Solutions to Worksheet on Discrete Envy Free Protocols

William Gasarch-U of MD

PROBLEM ONE

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A, B, C, D, E are the players.

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A, B, C, D, E are the players. P is the pie.

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Find a protocol FIRST-PLAYER-HAPPY where:

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1) All but T of the pie is allocated.

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3) The allocation is envy-free.

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Find a protocol FIRST-PLAYER-HAPPY where:

1) All but T of the pie is allocated.

2) 1st player gets $\geq X$ (You fill in the X)

3) The allocation is envy-free.

4) We do not know what B, C, D, E think of T.

First Attempt at Solution

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1) A cuts P into 6 pieces (all equal).

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2) B trims ≤ 3 pieces (create 4-way tie for biggest)

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Trimming put aside.
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Can't Work B, C, D might have trimmed all 6 pieces.

Second Attempt at Solution

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1) A cuts P into 7 pieces (all equal).

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Trimming put aside.
4) D trims ≤ 1 piece (creates tie for biggest)
5) E takes a piece. Went first so no envy.
DOES NOT WORK If E took untrimmed then all the rest are trimmed so A will get trimmed piece.

Third Attempt at Solution

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1) A cuts P into 8 pieces (all equal).

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DOESN"T WORK Scenario:

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B trims 3 pieces so have P_1', P_2', P_3', P_4 tied for biggest.

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DOESN"T WORK Scenario:
B trims 3 pieces so have P_1', P_2', P_3', P_4 tied for biggest.
C trims 2 P_1' and P_2'. So B only like P_3' and P_4.

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C trims 2 P_1' and P_2'. So B only like P_3' and P_4.
D trims 1 P_3'. So B only likes P_4.
E takes P_4. B cannot get a piece he liked.

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C trimmed 2 pieces to create a 3-way tie. We insiste she take one if avialable. But there is the other one. So C makes two pieces bad for B.

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B must create a 5-way tie, so he trims four.

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Total number of trimmed or taken pieces is $1 + 2 + 4 = 7$.

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C trimmed 2 pieces to create a 3-way tie. We insiste she take one if avialable. But there is the other one. So C makes two pieces bad for B.
B must create a 5-way tie, so he trims four.
Total number of trimmed or taken pieces is $1 + 2 + 4 = 7$.
After B takes a piece, we need to have an untrimmed piece left for A.

Fourth Attempt at Solution (Part I)

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1) A cuts P into 9 pieces (all equal).

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Trimming put aside.
7 pieces have been trimmed. 2 untrimmed left.

Fourth Attempt at Solution (Part II)

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5) E takes a piece. Went first so no envy.

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5) E takes a piece. Went first so no envy.

6) D takes a piece. Must take piece she trimmed if available.
Created tie for first so no envy.

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5) E takes a piece. Went first so no envy.

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Worst case: 1 untrimmed taken, 1 trimmed that B liked taken.

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5) E takes a piece. Went first so no envy.
6) D takes a piece. Must take piece she trimmed if available.
Created tie for first so no envy.
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Worst Case: C trimmed two pieces that B liked. Took one, ruined the other.

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6) D takes a piece. Must take piece she trimmed if available.
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Worst case: 1 untrimmed taken, 1 trimmed that B liked taken.
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Worst Case: C trimmed two pieces that B liked. Took one, ruined the other.
Worst Case: B only has 1 piece left that he likes.

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5) *E* takes a piece. Went first so no envy.
6) *D* takes a piece. Must take piece she trimmed if available.
Created tie for first so no envy.
Worst case: 1 untrimmed taken, 1 trimmed that *B* liked taken.
7) *C* takes a piece. Must take piece she trimmed if available.
Created 3-way tie for first so no envy.
Worst Case: *C* trimmed two pieces that *B* liked. Took one, ruined the other.
Worst Case: *B* only has 1 piece left that he likes.
8) *B* takes a piece. Must take piece she trimmed if available.
Created 5-way tie for first so no envy.

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Created tie for first so no envy.
Worst case: 1 untrimmed taken, 1 trimmed that B liked taken.
7) C takes a piece. Must take piece she trimmed if available.
Created 3-way tie for first so no envy.
Worst Case: C trimmed two pieces that B liked. Took one, ruined the other.
Worst Case: B only has 1 piece left that he likes.
8) B takes a piece. Must take piece she trimmed if available.
Created 5-way tie for first so no envy.
9) A takes a piece. Since 7 might be trimmed, and E might have taken trimmed, set $Y = 9$.

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Created 5-way tie for first so no envy.
9) A takes a piece. Since 7 might be trimmed, and E might have taken trimmed, set $Y = 9$.
Case work out, but we omit that.

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8) B takes a piece. Must take piece she trimmed if available.
Created 5-way tie for first so no envy.
9) A takes a piece. Since 7 might be trimmed, and E might have taken trimmed, set $Y = 9$.
Case work out, but we omit that.
A thinks his piece is $\frac{1}{9}$ so $X = \frac{1}{9}$.

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8) B takes a piece. Must take piece she trimmed if available.
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9) A takes a piece. Since 7 might be trimmed, and E might have taken trimmed, set $Y = 9$.
Case work out, but we omit that.
A thinks his piece is $\frac{1}{9}$ so $X = \frac{1}{9}$.
There are 2 pieces and trim left, called S. E thinks that $S \leq \frac{8}{9}$.

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6) F takes a piece. Went first so no envy.

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10) B takes a piece. Must take .... Created 9-way tie for first so no envy.

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3) The allocation is envy-free.

The All-Players-Happy Protocol

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1) FIRST-PLAYER-HAPPY($A, B, C, D, E; P$).

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The trim and piece set aside is S_2.

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The trim and piece set aside is S_2.

3) FIRST-PLAYER-HAPPY($C, D, E, A, B; S_2$).

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3) FIRST-PLAYER-HAPPY($C, D, E, A, B; S_2$).
The trim and piece set aside is S_3.

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3) FIRST-PLAYER-HAPPY($C, D, E, A, B; S_2$).
The trim and piece set aside is S_3.

4) FIRST-PLAYER-HAPPY($D, E, A, B, C; S_3$).

The All-Players-Happy Protocol

1) FIRST-PLAYER-HAPPY$(A, B, C, D, E; P)$.
The trim and piece set aside is S_1.

2) FIRST-PLAYER-HAPPY$(B, C, D, E, A; S_1)$.
The trim and piece set aside is S_2.

3) FIRST-PLAYER-HAPPY$(C, D, E, A, B; S_2)$.
The trim and piece set aside is S_3.

4) FIRST-PLAYER-HAPPY$(D, E, A, B, C; S_3)$.

One can show that all players think they have $\geq \frac{1}{7}$ so that whats left is $\leq \frac{6}{7}$.

PROBLEM FIVE-All-but ϵ Envy-Free

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For all ϵ

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For all ϵ
there is a 5-person protocol All-but-ϵ Envy-Free where:

1) All but T of the pie is allocated.
2) The allocation is Envy-Free
3) All of the players think $T \leq \epsilon$.

All-butϵ Envy-Free

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$S_0 = P$.

All-butϵ Envy-Free

$S_0 = P$.
For $i = 0$ to N
    ALL-PLAYER-HAPPY$(A, B, C, D, E; S_i)$.
    Let S_{i+1} be all that was set aside.

All-butϵ Envy-Free

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For $i = 0$ to N
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After N iterations all of the players think $S_N \leq (\frac{6}{7})^N$.

All-butϵ Envy-Free

$S_0 = P$.
For $i = 0$ to N
  ALL-PLAYER-HAPPY$(A, B, C, D, E; S_i)$.
  Let S_{i+1} be all that was set aside.
After N iterations all of the players think $S_N \leq (\frac{6}{7})^N$.
For N large enough get $S_N \leq \epsilon$.