

Homework 7

Morally due FRI Apr 5, 5:00PM. DEAD CAT Monday Apr 8 5:00

(NOTE- Change of when its due is so that we can go over it in rec BEFORE the exam.)

THE HW IS TWO PAGES LONG!!!!!!!!!!!!!!

1. (20 points)
 - (a) (10 points) WG, Jtwitty, and K are taking the class on a field trip to the Combinatorics Museum! There are 32 students in the class
WG will drive 18 of them.
Jtwitty will drive 7 of them.
K will drive 7 of them.
How many ways can the students choose which cars they want to be in?
 - (b) (15 points) Generalize the problem as follows. A_1, \dots, A_n are taking the class on a field trip! There are S students in the class.
 A_1 will drive a_1 of them.
 \vdots
 A_n will drive a_n of them.
(Note that $a_1 + \dots + a_n = S$.)
How many ways can the students choose which cars they want to be in?
2. (25 points) Use a combinatorial argument (NOT algebraic, NOT by induction) to show that if $S = a + b + c$ then

$$\frac{S!}{a!b!c!} = \frac{(S-1)!}{(a-1)!b!c!} + \frac{(S-1)!}{a!(b-1)!c!} + \frac{(S-1)!}{a!b!(c-1)!}$$

GOTO NEXT PAGE

3. (25 points) Fill in the blanks in the following statement. Describe your reasoning. BLANK will be a function of k, n .

If $A \subseteq \{1, \dots, n\}$ and $|A| = k$ then at least BLANK subsets of A have the same SUM.

4. (25 points) Show that no matter how you 3-color the 4×19 grid there will be a monochromatic rectangle.