hotel, nor (2) getting 5 heads in a row does not increase your chance of getting tails, as paradoxes.

There are 6 chapters all of which contain many articles. They are titled *Logic, Number, Geometry, Probability, Statistics, and Time*.

Many of the logic paradoxes are either variants on *The Liar's Paradox* or are about Truth-tellers and Liars. There was more variety in other chapters. There were a few things I did not already know. Many of the probability paradoxes, such as *The Wallet Paradox* are explained enough to intrigue, but not enough to enlighten. Granted, probability paradoxes are hard to unravel.

Some of the articles (though not many) asked the reader a question. The answers are in the last chapter.

5 Compare and Contrast Prof. Stewart's Cabinet, Five-Minute Math, and Aha!

All three books use the shortness and abundance of articles to their advantage: the articles makes their point and shut up (we can all learn from that). Which topics will be of interest to your great niece is hard to say; however, there are alot of different topics so surely something will appeal.

Professor Stewart's Cabinet will introduce a topic and is not timid about doing some of the math involved. This tends to work— they explain just enough of the math to engage the reader. The choice of topics is somewhat random, but this means there will be more topics that they have not seen elsewhere.

Five-Minute Mathematics will introduce a topic but not really go that far with it. Part of the reason is that it tends to do recent topics. Also, these articles originally appeared in the Newspaper so they had a wider audience.

Aha! seems to pack a lot of information into the 2 pages and a cartoon. Since these are reprints of older books these books do not have more recent topics. This does not matter one wit for your great niece. And it means that the topics are not too hard. For a more sophisticated reader the topics may be somewhat repetitive.

All three of these books are well written and are on a level that a High School Student who has had algebra and *already likes math* will be able to read. If your great niece does not already like math then you can use this book to find things to *tell her* that may intrigue her, but put off getting it as a gift.

Professor Stewart's Cabinet does not have an index, nor does it number its chapters. This makes it hard to find some things, but the table of contents helps pin things down pretty well. *Five Minute Mathematics* has a good index and chapter numbers. *Aha!* does not have an index, though it does have the articles in chapters laid out nicely.

One word of advice: I read these books straight through since I wanted to review them. That made me dizzy. Dip into them and read one article at a time—perhaps one a day, is a better way to appreciate them.

6 Review of Hexaflexagons... and Origami...

Martin Gardner is well known for writing a Math Column in Scientific American from 1956 to 1981. He has several books consisting of collecting up columns and adding commentary. These *are* the first two books!

The reader may have a sense of what the Klingons call *Nim Pah*, and the French call *Deja Vu*, and the English call *Deja Vu*. Didn't Martin Gardner already have these books reprinted recently? He did indeed! To paraphrase the *Preface to the Second Edition*

In 2005 when the MAA put all fifteen books on a CD, type was not reset. This severely limited what I could add to update the columns and expand bibliographies. Because Cambridge University Press is resetting type, I am now happily free to add as much fresh material as I please.

When I first read Martin Gardner's book (in 1975) one of the best things about them was that they included the original articles *and* updates, commentaries, etc. Hence I applaud the effort to update further.

The book *Hexaflexagons...* has 16 chapters and is roughly 190 pages. The book *Origami...* has 20 chapters and roughly 220 pages

Since Math is somewhat timeless, none of the material is dated. Some of it is often in other such books. For example, *Hexaflexagons...* has Probability Paradoxes, Towers of Hanoi, Fallacies, and NIM; and, *Origami...* has Platonic Solids, Phi, and Mazes. However, note that some of these topics appeared in popular form for the first time in Gardner's column. Criticizing the content for being overly familiar is like calling the original *Dracula*, *just another vampire story*. More important, he *still* has a different take on these subjects than in later books. For example, in *Hexaflexagons...* chapter 3, *Nine Problems*, one of the problems is the classic truth-tellers-and-liars. He actually discusses the issue of liars not being so clever or so naive (depending on your point of view) to fall for the old *if I was to ask you the following question what would you say* trick. More generally, after

reading it you can tell you are reading an original, not a rehash. Due to when they were written the books do not contain modern topics like Cryptography, P vs NP, or Mathematics of Finance.

One of the chapters in *Origami*... struck me as being about a problem that was once very popular but I haven't seen it recently. Its the *Monkey and the Coconuts* problem (only 268 hits on Google).

Five men and a monkey are on a desert island. The five men gather up coconuts and agree to split them up 5 ways the next day. In the middle of the night one of them gets up and splits them in 5 ways and has one left over. He gives the monkey the one and hides 1/5 of what is left for himself. Over the course of the evening the rest of the men do the same thing, always finding that the number of coconuts when divided by 5 leaves a remainder of 1. In the morning they split up the remaining coconuts and they divide by 5 exactly. What is the smallest amount of coconuts there could have been in the original pile.

I leave this to the reader; however, keep in mind that you should solve this with elementary methods. If you find yourself using Mathematica or advanced Diophantine analysis you may end up solving the problem but not in the spirit it is intended.

Many articles may be of interest even to the (mathematically sophisticated) readers of my column since (1) they are so well written, (2) they are classic, and (3) there is often a kernel of knowledge that you didn't quite know.

Some of the articles will interest you, some will not. But that is a matter of personal taste. However, I can personally attest to this being a great book for your great niece to learn some math of interest, since my great uncle gave it to his great nephew (me) back in 1975. And the rest is history.

7 Review of Group Theory in the Bedroom

Brian Hayes has a column in *American Scientist* on computer science and math. All but one of the articles in this book are columns he published there. The other one is from *The Sciences* which was a magazine of the New York Academy of Sciences, now defunct. The book has 12 articles in roughly 240 pages.

None of the old standards are here. No truth-tellers who play NIM with counterfeit coins. Hence I learned the most from this book by far.

I describe 5 of the 12 chapters.

Chapter *Clock of Ages* is about the Astronomical clock of Strasbourg which has kept the correct time for 160 years. How does it do such a good job. This chapter taught me things I didn't know.

Chapter *Randomness* was about the positive aspects of randomness in both algorithms and cryptography. It also discusses Quantum Computing and Quantum Crypto. Some of the Quantum material was new for me.

Chapter *The Easiest Hard Problem* begins as follows: A group of (say) 18 friends want to divide up to play a baseball game. They all want to have roughly equal teams. Two captains are chosen. They then alternate choosing people for their teams. What of it? They are trying to solve the *subset sum* problem which is NP complete! The chapter is on NP completeness but also on when they happen to have easy instances, or ones that are easy to approximate. The choose algorithm for teams works pretty well.

Chapter *Third Base* is on base 3. Base 3? Yes Base 3! Base 3 has some properties that make it mathematically interesting. There is more here than I thought.

The chapter that the book gets its title from, *Group Theory in the Bedroom*, is about flipping or rotating your mattress so that all parts of it get equal wear and tear. Note that the set of rotations and flips forms a group. If your mattress is a perfect square with no distinguishing marks, and you flip or rotate it at random, will the wear and tear be evenly divided? Do the mattress companies care? This chapter should be incorporated into group theory courses which often lack motivating examples until late in the semester.

8 Comparing and Contrasting Hexaflexagons . . . , Origami . . . , Group Theory . . .

All three of these books are excellent. There is sparingly little overlap since *Group Theory in the Bedroom* seems to intentionally focus on modern topics, whereas the books by Martin Gardner are, of course, old books and hence have old topics.

But I am not going to pick a winner. These are all good books, well written, correct, and worth reading. Any would be fine for your great niece. And *Group Theory in the Bedroom* might teach you a thing or three.