Review of

Infinitesimal

How a dangerous mathematical theory shaped the modern world[1]

by Amir Alexander

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1 Introduction

We have probably all heard the following:

- In the late 1600's Newton, Leibniz, others based calculus on infinitesimals which were not rigorous, but calculus worked
- In the early 1800's Cauchy, Dedekind, and others put calculus on a rigorous basis with the formal definition of the reals and of limits.
- Abraham Robinson made infinitesimals rigorous in the 1960's using techniques of model theory that were not avaiable to the mathematicians in either the 1600's or the 1800's.

Hence infinitesimals hardly seem like *a dangerous mathematical theory*. This book does not really talk about those historical episodes. Instead it talks about a slightly earlier time when some people thought reasoning using infinitesimals was *dangerous*.

How could a non-rigorous method of proof that seemed to produce the right answers be dangerous? One might wonder if such techniques can be made rigorous, or one can wonder if in some sense they already are rigorous, or one can also wonder if the results proven really are true. But this would seem to be a discussion among mathematicians, and hardly dangerous.

That last paragraph was written by a 21st century person (me). To truly understand how a mathematical theory can be considered dangerous in the mid 1500's we need to know the history and context. This book provides that as it tells two tales:

- How infinitesimals lost the battle of ideas in Italy.
- How infinitesimals won the battle of ideas in England.

2 Summary

2.1 Part I: The War Against Disorder: The Jesuits Against the Infinitely Small

Before the reformation most (perhaps all) Christians were Catholic. The reformation (1517) cast doubt on long held religious beliefs. How could the Catholic Church regain its position? The Jesuits were formed (1540) to strengthen the Catholic position by teaching and argument. They founded many colleges and succeeded to some extend. Their colleges would look very strange from a modern viewpoint: they *discouraged* original thought since there was no room for debate on the

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truth which was absolute. Expositions of older work (e.g., Euclid) were fine but original research was frowned upon.

They sought to put theology on as rigorous a basis as mathematics. Hence any mathematics that was not rigorous was considered dangerous. Several Italian mathematicians (including Galileo) used infinitesimals in a variety of ways and their work was condemned. The reasons for condemning it were a strange mix of theology and mathematics- topics that are separate today but were more connected back then. In the end the theory died in Italy.

2.2 Part II: Leviathan and the Infinitesimal

The English Civil war devastated England and raised many questions about what is the best form of government. David Hobbes thought he had the answer. In his book *Leviathan* he argues that the best government is one which has an absolute ruler and absolutely no dissent. That way there could be no rebellions or civil wars. He also thought that this could be proven *mathematically*. Moreover he thought that in order to do this, geometry must be able to resolve every question. Hence he worked on the problem of using a straightedge and compass to square the circle and had a construction that he was convinced worked (it did not, and in fact the problem is impossible). As a 21st centuery person I think that either squaring the circle or proving it could not be done would settle the question; however, I doubt he would see it that way.

He despised infinitesimals since they did not fit how he thought proofs should go. Unlike the Jesuits, his arguments against infinitesimals were math-based. However, his arguments did not carry the day. Partially because he also defended his incorrect construction of squaring the circle and partially because John Wallis, a far better mathematrician, was using infinitesimals with great success. In the end infinitesimals were accepted in England.

2.3 What to Make of All This?

In the mid 1500's if you were to ask which if Italy or England would go on to make great contributions to Mathematics and Science, the answer would clearly be Italy. What went wrong? The author claims that Italy's rejection of infinitesimals, and England's embracing of them, was the cause. This seems a bit simplistic; however, there is clearly some truth in it. It may also be that the kind of society that lets religion (or other non-science factors) dictate science will fall behind those that do not.

2.4 What I Hope is in a Sequel

In the 1600's the Church had an opinion about techniques of proof in mathematics. In the late 1800's Cantor inquired of the Catholic church if they were okay with his theory of cardinals. They were. I wonder if they cared. I'd be curious what the Catholic Church would have thought of the Banach-Tarski paradox had it been discovered in the 1600's.

Doren Zeilberger (whose blog entry about this book inspired me to review it) believes that experimental mathematics can and should be the basis of proofs. What does the Catholic Church thing of this? We cannot imagine they have an opinion! In short, in 2014 the Church does not have an opinion about techniques of proof in mathematics.

By the intermediate value theorem there must exist a year $1600 \leq x \leq 2014$ such that in year x the Church cared but in year $x+1$ they did not (I know that this is simplistic). I would want to

see a sequel that describes when the Catholic Church stopped caring about these things.

3 Opinion

This is a history book with some math in it (of interest!). Very little math background is needed for the book; however, one has to care about the issues involved.

The book tells a fascinating story of an age when some people thought that (1) mathematical proofs can apply to religion and politics, and (2) once this is done ther opinion will be proven correct. These points of view look absurd today; however, we must keep in mind that we have the benefit of their failures to learn from.