## 250 EXTRA

DUE TH May 9 at 3:30. DEAD CAT DAY: TU May 14 at 3:30
On Tu May 14 I may go over it in class
On Tu May 14 I will post solutions but you need to remind me
If you do this I WILL NOT GRADE IT unless you get a grade of $F, D, C$ in the class. If it is GOOD then I will bump you up:

F to D-
D to C-
C to B-
DO NOT ask how good is good. DO IT and DO IT WELL.
ADVICE: You should ALL do this as it is a good study aid.
NOTE: It does not cover topics asked after the second midterm.

1. (25 points) Let $T$ be defined by:
$T(0)=1$
$T(1)=3$
$(\forall n \geq 2)[T(n) \leq 4 T(n-1)+5 T(n-2)]$.
Using constructive induction find $A$ and $B$, both natural numbers, such that

$$
(\forall n \geq 1)\left[T(n) \leq A B^{n}\right]
$$

2. (25 points)
(a) (10 points) For $0 \leq x \leq 15$ determine $x^{2}(\bmod 16)$. (You can use a calculator. You will need this for part 2 of this problem.)
(b) (15 points) Prove that there is no $\left(x_{1}, \ldots, x_{14}\right)$ (all natural numbers) such that:

$$
x_{1}^{4}+x_{2}^{4}+\cdots+x_{14}^{4}=1599
$$

3. (25 points) For each of the statements below (the ones labelled (a) and (b)) STATE if it is TRUE or FALSE. If you write TRUE then give a PROOF. If your write FALSE then give a COUNTEREXAMPLE.
You must clearly state TRUE or FALSE before giving your proof or counterexample.
(a) There is a rational that when cubed equals 15 .
(b) If $A$ is a set whose powerset has size 15 then $A$ is infinite.
4. (25 points) Show that there exists $a, b$ such that for ALL 17-colorings of the $a \times b$ grid there is a monochromatic rectangle.
