Excerpt from SIGACT NEWS book review column
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Column Edited by William Gasarch

Joint Review of[1] of
The Mathemagician and Pied Piper:
A Collection in Tribute to Martin Gardner
Edited by Elywn Berlekamp and Tom Rodgers
Published by A.K.Peters, 1999
265 pages, Hardcover, $40.00

Puzzlers' Tribute:
A Feast for the Mind
Edited by David Wolfe and Tom Rodgers
Published by A.K.Peters, 2002
420 pages, Hardcover, $40.88

Tribute to a Mathemagician
Edited by Barry Cipra, Erik Demaine, Martin Demaine, Tom Rodgers
Published by A.K.Peters, 2004
262 pages, Hardcover, $40.00

Mathematical Wizardry for a Gardner
Edited by Ed. Pegg Jr, Alan Schoen, Tom Rodgers
Published by A.K.Peters, 2005
262 pages, Hardcover, $40.00

A Lifetime of Puzzles:
A Collection of Puzzles in Honor of Martin Gardner's 90th Birthday
Edited by Erik Demaine, Martin Demaine, Tom Rodgers
Published by A.K.Peters, 2008
262 pages, Hardcover, $40.00

Homage to a Pied Puzzler
Edited by Ed. Pegg Jr, Alan Schoen, Tom Rodgers
Published by A.K.Peters, 2009
262 pages, Hardcover, $50.00

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1 Introduction

Martin Gardner is well known for writing a Math Column in Scientific American from 1956 to 1981. His influence on mathematics is immense. Many (including me) have realized that mathematics can be fun and interesting via his columns. Alas he passed away at the age of 95 in 2010.

To show their appreciation there have been several *Gatherings for Gardner* which are gatherings of mathematicians, puzzlers, and magicians. They give talks on work that is related to what Gardner has written about- recreational mathematics which sometimes connects to more serious topics. The first two gatherings were in 1993 and 1995. They had no proceedings. However, the ones since 1998 have. We give reviews of the six proceedings that are available.

All of the proceedings have a similar character. They all have a large variety of types of articles: math problems, recreational math, history of math or puzzles or magic, and some personal stories about Martin Gardner and other mathematicians. One proceeding had an article by Frank Harary, and a later one had an article about him in Memoriam

The writing quality of the articles is surprisingly good for a collection of articles by different people. Some topics intrigued me and others did not; however, when a topic did not intrigue me it may well intrigue someone else.

I will review each book separately; however, since they are similar (this is not a criticism) I will give (roughly) less detail on a book the further down the list they are. I will then review them as a whole.

2 The Mathemagician and Pied Puzzler

Personal Magic: This section has chapters about Martin Gardner directly.

Martin Gardner: "A Documentary" by Dana Richards is a documentary in words. It is a collection of facts and quotes about and by Martin Gardner from various interviews. In just 10 pages this tells how he got to be where he is and what kind of man he was. *Ambrose, Gardner, and Doyle* by Raymond Smullyan is a science fiction story about people in the future looking back at an article that Martin Gardner had written. I would call it a satire of academia except that it is too close to being true.

Puzzlers

Card Game Trivia by Stewart Lamle is a one page list of facts about cards. Here is one: The Joker was devised by a Mississippi riverboat gambler to increase the odds of getting good poker hands. *Creative Puzzle Thinking* by Nob Yoshigahara is a list of 11 puzzles without the answers. Here is an example:

Which two numbers come at the end of this sequence?

2,4,5,30,32,34,36,40,42,44,46,50,52,54,56,60,62,64,x,y

Number Play, Calculators, and Card Tricks: Mathemagical Black Holes by Michael W. Ecker. Take a number, say 100. Now write down all of its divisors (include 1 and the number itself): $1, 2, 4, 5, 10, 20, 25, 50, 100$. Now take the sum of the digits of the divisors: $1 + 2 + 4 + 5 + 1 + 0 + 2 + 0 + 2 + 5 + 5 + 0 + 1 + 0 + 0 = 28$.

Now repeat the process. The divisors of 28 are $1, 2, 4, 7, 14, 28$. The sum of the digits of the divisors is $1 + 2 + 4 + 7 + 1 + 4 + 2 + 8 = 29$.

The divisors of 29 are $1, 29$. The sum of the digits of the divisors is $1 + 2 + 9 = 12$.

The divisors of 12 are $1, 2, 3, 4, 6, 12$. The sum of the digits of the divisors is $1+2+3+4+6+1+2 = 19$.

The divisors of 19 are $1, 19$. The sum of the digits of the divisors is $1 + 1 + 9 = 11$.

The divisors of 11 are $1, 11$. The sum of the digits of the divisors is $1 + 1 + 1 = 3$.

The divisors of 3 are $1, 3$. The sum of the digits of the divisors is $1 + 3 = 4$.

The divisors of 4 are $1, 2, 4$. The sum of the digits of the divisors is $1 + 2 + 4 = 7$.

The divisors of 7 are $1, 7$. The sum of the digits of the divisors is $1 + 7 = 8$.

The divisors of 8 are $1, 2, 4, 8$. The sum of the digits of the divisors is $1 + 2 + 4 + 8 = 15$.

The divisors of 15 are $1, 3, 5, 15$. The sum of the digits of the divisors is $1 + 2 + 4 + 8 = 15$.

AH-Ha- 15 goes to itself. Of more interest is that *every number eventually goes to 15*. This is called a *black hole*. This chapter discusses many such black holes and applies them to a magic trick.

Puzzles From Around the World by Richard Hess. This chapter has 17 easy problems, 19 medium problems, 20 hard problems, and solutions to all of them. Here is a Hard Problem:

An irrational punch centered on point p in the plane removes all points that are an irrational distance from p. How many irrational punches are needed to remove all points from the plane.

Mathemagics

Misfiring Tasks by Ken Knowlton. There are theorems in math that hold for all but a finite number of numbers or even all but one number. Here is one (not mentioned in the article) *Every number is the sum of at most 8 cubes except 23 which needs 9.* This chapter was about such sequences. One point of interest is that it's hard to define rigorously.

Some Diophantine Recreations by David Singmaster. This is actually a nice history of Diophantine problems. Here is an old one that seems to recur alot: Three sisters sell apples in the marketplace. The oldest one sells 50, the middle one sells 40, the youngest one sells 10. They each bring home the same amount of money. How is that possible? Well, they each sold the apples at two different prices (one price in the morning, and a lower price in the afternoon when they wanted to make sure they cleared their stock). How much did they charge and how much did they bring home? There may be several answers so other constraints may be given. You can replace the three sisters with any number of sisters and the numbers 50, 40, 10 with other numbers.

Who Wins Misere Hex? by Jeffery Lagarias and Daniel Sleator. Hex is a well known game where it is know that player I wins. There is a parameter n- the size of the board. Misere Hex is (as is usual for Misere games) reverses the criteria for who wins and loses. This chapter solves the game completely! Player I wins iff n is even. They also show (it is part of the proof) that whoever loses can force the game to be played to the bitter end.

How Random are $3x + 1$ *Iterates?* by Jeffery Lagarias. Consider the function (1) $f(x) = x/2$ if x is even, (2) $f(x) = 3x + 1$ if x is odd. The *Collatz Conjecture* states that for all $x \in N$ the sequence $f(x), f(f(x)), \ldots$ will eventually reach 1. (Note that $f(1) = 4$, $f(4) = 2$, $f(2) = 1$ so the sequence will be periodic.) This is still open and seems hard. Jefferey Lagarias is an expert on the problem (see his website of papers on it). In this paper he looks at estimations of when the sequence gets to 1.

3 Puzzlers' Tribute

The Toast Tributes This section has many chapters that are toasts to people who have recently passed away. Some are biographical, some are problem sets. All are touching.

Tantalizing Appetizers: Challenges for the Reader.

A Clock Puzzle by Andy Latto. There is a clock where the hour hand and the minute hand are the same length. Note that I can usually still tell what time it is. For example if one hand is on the 12:00 and the other on the 3:00 then it has to be 3:00 and not 12:15, since at 12:15 the hour hand would not be exactly on the 12:00. How many times of the day can I NOT tell what time it is?

Six Off-Beat Chess Problems by John Beasley. Usually in a chess problem it is not important to know what the *prior* move was. For all six of these problems you can deduce something about the prior move, and what you deduce is important. These problems require more cleverness than chess ability. And they are all fun! (Solutions are included).

Smoked Ham: A Course in Magic I am not sure what the title of this section means. However, the articles in it are about magic. Most of them are about cards, dice, and knots. One is a nice poem *Casey at the Fox* (The Fox is a magic show) about a magician who disappoints.

Chef's Caprice This section does not have much of a theme. One interesting article in it was *Coincidences* by A. Ross Ecker who investigates using math whether some phenomena is a con incidence or not (he admits the term is not that well defined). For example, Abraham Lincoln and Charles Darwin were born on the same day. Is that a coincidence? What is the probability that two famous people will be born on the same day? He deduces that the probability is high enough that we should not be surprised it happened.

Wild Games (and puzzles)

Early Japanese Export Puzzles: 1860's to 1960's by Jerry Slocum and Rik van Grol is a historical review of such puzzles. It is interesting though there is not much math in it. *The Partridge Puzzle* by Robert Wainwright is the following: Recall that

$$1 \cdot 1^2 + 2 \cdot 2^2 + 3 \cdot 3^2 + \cdots + i \cdot i^2 = N^2$$

where $N = \frac{i(i+1)}{2}$. Hence you might be able to use one 1×1 tile, two 2×2 tiles, ..., i $i \times i$ tiles to tile the $N \times N$ grid. For which i can you do this? For $i = 2, 3, 4, 5$ this is not possible. For $i = 11$ such a tiling is possible. The cases of $i = 6, 7, 8, 9, 10$ are unknown. The article discusses all of this and variants.

Mathematical Entrees and Mathematical Treats (two different but similar sections)

Fermat's Last Theorem and the Fourth Dimension By Jim Propp gives a nice history of FLT including Wiles result. It is probably the most sophisticated mathematical article in all 6 books. *Games People Don't Play* by Peter Winkler was an excellent series of games. I originally thought people didn't play them because they weren't real games (most uses of the word *game* in mathematics are not really fun- see my blog entry http://blog.computationalcomplexity.org/2010/06/whats-your-game-mr-bond-sequel.html). But actually these are games where one of the player has a an unfair advantage, which is why they are not played. Here is an example: Paula writes down two integers on two different pieces of paper. They must be different. Victor picks one of the pieces and looks at it. He then bets \$1.00 that either the number he has seen is bigger or smaller than the number he has not seen. Does Victor have a strategy that is better than 1/2 probability of being right. Surprisingly yes. *Some Tricks and Puzzles* by Raymond Smullyan contains his usual brand of paradoxes. A nice light read. *How Recreational Mathematics can Save the World* by Keith Devlin is about how applied pure math can be (or perhaps how pure applied math can be). He does end up with some real applications- of hard math. In the end, alas, Recreational math cannot save the world. *Sum Free Games* by Frank Harary are games where players pick numbers and either hope to achieve or avoid being the one who causes there to be 3 numbers x, y, z where $x + y = z$. This kind of game has its roots in Ramsey Theory.

4 Tribute to a Mathemagician

This books section titles are **In Memoriam**, **Braintreasures**, **Brainticklers**, **Brainteasers**, **Braintempters**, **Braintauters**, **Braintools**. I will dispense with telling you which chapters I describe came from which section. Telling the different between a Braintempter and a Braintauter is an unsolved Brainteaser.

The Incredible Swimmer Puzzle by Stewart Coffin is a charming story about the puzzle and includes the puzzle itself. Apparently Stewart made up the puzzle but it had a typo and this lead to confusion that was finally straighted out after about 40 years. *A History of the Ten-Square* by A. Ross Eckler is about the quest for a 10 by 10 grid of letters so that every row and column is a word. Or Phrase. Or something. They give evidence that if you use a standard dictionary you will not succeed. However, they manage with larger sources of words such as peoples names or places. Lower order squares were known and found by hand; however, the 10-square was only possible with modern computers.

Underspecified Puzzles by David Wolfe and Susan Hirshberg are really really underspecified. Fortunately solutions were provided and were amusing. *Five Algorithmic Puzzles* by Peter Winkler is, as the title says, Five Algorithmic Puzzles. Most involved a parameter that was not obvious that you had to note was always increasing. *Upstart Puzzles* by Dennis Shasha is about puzzles where the originator had a solution in mind for the original puzzle, which is correct, but more advanced forms the originator does not know the solution. In some cases nobody does.

The Burnside Di-Lemma: Combinatorics and Puzzle Symmetry by Nick Baxter blurs the line between recreational and serious mathematics. Combinatorics in general does that. Usually recreational mathematics uses simple tools. The Burnside Lemma is not a simple tool. However, this article presents it well. For those who do not know, Burnside's Lemma uses group theory to count objects that have unusual symmetries. The classic example is counting the symmetries of a cube that has 3 sides RED and 3 sides BLUE (it matters where the sides are).

5 Mathematical Wizardry

The Ig Nobel Prizes by Stanley Eigen is about the Ig Nobel Prize which goes to science that makes us laugh but also think. The criteria is somewhat loose in that it goes to false science (e.g., Calculating the odds that Mikhail Gorbachev is the Anti-Christ) and to funny but valid science (e.g., an experiment involving a black tar in a funnel which outputs one drop every nine years.)

Packing Equal Circles in a Square by Peter Gabor Szabo talks about the following problem: We want to put n overlapping identical circles into a square of side 1. What is the max radius? For $1 \le n \le 30$ the answers are known. Some use some math of interest and some use computer work. Some of the numbers are nice ($r_4 = 0.25$) but some are nasty $r_3 = \frac{8-5\sqrt{2}+4\sqrt{3}-3\sqrt{6}}{2}$. r_{11} is far nastier in that it involves square roots of linear combinations of square roots; however, r_{19} is even worse since we only know that it is the root of a particular polynomial of degree 10. This chapter tells us some information but also has good references. This is appropriate since this material involves serious math.

Uncountable Sets and an Infinite Real Number Game by Matthew Baker gives an alternative proof that the reals are uncountable via games. He also uses games to prove other theorems in analysis.

The Association Method for Solving Certain Coin-Weighting Problems by Dick Hess gives a *general method* that works to solve several coin weighting problems. The classic version, 12 coins one is counterfeit (heavier or lighter but we don't know which) and a balance scale, falls out of his methods. He looks at different numbers-of-coins and different types of weightings.

6 A Lifetime of Puzzles

The first section has four articles on magic. Surprisingly (at least to me) is that one of these articles had serious mathematical content. Diaconis and Graham have an article *Products of Universal Cycles* where they look at generalizations of de Bruijn cycles and apply them to magic. A k-ary de Bruijn cycle (also called a de Bruijn sequence) is a sequence of 0's and 1's such that each window of width k shows a different element of $\{0,1\}^k$. For example 10111000 is a 3-ary de Bruijn sequence. How should this be defined if you have more than 2 letters in your alphabet? You might think that if your alphabet is $\Sigma = \{0,1,2,4\}$ and $k = 3$ you want different elements of Σ^3. Not quite. You want different orderings. For example, if you have already seen 124 you do not want to see another 3-sequence that is increasing. As an example 132134 is a 3-ary de Bruijn cycle. These object seem to be very interesting mathematically and apply to Magic.

Can Ants be interesting mathematically? Peter Winkler thinks so. He has a chapter *The Adventures of Aunt Alice* that is a sequence of problems involving ants. Here is one: *Twenty-five ants are placed randomly on a meter-long rod, oriented east-west. The thirteenth ant from the west end of the rod is our friend Aunt Alice. Each ant is facing east or est with equal probability. They proceed to march forward (that is, in whatever direction they are facing) at 1 cm/sec; whenever to ants collide, they reverse directions. How long does it take before we can be certain that Alice is off the rod?* There are 10 puzzles of this type in the chapter, with the answers. They are of the type that once you see the answer it is obvious but before that it might not be.

7 Homage to a Pied Puzzler

Martin Gardner called Sam Loyd *America's Greatest Puzzlist.* Loyd is credited with many puzzles and indeed deserves credit for ... some of them. Jerry Slocum in *Sam Loyd's Most Successful Hoax* gives strong evidence that Sam Loyd did not invent the 15-puzzle which is often attributed to him. This is not just people attributing the puzzle to Sam Loyd after the fact. Sam Loyd himself claimed to have invented it. I am not quite sure what to make of it. On the one hand, it is immoral to claim to be a puzzle's inventor when you are not. On the other hand, this is a a Hoax, and that is what puzzlers do.

There are seven articles in this book with the word *seven* in the title. I doubt this is a coincidence. One of them is *Seven Staggering Sequences* by Neil Sloane. I describe one that is truly staggering. Let $b(n)$ be defined as follows:

- $b(1) = 1$.
- To find $b(n+1)$ you write down
$$b(1), b(2), \ldots, b(n)$$
and find an X, Y sequences of naturals, $k \in N$ such that
$$b(1), b(2), \ldots, b(n) = XY^k$$
and k is as large as possible. Let $b(n+1) = k$.

We work out the first few.

1. $b(1) = 1$

2. $b(2) = 1$ using X is the empty string, $Y = 1$, and $k = 1$.

3. Look at the sequence $b(1), b(2) = 1, 1$. Let X be the empty string, $Y = 1$, and $k = 2$. So $b(3) = 2$.

4. Look at the sequence $b(1), b(2), b(3) = 1, 1, 2$. Let $X = 1, 1$, $Y = 2$, $k = 1$, So $b(4) = 1$.

The first few terms are

$$1, 1, 2, 1, 1, 2, 2, 2, 3, 1, 1, 2, 1, 1, 2, 2, 2, 3, 2, 1, 1, 2$$

Does 4 ever appear? Yes, $b(220) = 4$. Does 5 ever appear? Yes. They proved (in a different article) that there is some $x < 10^{10^{23}}$ such that $b(x) = 5$. More generally they prove that you get x within roughly $2^{2^{3^{\cdots m}}}$.

Only one word describes such a sequences: Staggering!

8 Opinion

There are chapters on a large range of topics: Magic, History, Recreational Mathematics, Serious Mathematics, and some articles that are hard to classify. Clearly the line between recreational and serious mathematics is hard to draw; however, in this book, the line between magic and math of any sort is also hard to draw.

Even though there is a large range of topics they all have a certain spirit. They are all fun! (The articles in Memoriam may be an exception. Or not.) They are all by authors who love their subject and want to share it with you. And, as noted 6 pages ago, they are well written.

How can you, my readers who are somewhat sophisticated mathematically, use these books?

- Many of the chapters can be the basis for class projects on a variety of levels.
- The math presented in these books is pretty easy but the pointers to other papers and open problems is at times deeper.
- There is so much variety in these books that I suspect there are mathematical things in them that you simply did not know and can learn. So do so!