Homework 7
Morally due FRI Apr 5, 5:00PM. DEAD CAT Monday Apr 8 5:00
(NOTE- Change of when its due is so that we can go over it in rec
BEFORE the exam.)

THE HW IS TWO PAGES LONG!!!!!!!!!!!!!!!

1. (20 points)

(a) (10 points) WG, Jtwitty, and K are taking the class on a field trip
to the Combinatorics Museum! There are 32 students in the class
WG will drive 18 of them.
Jtwitty will drive 7 of them.
K will drive 7 of them.
How many ways can the students choose which cars they want to
be in?

(b) (15 points) Generalize the problem as follows. A₁, . . . , Aₙ are tak-
ing the class on a field trip! There are S students in the class.
A₁ will drive a₁ of them.
...
Aₙ will drive aₙ of them.
(Note that a₁ + · · · + aₙ = S.)
How many ways can the students choose which cars they want to
be in?

2. (25 points) Use a combinatorial argument (NOT algebraic, NOT by
induction) to show that if S = a + b + c then

\[
\frac{S!}{a!b!c!} = \frac{(S - 1)!}{(a - 1)!b!c!} + \frac{(S - 1)!}{a!(b - 1)!c!} + \frac{(S - 1)!}{a!b!(c - 1)!}
\]
3. (25 points) Fill in the blanks in the following statement. Describe your reasoning. BLANK will be a function of $k, n$.

If $A \subseteq \{1, \ldots, n\}$ and $|A| = k$ then at least BLANK subsets of $A$ have the same SUM.

4. (25 points) Show that no matter how you 3-color the $4 \times 19$ grid there will be a monochromatic rectangle.