## 250 EXTRA

## DUE TH May 9 at 3:30. DEAD CAT DAY: TU May 14 at 3:30 On Tu May 14 I may go over it in class

## On Tu May 14 I will post solutions but you need to remind me

If you do this I WILL NOT GRADE IT unless you get a grade of F, D, C in the class. If it is GOOD then I will bump you up:

F to D-

D to C-

C to B-

DO NOT ask how good is good. DO IT and DO IT WELL.

ADVICE: You should ALL do this as it is a good study aid.

NOTE: It does not cover topics asked after the second midterm.

- 1. (25 points) Let T be defined by:
  - T(0) = 1 T(1) = 3  $(\forall n \ge 2)[T(n) \le 4T(n-1) + 5T(n-2)].$ Using constructive induction find A and B, both natural numbers, such

that

 $(\forall n \ge 1)[T(n) \le AB^n].$ 

## 2. (25 points)

- (a) (10 points) For  $0 \le x \le 15$  determine  $x^2 \pmod{16}$ . (You can use a calculator. You will need this for part 2 of this problem.)
- (b) (15 points) Prove that there is no  $(x_1, \ldots, x_{14})$  (all natural numbers) such that :

$$x_1^4 + x_2^4 + \dots + x_{14}^4 = 1599.$$

- 3. (25 points) For each of the statements below
  (the ones labelled (a) and (b)) STATE if it is TRUE or FALSE.
  If you write TRUE then give a PROOF.
  If your write FALSE then give a COUNTEREXAMPLE.
  You must clearly state TRUE or FALSE before giving your proof or counterexample.
  - (a) There is a rational that when cubed equals 15.
  - (b) If A is a set whose powerset has size 15 then A is infinite.

4. (25 points) Show that there exists a, b such that for ALL 17-colorings of the  $a \times b$  grid there is a monochromatic rectangle.