Homework 2, MORALLY Due Feb 11
WARNING: THIS HW IS TWO PAGES LONG!!!!!!!!!!!!!!!!!!

1. (25 points)

    (a) (15 points) Write a truth table with 3 inputs and 2 outputs for the following function:

    $f(x, y, z) = x + y + z \pmod 3$.

    (b) (10 points) Write a circuit for f using AND, OR, and NOT using the method shown in class (do not simplify- that would make it harder for the TA's to grade!)

    SOLUTION TO PROBLEM 1 OMITTED
    GO TO NEXT PAGE

2. (25 points)

    (a) (10 points) Use truth table so show that

$$\neg(p \wedge q \wedge r) \equiv \neg p \vee \neg q \vee \neg r$$

    (This is called *DeMorgan's law on three variables*.)

    (b) (15 points) Consider the statement:

$$\text{for all } n \; [\neg(p_1 \wedge \cdots \wedge p_n) \equiv \neg p_1 \vee \cdots \vee \neg p_n]$$

    Prove it. Note that you cannot use Truth Table since we want it
    for all n. Do not use Induction (later when we learn induction we
    will do that). Use reasoning about what the truth table for both
    sides must look like.

SOLUTION TO PROBLEM 2

Part a omitted.

Part b:

First consider: for which rows will $\neg(p_1 \wedge \cdots \wedge p_n)$ be FALSE? Only if
$p_1 \wedge \cdots \wedge p_n$ is TRUE. Since its a $\wedge$ that only happens if ALL of the
p_i's are TRUE. So we have:

$\neg(p_1 \wedge \cdots \wedge p_n)$ FALSE if and only if ALL p_i's are TRUE.

Second consider: for which rows will $\neg p_1 \vee \cdots \vee \neg p_n$ be FALSE? Since
its a $\vee$ this only happens if each literal is FALSE. So we need, for all
i, $\neg p_i$ is FALSE. So we need, for all i, p_i is TRUE. So we have:

$$\neg p_1 \vee \cdots \vee \neg p_n$$

FALSE if and only if all p_i's are TRUE.

Since the two statements are FALSE for all the same rows, they are
also TRUE for all the same rows. Hence they are equivalent.

3. (25 points – 5 points each) For each of the following statements write the negation without using any negations signs.

  (a) $x \leq 4$

  (b) $1 < x < 2$

  (c) $x_1 < x_2 < \cdots < x_n$

  (d) $x \leq 5$ OR $x \geq 10$

  (e) $x \leq 5$ AND $x \geq 10$

SOLUTION TO PROBLEM 3

a) $x > 4$.

b) $x \leq 1$ OR $x \geq 2$.

c) This can be rewritten as

$$\bigwedge_{1 \leq i \leq n-1} x_i < x_{i+1}$$

So its negation is

$$\bigvee_{1 \leq i \leq n-1} x_{i+1} \leq x_i$$

d) $1 < x < 10$. Also a fine answer: $1 < x$ AND $x < 10$.

e) $x > 5$ OR $x < 10$. This one is a bit odd since the original statement can never happen. But the negation of something that can't happen is something that can happen.

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4. (25 points) Look up the lyrics to the song *If this isn't love* from the musical Finian's Rainbow. (You might also want to listen to it on You Tube.) Assume the song is true. For each simple proposition in the song assign a letter (try to make it mnemonic). For example

L is *This is Love*

so

$\neg L$ is *This isn't Love*

and

W is *the whole world is crazy*

Write down a proposition that expresses what happens if this isn't love. It will start:

$$\neg L \Rightarrow (W \wedge \text{ other stuff}).$$

(Note- one of the parts of the sentence doesn't make sense to me, we'll discuss later.)

SOLUTION TO PROBLEM 4

L is *this is love*

W is *the whole world is crazy*

D is *I'm daft as a daisy*

A is *Something is a-miss*

H is *I'll eat my hat*

S is *Winter is Summer*

P is *My heart needs a plumber*

K is *I'll kiss your hand*

$$\neg L \Rightarrow (W \wedge D \wedge A \wedge H \wedge S \wedge P \wedge K)$$

GOTO next page for more of this solution

The last part,

$$\neg L \Rightarrow K$$

seems odd to me since the singer is saying

If what I feel is NOT love then I'll kiss your hand.

Really? Um, the singer might get his face slapped with that hand ze just kissed!

(The word *ze* is a genderless pronoun. Appropriate here since both males, females, and nonbinaries have all sung the song.)

Also note: There are longer versions out there, including

If this isn't love, there's no Glocca Morra.

Glocca Mora is a town in Ireland. The Musical is about Irish Immigrants who want to come to America and bury gold near Fort Knox to get more gold. They stole the gold from a Leprechaun. Doesn't make a whole lot of sense. But the music is good! And there is also some very strong (for its day, 1947 debut) anti-racism stuff in it. Still, odd to have a strong and good social message in a musical that otherwise is about magic and Leprechauns.