Moving Knife Protocols

William Gasarch-U of MD

What is a Moving Knife Protocol?

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Even so We won't be defining **Discrete Protocol** or proving the $cn \log n$ lower bound here.

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MK can do a Protocol that Discrete Cannot

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Even So Is there a 2-person protocol where both get exactly $\frac{1}{2}$?

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Bonus Later this protocol will be used within an Envy-Free protocol.

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Unknown For $n \geq 5$ no MK protocol is known where the number of cuts is reasonable.

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1) Ref moves knife until someone yells (when they think $V(P) = \frac{1}{3}$). We will assume A yelled.
2) Cake is cut $P \cup Q$. Note $V_A(P) = \frac{1}{3}$, $V_A(Q) = \frac{2}{3}$, $V_B(P), V_C(P) \le \frac{1}{3}$, $V_B(Q), V_C(Q) \ge \frac{2}{3}$.
3) One of B, C is determined by coin flip. We assume B.
4) Do $\mathrm{H}(A, B; Q)$ to create Q_1, Q_2. Note
$V_A(P) = V_A(Q_1) = V_A(Q_2) = \frac{1}{3}$.
$V_B(P) \le V_B(Q_1) = V_B(Q_2)$.
5) C picks one of P, Q_1, Q_2. No envy since went first.
6) B picks one of whats left. No envy since
$V_B(P) \le V_B(Q_1) = V_B(Q_2)$.
7) A gets whats left. No envy since
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On Worksheet you will prove the protocol works.

3-Person MK Envy Free Protocol: Maybe Simpler

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We are cutting a pie so its a circle.

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1) A takes 3 knives and has each one go out from the center of the cake to the edge (wedges). She rotates them clockwise (keeping wedges equal).

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5) B takes. No envy since thinks two tied for biggest.

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6) A takes. No envy since thinks all pieces equal.

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Is this simpler? Discuss.

4-Person MK Envy Free Protocol

PHASE ONE:

4-Person MK Envy Free Protocol

PHASE ONE:
1) Do $\mathrm{HH}(A, B; P)$.

4-Person MK Envy Free Protocol

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2) C trims (or not) a piece (create tie for max). Put trim T aside.

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2) C trims (or not) a piece (create tie for max). Put trim T aside.

3) D pick a piece. No envy since went first.

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3) D pick a piece. No envy since went first.
4) C picks a piece. Take trimmed piece if can. No envy: tie for biggest.

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5) B picks a piece, A picks a piece. No envy since all pieces except trimmed are $\frac{1}{4}$.

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6) Still have to deal with Trim on next page.

4-Person MK Envy Free Protocol

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PHASE TWO: We assume C got a trimmed piece.
A and B do not care how much of T C gets.

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