Homework 1, MORALLY Due Feb 5, DEAD-CAT DAY 7

1. (10 points) When is the first midterm (give date and time)? When is the second midterm (give date and time)? When is the final (give date and time)? By when do you inform Professor Gasarch that you cannot make the timeslot of the first or the second midterm? Of the final?

2. (30 points) Give a Propositional Formula on four variables that has exactly three satisfying assignments. Give the satisfying assignments.

   SOLUTION TO PROBLEM 2

   $(x_1 \wedge x_2 \wedge x_3 \wedge x_4) \vee (\neg(x_1) \wedge x_2 \wedge x_3 \wedge x_4) \vee (\neg(x_1) \wedge \neg(x_2) \wedge x_3 \wedge x_4)$

   Only satisfying: (T, T, T, T) and (F, T, T, T) and (F, F, T, T).

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3. (30 points)

- Do a truth table for $(p \Rightarrow q) \Rightarrow r$.
- Do a truth table for $p \Rightarrow (q \Rightarrow r)$.
- Are they equivalent? If NOT then state a row where they differ.

SOLUTION TO PROBLEM 3

SHORT CUT: Only way that $x \Rightarrow y$ is F is if x is T and y is F.

The only way $p \Rightarrow (q \Rightarrow r)$ is F is if p is T and $(q \Rightarrow r)$ is F. The latter can only happen if q is T and r is F. Hence the only way $p \Rightarrow (q \Rightarrow r)$ is F is if p is T, q is T, and r is F.

The only way $(p \Rightarrow q) \Rightarrow r$ is F is if $(p \Rightarrow q)$ is T and r is F. $(p \Rightarrow q)$ is T when (p, q) is either (T, T), (F, T) or (F, F). Hence $(p \Rightarrow q) \Rightarrow r$ is F when (p, q, r) is either (T, T, F), (F, T, F), (F, F, F).

p	q	r	$p \Rightarrow (q \Rightarrow r)$	$(p \Rightarrow q) \Rightarrow r$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

NOT equiv: they differ on the row (F,T,F).

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4. (30 points) Show that, for all $n \geq 1$, there exists a formula that is satisfied by exactly n satisfying assignments. Give the satisfying assignments. (This is NOT by induction. Just give the Formula. You may use DOT DOT DOT (that is "...") though it should be clear what you mean.)

   SOLUTION TO PROBLEM 4.

   Let $\phi(i, n)$ be the Boolean formula on $x_1, \ldots, x_n$ where $x_1, \ldots, x_i$ are NEGATED but the rest are not. For examples

   $\phi(0, n)$ is $(x_1 \wedge x_2 \wedge \cdots \wedge x_n)$

   $\phi(3, n)$ is $(\neg x_1 \wedge \neg x_2 \wedge \neg x_3 \wedge x_4 \wedge x_5 \wedge \cdots \wedge x_n)$

   Our formula is

$$\phi(0, n) \vee \phi(1, n) \vee \cdots \vee \phi(n-1, n)$$

   The satisfying assignments are

   $(T, \ldots, T)$ (n T's)

   $(F, T, \ldots, T)$ (1 F and then $n-1$ T's)

   $(F, F, T, \ldots, T)$ (2 F's and then $n-2$ T's)

   $\vdots$

   $(F, F, F, \ldots, F, T)$ ($n-1$ F's and then 1 T)

   There are other options. Perhaps more pleasing would be $\phi(1, n) \vee \cdots \vee \phi(n, n)$.