Equitable Allocationss

William Gasarch-U of MD

An Envy Free Allocation

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	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	**2**	**2**
B	1	1	**2**	**2**	3	3
C	**6**	**4**	1	1	0	0

An Envy Free Allocation

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	2	2	2	**2**	**2**
B	1	1	**2**	**2**	3	3
C	**6**	**4**	1	1	0	0

A's View: A gets 4.

An Envy Free Allocation

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	2	2	2	**2**	**2**
B	1	1	**2**	**2**	3	3
C	**6**	**4**	1	1	0	0

A's View: A gets 4. B gets 4.

An Envy Free Allocation

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	2	2	2	**2**	**2**
B	1	1	**2**	**2**	3	3
C	**6**	**4**	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

An Envy Free Allocation

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	2	2	2	**2**	**2**
B	1	1	**2**	**2**	3	3
C	**6**	**4**	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4.

An Envy Free Allocation

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	2	2	2	**2**	**2**
B	1	1	**2**	**2**	3	3
C	**6**	**4**	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4.

An Envy Free Allocation

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	2	2	2	**2**	**2**
B	1	1	**2**	**2**	3	3
C	**6**	**4**	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

An Envy Free Allocation

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	2	2	2	**2**	**2**
B	1	1	**2**	**2**	3	3
C	**6**	**4**	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.
B's View: A gets 4. B gets 4. C gets 2.
C's View: A gets 0.

An Envy Free Allocation

	I_1	I_2	I_3	I_4	I_5	I_6
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B's View: A gets 4. B gets 4. C gets 2.
C's View: A gets 0. B gets 2.

An Envy Free Allocation

	I_1	I_2	I_3	I_4	I_5	I_6
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A's View: A gets 4. B gets 4. C gets 4.
B's View: A gets 4. B gets 4. C gets 2.
C's View: A gets 0. B gets 2. C gets 10.

An Envy Free Allocation

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A's View: A gets 4. B gets 4. C gets 4.
B's View: A gets 4. B gets 4. C gets 2.
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Nobody is envious. So is there still a reason to be unhappy? Discuss.

What If They Find Out The Others Val?

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A's View: A gets 4. B gets 4.

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A's View: A gets 4. B gets 4. C gets 4.
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Later A finds out C's valuation–realizes $V_C(I_1 \cup I_2) = 10$.

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A's View: A gets 4. B gets 4. C gets 4.
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Later A finds out C's valuation–realizes $V_C(I_1 \cup I_2) = 10$.
$V_C(I_1 \cup I_2) > V_A(I_5 \cup I_6)$. So A is mad!

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Later A finds out C's valuation–realizes $V_C(I_1 \cup I_2) = 10$.
$V_C(I_1 \cup I_2) > V_A(I_5 \cup I_6)$. So A is mad!

Editorial Note I think A is being petty and stupid.

Equitable Allocations

Def The allocation is **Equitable** if $V_A(P_A) = V_B(P_B) = V_C(P_C)$.

Can You Give an Equitable Allocation

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Consider this valuation

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Consider this valuation

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Can You Give an Equitable Allocation

Consider this valuation

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Do With Neighbor Find an Equitable Allocation.

Can You Give an Equitable Allocation

Consider this valuation

	I_1	I_2	I_3	I_4	I_5	I_6
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Do With Neighbor Find an Equitable Allocation.

Then Ask Are A, B, C all happy with this?

An Equitable Allocation

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A's view: A gets 4.

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A's view: A gets 4. B gets 4.

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A's view: A gets 4. B gets 4. C gets 4.

B's view: A gets 4.

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A's view: A gets 4. B gets 4. C gets 4.
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C's view: A gets 6.

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A's view: A gets 4. B gets 4. C gets 4.
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C's view: A gets 6. B gets 4. C gets 4.

Everyone gets 4 in their own view so equitable.

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B	1	1	**2**	**2**	3	3
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A's view: A gets 4. B gets 4. C gets 4.
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Everyone gets 4 in their own view so equitable. **Yeah!**

An Equitable Allocation

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Everyone gets 4 in their own view so equitable. **Yeah!**

C envies A so not envy-free

An Equitable Allocation

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A's view: A gets 4. B gets 4. C gets 4.
B's view: A gets 4. B gets 4. C gets 4.
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Everyone gets 4 in their own view so equitable. **Yeah!**

C envies A so not envy-free **Boo!**

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A's view: A gets 4. B gets 4. C gets 4.
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C's view: A gets 6. B gets 4. C gets 4.

Everyone gets 4 in their own view so equitable. **Yeah!**

C envies A so not envy-free **Boo!**

Do With Neighbor Find an allocation that is both envy-free and equitable?

An Allocation that is Both Equitable and Envy-Free

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A	2	**2**	**2**	**2**	2	2
B	1	1	2	2	**3**	**3**
C	**6**	4	1	1	0	0

An Allocation that is Both Equitable and Envy-Free

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	**2**	**2**	**2**	2	2
B	1	1	2	2	**3**	**3**
C	**6**	4	1	1	0	0

A's view: A gets 6.

An Allocation that is Both Equitable and Envy-Free

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	**2**	**2**	**2**	2	2
B	1	1	2	2	**3**	**3**
C	**6**	4	1	1	0	0

A's view: A gets 6. B gets 4.

An Allocation that is Both Equitable and Envy-Free

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	**2**	**2**	**2**	2	2
B	1	1	2	2	**3**	**3**
C	**6**	4	1	1	0	0

A's view: A gets 6. B gets 4. C gets 2.

An Allocation that is Both Equitable and Envy-Free

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	**2**	**2**	**2**	2	2
B	1	1	2	2	**3**	**3**
C	**6**	4	1	1	0	0

A's view: A gets 6. B gets 4. C gets 2.

B's view: A gets 5.

An Allocation that is Both Equitable and Envy-Free

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	**2**	**2**	**2**	2	2
B	1	1	2	2	**3**	**3**
C	**6**	4	1	1	0	0

A's view: A gets 6. B gets 4. C gets 2.
B's view: A gets 5. B gets 6.

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	I_1	I_2	I_3	I_4	I_5	I_6
A	2	**2**	**2**	**2**	2	2
B	1	1	2	2	**3**	**3**
C	**6**	4	1	1	0	0

A's view: A gets 6. B gets 4. C gets 2.
B's view: A gets 5. B gets 6. C gets 1.

An Allocation that is Both Equitable and Envy-Free

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	**2**	**2**	**2**	2	2
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A's view: A gets 6. B gets 4. C gets 2.
B's view: A gets 5. B gets 6. C gets 1.
C's view: A gets 6.

An Allocation that is Both Equitable and Envy-Free

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C	**6**	4	1	1	0	0

A's view: A gets 6. B gets 4. C gets 2.
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C's view: A gets 6. B gets 0.

An Allocation that is Both Equitable and Envy-Free

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	**2**	**2**	**2**	2	2
B	1	1	2	2	**3**	**3**
C	**6**	4	1	1	0	0

A's view: A gets 6. B gets 4. C gets 2.
B's view: A gets 5. B gets 6. C gets 1.
C's view: A gets 6. B gets 0. C gets 6.

This is equitable and envy free